# Identification of flutter derivatives from the forced and free vibration tests using EEE method

*Y.H. Hong[1], Y.C. Hwang[2], H.K. Kim[3] and H.S. Lee[4]

[1), 2), 3), 4)] *Department of Civil & Environmental Engineering, Seoul National University, Seoul 110-799, Korea*
[1)] *hyh99@snu.ac.kr*

## ABSTRACT

This paper proposes a new approach to identify the flutter derivatives by minimizing an equation error estimator (EEE) which is defined as the least-square errors between structural resistance forces and aeroelastic forces induced by wind. Dissimilar to the other SI algorithms, the proposed method can successfully employed for the identification of flutter derivatives regardless of the experimental procedure and does not require any complicated sensitivity analysis or complex eigen-value analysis to identify the unknown system parameters. The EEE requires complete information on the state variables at all-time steps. In the proposed method, accelerations of a section model are measured with accelerometers in wind tunnel tests while the velocities and displacements corresponding to the measured accelerations are reconstructed by the FEM-based finite impulse response filter (FFIR filter). The validity of the proposed method is demonstrated through both free vibration test and forced vibration test of bridge sectional models in the wind tunnel. It is shown that the flutter derivatives identified by the proposed method agree well with those by previous methods.

## 1. INTRODUCTION

After the flutter derivative-based aeroelastic formula had been by Scanlan, great number of efforts have been made to estimate the flutter derivatives from the test of bridge model in wind tunnel. The flutter derivatives can be identified with different experimental procedures from an idealized 2-DOF section model, i.e. forced-vibration test and free-vibration test. The most widely adopted technique is the free-vibration method. Scanlan and Tomko proposed the extraction scheme for flutter derivatives from 2DOFs coupled motion tests (1971). Sarkar developed the Modified Ibrahim Time Domain (MITD) to estimate cross flutter derivatives along with direct flutter derivatives

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[1)] Ph.D.
[2)] Graduate Student
[3), 4)] Professor

(1994). The procedures of these approaches are generally based on the response error estimation, which minimizes the relative error between measured displacement and predicted displacement. Although free-vibration technique requires relatively complicated procedure to identify the flutter derivatives and need additional assumption because of the uncertainty of frequency similarity, but it is appealing for the simple setup and the possibility for the realization of interaction between the structure and the wind.

A more reliable procedure is forced-vibration method [Diana 2004, Falco 1992, Kim 2007 and Matsumoto 1993] in the sense of the law of similarity. As the selection of the experimental procedure, different numerical algorithms are employed for the extraction of the flutter derivatives. Numerous SI algorithms used for the free-vibration technique is inappropriate for the steady state response of the forced-vibration test and vice versa. As far as the author knows of, the general algorithm that can be employed for extraction of flutter derivatives regardless of the testing procedures has not been proposed yet.

This thesis proposes a new approach to identify the flutter derivatives by minimizing an equation error estimator (EEE) [Hjelmstad 1995] which is defined as the least-square errors between structural resistance forces and aeroelastic forces induced by wind. Dissimilar to the other SI algorithm, the proposed method can successfully employed for the identification of flutter derivatives regardless of the experimental procedure and do not require any complicated sensitivity analysis or complex eigen-value analysis to identify the unknown system parameters.

The EEE approach requires not only displacement response but also velocity and acceleration history for system identification. In this approach, a displacement and velocity reconstruction scheme [Hong 2010] is used to calculate displacement and velocity history from measured acceleration. Hence, both these reconstructed responses and the measured acceleration is used for EEE method.

## 2. ESTIMATION OF FLUTTER DERIVATIVES

2.1 Equations of motion for the free vibration test

In this paper, a section model for the wind induced vibration is assumed to have two degrees of freedom in vertical ($h$) and rotational ($\alpha$) direction. The equations of motion per unit length for the section model can be expressed by following equations.

$$m_h \ddot{h} + c_h \dot{h} + k_h h = L_h$$
$$m_\alpha \ddot{\alpha} + c_\alpha \dot{\alpha} + k_\alpha \alpha = M_\alpha \tag{1}$$

where $m_h$ and $m_\alpha$ are mass and the mass moment of inertia per unit length, respectively; $c_h$ and $c_\alpha$ are mechanical damping in bending and torsion, respectively; $k_h$ and $k_\alpha$ are the mechanical stiffness coefficients in vertical and rotational direction; $L_h$ and $M_\alpha$ are aeroelastic lift force and moment, respectively. Here, the material properties for each mode are indicated with the corresponding subscripts.

The aeroelastic force acting on a sinusoidal oscillating section model in a single mode is assumed as a linear function to the motion of the section and its first order derivative [Iwamoto 1995 and Scanlan 1971]:

$$\mathbf{F}_{ae}(t) = \begin{pmatrix} L_{ae}(t) \\ M_{ae}(t) \end{pmatrix} \approx \mathbf{C}_{ae}(\omega)\dot{\mathbf{U}}(t) + \mathbf{K}_{ae}(\omega)\mathbf{U}(t)$$

$$= \begin{bmatrix} H_1(\omega) & H_2(\omega) \\ A_1(\omega) & A_2(\omega) \end{bmatrix} \begin{pmatrix} \dot{h}(t) \\ \dot{\alpha}(t) \end{pmatrix} + \begin{bmatrix} H_4(\omega) & H_3(\omega) \\ A_4(\omega) & A_3(\omega) \end{bmatrix} \begin{pmatrix} h(t) \\ \alpha(t) \end{pmatrix}$$

(2)

where $L_{ae}$ and $M_{ae}$ are the aeroelastic lift force and moment, respectively, while     is the circular frequency of the oscillation, and $H_m$ and $A_m$ ($m = 1, 2, 3, 4$) are the flutter derivatives. It is customary to use normalized expressions of the flutter derivatives [Scanlan 1971]. For the simplicity of presentation, however, this thesis presents discussions with the un-normalized forms of the flutter derivatives.

2.2 Equation error estimation (EEE)

In case the complete time history of displacement, velocity and acceleration are available, the flutter derivatives are identified by employing the EEE proposed in section 4.1 as follows:

$$\underset{\mathbf{X}}{\text{Min}}\,\Pi(\mathbf{X}) = \frac{1}{2}\sum_{i=1}^{nt}\left\|\mathbf{F}_{kn}(t_i) - \mathbf{F}_{un}(\mathbf{X},t_i)\right\|_2^2$$

$$= \frac{1}{2}\sum_{i=1}^{nt}\left\|\mathbf{F}_{st}(t_i) - \mathbf{F}_{ae}(\mathbf{X},t_i)\right\|_2^2$$

(3)

here, X is the vector of the flutter derivatives to be identified.

$$\mathbf{X} = (\overline{H}_1 \quad \overline{H}_2 \quad \overline{H}_3 \quad \overline{H}_4 \quad \overline{A}_1 \quad \overline{A}_2 \quad \overline{A}_3 \quad \overline{A}_4)^T$$

(4)

$\mathbf{F}_{st}(t_i)$ and $\mathbf{F}_{ae}(\mathbf{X},t_i)$ are the structural resistance force and the aeroelastic force at time $t_i$, respectively.

$$\mathbf{F}_{st}(t_i) = \mathbf{M}\ddot{\mathbf{U}}(t_i) + \mathbf{C}\dot{\mathbf{U}}(t_i) + \mathbf{K}\mathbf{U}(t_i)$$

(5)

The aeroelastic force given in (4-28) is rewritten in terms of the vector of the flutter derivatives.

$$\mathbf{F}_{ae}(\mathbf{X},t_i) = \mathbf{s}(t_i)\mathbf{X}$$

$$= \begin{bmatrix} \dot{h}(t_i) & \dot{\alpha}(t_i) & \alpha(t_i) & h(t_i) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dot{h}(t_i) & \dot{\alpha}(t_i) & \alpha(t_i) & h(t_i) \end{bmatrix}\mathbf{X}$$

(6)

Following the solution procedure of the EEE method in Eq. (3) ~ Eq. (6), a unique solution is always determined by Eq. (6) as long as a sufficient amount of measured dynamic responses of a section model are provided.

## 3. RECONSTRUCTION OF DISPLACEMENT AND VELOCITY HISTORY FROM MEASURED ACCELERATION

Hong et al. have proposed a new class of FEM-based finite impulse response (FFIR) filter to reconstruct displacement and velocity simultaneously. In their approach, the displacement is reconstructed by solving the following minimization problem defined in a time interval, $T_1 < t < T_2$.

$$\underset{u}{\text{Min}}\,\Pi(u) = \frac{1}{2}\int_{T_1}^{T_2}(\frac{d^2u}{dt^2} - \overline{a})^2\,dt + \frac{\beta^2}{2}\int_{T_1}^{T_2}u^2\,dt \tag{6}$$

where $u$ and $\overline{a}$ are the displacement and the measured acceleration, respectively.
The direct discretization of the variation statement of Eq. (6) with the finite element method using $2k$-th elements leads to a FFIR filter.

$$\delta\Pi(u) = \sum_{e=1}^{2k}\int_{\Delta t}\frac{d^2\delta u^e}{dt^2}(\frac{d^2u^e}{dt^2} - \overline{a}^e)dt + \beta^2\sum_{e=1}^{2k}\int_{\Delta t}\delta u^e u^e\,dt = 0 \tag{7}$$

Here, $u^e$ and $\overline{a}^e$ denote the displacement and acceleration in element, e, respectively. The standard FEM formulation for a beam on an elastic foundation is adopted to derive the following matrix expression of Eq. (7).

$$(\mathbf{K} + \beta^2(\Delta t)^4\mathbf{M})\mathbf{u} = (\Delta t)^2\mathbf{Q}\overline{\mathbf{a}} \tag{8}$$

where $\mathbf{u}$ and $\overline{\mathbf{a}}$ denote the nodal unknown vector and the measured acceleration vector associated with all sampling points of measurement. The nodal unknown vector consists of the nodal displacements and the nodal velocities. The matrixes in Eq. (8) are defined as

$$\mathbf{K} = \sum_e\int_0^1\frac{d^2\mathbf{N}_H^T}{d\xi^2}\frac{d^2\mathbf{N}_H}{d\xi^2}d\xi \;,\; \mathbf{M} = \sum_e\int_0^1\mathbf{N}_H^T\mathbf{N}_H\,dt \;,\; \mathbf{Q} = \sum_e\int_0^1\frac{d^2\mathbf{N}_H^T}{d\xi^2}\mathbf{N}_L\,d\xi \tag{9}$$

where $\sum_e$ is the assembly operator of the FEM, and $\xi$ is the natural coordinate for the time variable ranging from 0 to 1, $\mathbf{N}_H$ and $\mathbf{N}_L$ are Hermitian shape function and the linear shape function, respectively. The nodal unknown vector is obtained by solving Eq. (8).

$$\mathbf{u} = (\Delta t)^2(\mathbf{K} + \beta^2(\Delta t)^4\mathbf{M})^{-1}\mathbf{Q}\overline{\mathbf{a}} = (\Delta t)^2\mathbf{C}\overline{\mathbf{a}} \tag{10}$$

where **C** is the coefficient matrix of order $2(2k+1) \times (2k+1)$.

Assuming the time step at the center of a time window represents time $t$, the reconstructed displacement and velocity are expressed as following equations

$$u(t) = u_{k+1} = (\Delta t)^2 \sum_{p=1}^{2k+1} C_{2k+1,p} \overline{a}_p = (\Delta t)^2 \sum_{p=-k}^{k} c_{p+k+1} \overline{a}(t + p\Delta t) \tag{11}$$

$$v(t) = v_{k+1} = \sum_{p=1}^{2k+1} C_{2k+2,p} \overline{a}_p = \Delta t \sum_{p=-k}^{k} \hat{c}_{p+k+1} \overline{a}(t + p\Delta t) \tag{12}$$

## 4. EXPERIMENTAL VERIFICATION

For the verification of the proposed method for the free-vibration test, the flutter derivatives are identified using measurements taken from a series of free-oscillation tests for the two types of representative sections of bridge decks; a thin rectangular plate with the width-to-depth (B/D) ratio of 20 and a bluff H-type section. Fig. 4 shows the dimensions of the sections. The flutter derivatives are identified by the proposed method using the measured accelerations and by the MITD method using the measured displacements for comparison. The identified results from both of the methods presented here are the representative values for multiple measurements. The flutter derivatives identified for each measurement are averaged for the MITD method, while multiple measurements are considered together in optimization for the proposed method.

## summary and CONCLUSION

This paper proposed a new identification method for flutter derivatives base on the EEE, and compared with conventional PEM.

Though EEE results in less compatible results between predicted and measured displacements than PEM, the authors believe that the flutter derivatives identified by EEE contain more physically reasonable meanings based on the governing equilibrium equation. Moreover, the EEE is free from the convergence of ill-posed problem, and give very stable identification results.

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