Frequency- and Time-domain Aeroelastic Analysis of Cable-Supported Bridges using Approximated Aerodynamic Transfer Functions

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Summary

An approximate approach for the frequency- and time-domain aeroelastic analysis of bridge decks is presented. The aerodynamic transfer functions are approximated as a linear function with the coefficients determined through minimization of the weighted error between the exact and approximated transfer function. The weighting function in optimization is introduced for the improved approximation. Using the proposed method, the frequency dependence of the aerodynamic transfer function is eliminated, and a popular time marching algorithm is adopted for the aeroelastic analysis in the time domain. For the frequency domain analysis, a complete set of modal frequencies and modal shapes can be evaluated in a single eigenvalue analysis.

Keywords: *Aeroelasticity; Weighted least square method; Cable-supported bridge; Self-excited force; Flutter analysis; Buffeting analysis.*

1. Introduction

For aeroelastic analyses of bridge decks under wind action, numerous approaches have been proposed since Scanlan and Tomko[1] defined self-excited forces using flutter derivatives. The difficulty in aeroelastic analysis basically arises from the frequency dependence of self-excited forces. For the frequency-domain aeroelastic analysis, an iterative procedure is required to solve the nonlinear eigenvalue problem, which is referred to as branch method[2]. The critical issue in the time-domain approach is the elimination of the frequency-dependent characteristics of the aerodynamic forces, for which the convolution integral is usually utilized. The impulse response functions are formed with the identified flutter derivatives through optimization in the frequency domain, and the aerodynamic forces are expressed as convolutions between the impulse response functions and the deck motion. The rational function approximation (RFA) has been the most popular approach for forming impulse response functions for the convolution integral[3]. Despite its popularity, however, Caracoglia and Jones[4] reported on the potential limitations of the RFA method on its applicability to bluff sections. Recently, Jung et al. [5] proposed a new algorithm for evaluating impulse response functions through a domain-discretization approximation to overcome the shortcomings of the RFA.

Although the convolution integral approach and the branch method can be successfully applied to time-domain and frequency-domain analyses, respectively, for various types of sections, they are based on different approximations. The impulse response functions used for the convolution integral become inconsistent with the given flutter derivatives of a section through optimization, while the aerodynamic forces are evaluated at only one assumed frequency in the branch method. Therefore, the consistency between results of a time-domain analysis and a frequency-domain analysis cannot be generally guaranteed.

This paper presents a new unified approach for the aeroelastic analysis of a bridge structure by approximating each component of the aerodynamic transfer functions in frequency domain. Using this approximation, the equation of motion for an aeroelastic system becomes a set of simple second-order differential equations in the time domain. The coefficients of the second-order polynomial are determined through minimization of weighted errors between the exact and approximated aerodynamic transfer function in the frequency domain. The validity of the proposed method is demonstrated by applying to an idealized cable-supported structure.

2. An Approximated Approach for the Aeroelastic Analysis

2.1 Equation of motion of a cable bridge under wind action

The dynamic virtual work expression of a discretized structure under the action of wind is given as follows:

$$\delta\mathbf{U}^T\mathbf{M}\ddot{\mathbf{U}} + \delta\mathbf{U}^T\mathbf{C}\dot{\mathbf{U}} + \delta\mathbf{U}^T\mathbf{K}\mathbf{U} = \delta\Pi_{ad} + \delta\mathbf{U}^T\mathbf{P}_{ex} \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$, $\mathbf{P}_{ex}$ and $\mathbf{U}$ are the mass, damping, stiffness matrix of the discretized structural system, the equivalent nodal force vector and the nodal displacements vector, respectively, and δ, when placed in front of a variable, indicates a virtual quantity. The external virtual work done by wind-induced aerodynamic forces is denoted as $\delta\Pi_{ad}$ in Eq. (1), and defined as the summation of the contribution from each element.

$$\delta\Pi_{ad} = \sum_e \delta\Pi_{ad}^e = \sum_e \int_{l^e} (\delta h^e \cdot L_{ad}^e + \delta\alpha^e \cdot M_{ad}^e)dx \tag{2}$$

where L_{ad}^e and M_{ad}^e are the aerodynamic lift force and torsional moment per unit length in the local coordinate system for element e, respectively, while h^e and α^e are the vertical and torsional displacement defined also in the local coordinate system, respectively.

In accordance with the Scanlan and Tomko's formulation[1], the self-excited forces acting on a sinusoidally oscillating section in a single frequency are defined as:

$$\begin{aligned} L_{ae} &= \frac{1}{2}\rho U^2 B[KH_1^*\frac{\dot{h}}{U} + KH_2^*\frac{B\dot{\alpha}}{U} + K^2H_3^*\alpha + K^2H_4^*\frac{h}{B}] \\ M_{ae} &= \frac{1}{2}\rho U^2 B^2[KA_1^*\frac{B\dot{h}}{U} + KA_2^*\frac{B\dot{\alpha}}{U} + K^2A_3^*\alpha + K^2A_4^*\frac{h}{B}] \end{aligned} \tag{3}$$

where ρ is the air density, U is the mean wind velocity, B is the width of the section model. $K=B\omega/U$ is the reduced frequency where ω is the angular frequency of the oscillation. The flutter derivatives are denoted as H_m^* and A_m^* (m=1,2,3,4), and are functions of the angular frequency. Because of this frequency dependence, the self-excited forces are usually defined in the frequency domain as follows [5]:

$$\begin{aligned} \begin{pmatrix} F(L_{ad}) \\ F(M_{ad}) \end{pmatrix} &= \frac{1}{2}\rho U^2 \begin{bmatrix} iK^2H_1^* + K^2H_4^* & B(iK^2H_2^* + K^2H_3^*) \\ B(iK^2A_1^* + K^2A_4^*) & B^2(iK^2A_2^* + K^2A_3^*) \end{bmatrix} \begin{pmatrix} F(h) \\ F(\alpha) \end{pmatrix} \\ &= \begin{bmatrix} \Psi_{hh} & \Psi_{h\alpha} \\ \Psi_{\alpha h} & \Psi_{\alpha\alpha} \end{bmatrix} \begin{pmatrix} F(h) \\ F(\alpha) \end{pmatrix} = \boldsymbol{\Psi}(\omega)F(\mathbf{u}^e) \end{aligned} \tag{4}$$

where F denotes the Fourier transform, and $\mathbf{u}^e$ is the displacement vector containing local displacement components h^e and α^e. Ψ_{mn} is the transfer function between the aeroelastic forces in the m direction and the motion in the n direction, and i is the imaginary unit. Using a standard finite element procedure for the interpolation and the coordinate rotation procedure, the displacement vector in Eq. (4) is expressed in terms of the nodal displacement vector of the discretized structure, $\mathbf{U}$, defined in the structural coordinate system.

$$\mathbf{u}^e = \mathbf{N}^e\boldsymbol{\Gamma}^e\mathbf{T}^e\mathbf{U} \tag{5}$$

where $\mathbf{N}^e$, $\boldsymbol{\Gamma}^e$ and $\mathbf{T}^e$ are the shape function matrix for the local displacement vector, the transformation matrix between the local and structural coordinate system and the compatibility matrix that relates h^e and α^e to the structural displacement vector, respectively. The virtual work done by the aerodynamic forces in the frequency domain can easily be obtained using Eq. (4) and Eq. (5).

$$F(\delta\Pi_{ad}) = F(\sum_e \delta\Pi_{ad}^e) = \sum_e \int_{l^e} (\delta h^e \cdot F(L_{ad}^e) + \delta\alpha^e \cdot F(M_{ad}^e))dx$$

$$= \sum_e \int_{l^e} (\delta\mathbf{u}^e)^T \mathbf{\Psi}^e F(\mathbf{u}^e))dx = \delta\mathbf{U}^T \sum_e (\mathbf{\Gamma}^e\mathbf{T}^e)^T \int_{l^e} (\mathbf{N}^e)^T \mathbf{\Psi}^e \mathbf{N}^e dx \mathbf{\Gamma}^e\mathbf{T}^e F(\mathbf{U}) \tag{6}$$

$$= \delta\mathbf{U}^T \mathbf{\Psi}(\omega) F(\mathbf{U})$$

where $\mathbf{\Psi}$ is referred to as the aerodynamic transfer function defining the aerodynamic force in terms of the displacement in the frequency domain.

The Fourier transform of Eq. (1) yields the dynamic virtual work expression of an aeroelastic system in the frequency domain.

$$F(\delta\Pi) = \delta\mathbf{U}^T([-\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K} - \mathbf{\Psi}(\omega)]F(\mathbf{U}) - F(\mathbf{P}_{ex})) = 0 \tag{7}$$

As Eq. (7) should hold for all admissible $\delta\mathbf{U}$, the equation of motion for a structure that is subject to the action of wind is derived in the frequency domain.

$$[-\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K} - \mathbf{\Psi}(\omega)]F(\mathbf{U}) = F(\mathbf{P}_{ex}) \tag{8}$$

The inverse Fourier transform of Eq. (9) yields the equation of motion in the time domain, which contains the well-known convolution expression for the aerodynamic force.

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{P}_{ex} + \int_0^t \mathbf{\Phi}(t-\tau)\mathbf{U}(\tau)d\tau \tag{9}$$

Here, $\mathbf{\Phi}$ is the aerodynamic impulse response function matrix of the structural system, which is the inverse Fourier transform of the aerodynamic transfer function matrix, $\mathbf{\Psi}$. The convolution integral in Eq. (9) is valid if and only if the aerodynamic impulse response function vanishes for $t < 0$, which is referred to as the causality condition[5]. Several approaches have been proposed for forming approximated transfer functions that satisfy the casualty condition using the measured flutter derivatives[3,5].

The aerodynamic transfer functions used for the convolution integral approach and the branch method become different from those formed by the given flutter derivatives. The transfer functions of a section in Eq. (4) are modified in the optimization to enforce the causality condition in the convolution integral approach in Eq. (9). Meanwhile, the aerodynamic transfer functions of Eq. (6) in the branch method represent aerodynamic information related to only one assumed frequency rather than on the whole frequency range. Therefore, the results of an aerodynamic analysis in one analysis domain are generally inconsistent with those for the other analysis domain, which is a crucial drawback of methodologies that are currently used in aeroelastic analyses.

A new unified approach for the aeroelastic analysis can be formulated in case the aerodynamic transfer function matrix can be reasonably approximated by a linear function with respect to frequency as follows:

$$\mathbf{\Psi}(\omega) \approx \tilde{\mathbf{\Psi}}(\omega) = i\omega\tilde{\mathbf{C}} + \tilde{\mathbf{K}} \tag{10}$$

where $\tilde{\mathbf{\Psi}}(\omega)$ is the approximated aerodynamic transfer function while $\tilde{\mathbf{C}}$ and $\tilde{\mathbf{K}}$ are unknown coefficient matrices. By use of Eq. (10) and the inverse Fourier transform of Eq. (8), the approximated equation of motion is defined as a usual second order differential equation in the time domain.

$$\mathbf{M}\ddot{\mathbf{U}} + \tilde{\mathbf{C}}_{ae}\dot{\mathbf{U}} + \tilde{\mathbf{K}}_{ae}\mathbf{U} = \mathbf{P}_{ex} \tag{11}$$

where $\tilde{\mathbf{C}}_{ae} = \mathbf{C} - \tilde{\mathbf{C}}$ and $\tilde{\mathbf{K}}_{ae} = \mathbf{K} - \tilde{\mathbf{K}}$. The unknown coefficient matrices in Eq. (11) are determined through minimization in the frequency domain.

2.2 Minimization of the weighted least square error

The weighted error matrix of the approximation in Eq. (10) is defined by the modulus of a complex number of each component.

$$E_{kl} = \left|\Psi_{kl} - \widetilde{\Psi}_{kl}\right| w_{kl} = ((\Psi_{kl}^R - \widetilde{K}_{kl})^2 + (\Psi_{kl}^I - \omega\widetilde{C}_{kl})^2)^{0.5} w_{kl} \tag{12}$$

where w_{kl}, Ψ_{kl}^R and Ψ_{kl}^I are predefined weighting functions, the real and imaginary part of Ψ_{kl}, respectively, while $|\cdot|$ denotes the modulus of a complex number. $\widetilde{C}_{kl}$ and $\widetilde{K}_{kl}^r$ are the *kl*-components of the corresponding coefficient matrices. The unknown coefficient matrices are determined by minimizing the norm of the weighted errors in Eq. (12).

$$\underset{\widetilde{C}_{kl},\widetilde{K}_{kl}}{\mathrm{Min}}\,\Pi = \frac{1}{2}\int_0^{\omega_{\max}} E_{kl}^2 d\omega = \frac{1}{2}\int_0^{\omega_{\max}} (\Psi_{kl}^R - \widetilde{K}_{kl})^2 w_{kl}^2 d\omega + \frac{1}{2}\int_0^{\omega_{\max}} (\Psi_{kl}^I - \omega\widetilde{C}_{kl})^2 w_{kl}^2 d\omega \tag{13}$$

where $\omega_{\max}$ is the maximum frequency that defines the maximum frequency range of the aerodynamic transfer function. The first-order optimality condition yields the following linear algebraic equations:

$$\widetilde{C}_{kl} = \int_0^{\omega_{\max}} \omega \Psi_{kl}^I w_{kl}^2 d\omega \Big/ \int_0^{\omega_{\max}} \omega^2 w_{kl}^2 d\omega, \quad \widetilde{K}_{kl} = \int_0^{\omega_{\max}} \Psi_{kl}^R w_{kl}^2 d\omega \Big/ \int_0^{\omega_{\max}} w_{kl}^2 d\omega \tag{14}$$

The weighting functions in the error matrix are introduced to consider the responses of an aeroelastic system in the approximated transfer function of the aerodynamic forces. The approximation errors at frequencies where the responses of an aeroelastic system become small have little effect on the final solution of the aeroelastic analysis, regardless of their magnitude, and thus may be safely neglected in the minimization. Once the weighting functions are properly defined, the frequency-domain integrals in Eq. (14) can be evaluated by means of a numerical integration scheme such as the trapezoidal rule, and the *kl*-components of the coefficient matrices are easily obtained by solving the numerically integrated form of Eq. (14).

3. Numerical Examples of the Bridge Model

The validity of the proposed method is demonstrated for an idealized model bridge with 8 stay cables, which simulates the center span of a cable-stayed bridge. The span length of the model bridge is assumed to be 200 m. Since the aeroelastic behaviors of a bridge are mainly dependent on the cross-sectional shape of the bridge deck, two extreme types of deck sections are considered. One is a thin rectangular section with a width-to-depth (B/D) ratio of 20 representing a streamlined box section, and the other is a bluff H-type section simulating a slab-on-stringer type girder.

3.1 A thin rectangular section of B/D=20

The proposed method is applied to the aeroelastic analysis of a bridge model with a thin rectangular section. Fig. 1 shows the damping ratios of the aeroelastic system obtained by the proposed method and by the branch method. The results obtained by the proposed method are in good agreement with the values obtained by the branch method. The time-domain aeroelastic analysis is performed for the forced vibration at a wind velocity of 55 m/s. Although the aeroelastic analysis is performed near the flutter onset velocity, the proposed method accurately yields displacements compared with those by the convolution integral.

3.2 A bluff H-type section

The aeroelastic analysis of the section model with a bluff H-type section is performed. Fig. 3 shows the damping ratios of the aeroelastic system, and Fig. 4 illustrates the free vibration responses at the wind velocity of 8 m/s. Even in the case of the bluff H-type section, the results obtained by the proposed method are also in good agreement with the values obtained by the branch method as well as the convolution integral.

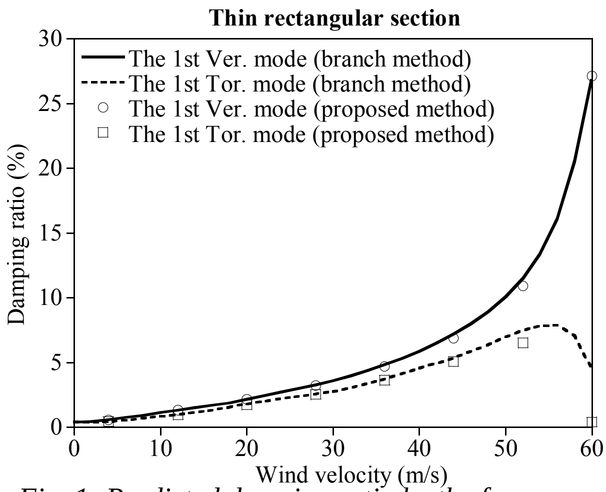

Fig. 1: Predicted damping ratio by the frequency-domain analysis of the bridge model

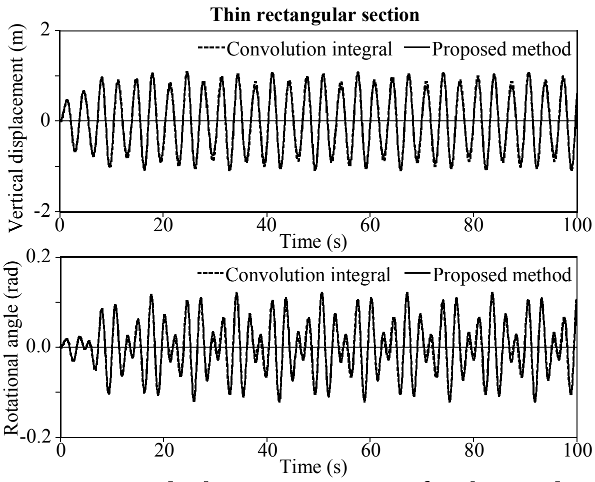

Fig. 2: Forced vibration responses for the wind velocity of 55m/s of the bridge model

4. Conclusions

A unified approach for the aeroelastic analysis is proposed. The aerodynamic transfer functions are approximated as a linear function with coefficients determined through the minimization of the weighted error between exact and approximated transfer functions. With the proposed approximation, the frequency dependence of the aerodynamic transfer function is eliminated, and, as a result, the equation of motion for an aeroelastic system can be simply expressed as the same type of second-order differential equation in the time domain as that for a structural system. The validity of the proposed method is demonstrated for an section model with two extreme types of deck sections. Based on the results of the numerical simulations presented in this study, it can be concluded that the proposed approximation of the aerodynamic transfer function works well, even for the case of a bluff H-type section.

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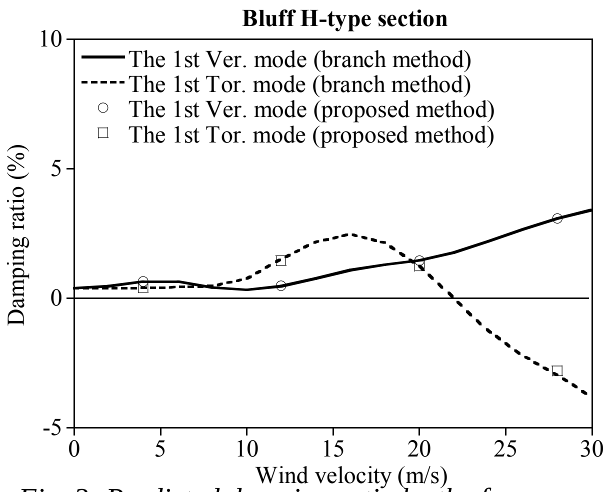

Fig. 3: Predicted damping ratio by the frequency-domain analysis of the bridge model

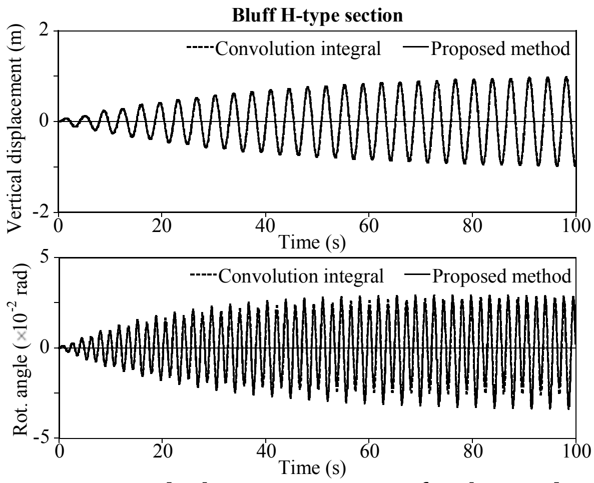

Fig. 4: Forced vibration responses for the wind velocity of 20m/s of the bridge model

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