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Time-domain Aeroelastic Analysis of Bridge using a Truncated Fourier Series of the Aerodynamic Transfer Function

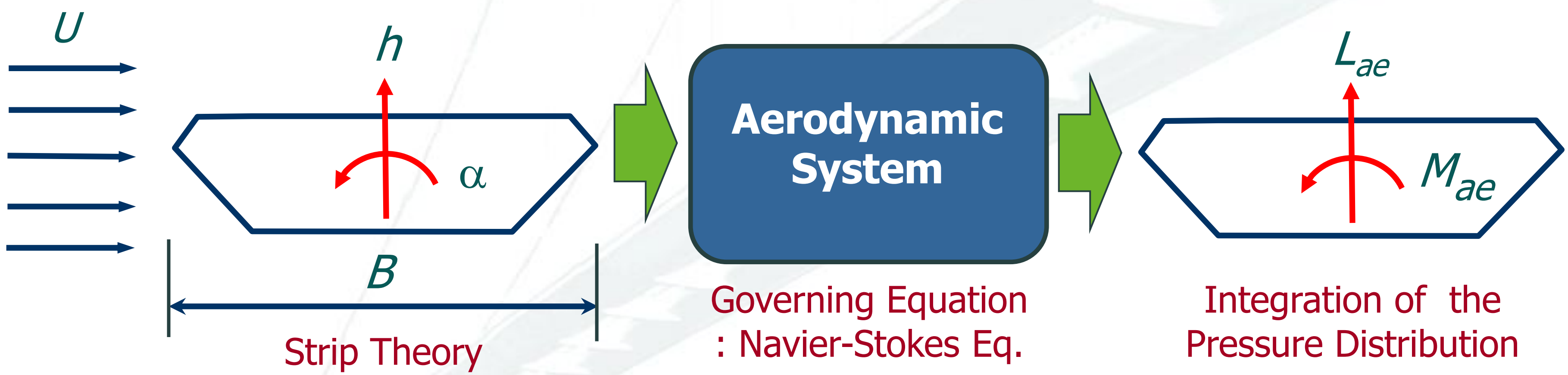
Jinwook Park, Seoul National University, Korea
Kilje Jung, University of Notre Dame, USA
Ho-Kyung Kim, Seoul National University, Korea
Hae Sung Lee, Seoul National University, Korea

S&U Consortium

Contents

◆ Aerodynamic Forces induced by Motion of a Bridge

- Self-exited forces



- No analytical solutions of the aerodynamic forces
- Usually assumed as a linear system in frequency domain
  - Determined through a Wind tunnel test or CFD simulation
- For the time-domain analysis, the aerodynamic system should satisfy the Causality Requirement.

2. Causality Requirement

◆ Linearized Aerodynamic System in Frequency Domain

- Adopting the Flutter Derivatives proposed by Scanlan

- Mixed frequency/time domain
  - Small-amplitude sinusoidal motion with a single frequency

$$\begin{pmatrix} L_{ae}(t) \\ M_{ae}(t) \end{pmatrix} = \frac{B}{U}\begin{bmatrix} H_1(K) & H_2(K) \\ A_1(K) & A_2(K) \end{bmatrix}\begin{pmatrix} \dot{h}(t) \\ \dot{\alpha}(t) \end{pmatrix} + \begin{bmatrix} H_4(K) & H_3(K) \\ A_4(K) & A_1(K) \end{bmatrix}\begin{pmatrix} h(t) \\ \alpha(t) \end{pmatrix}$$

where, $K = \dfrac{B\omega}{U}$: Reduced frequency

- Basically based on the Aerodynamic Transfer Functions

$$\begin{pmatrix} F(L_{ae}) \\ F(M_{ae}) \end{pmatrix} = \begin{bmatrix} iKH_1(K)+H_4(K) & iKH_2(K)+H_3(K) \\ iKA_1(K)+A_4(K) & iKA_2(K)+A_3(K) \end{bmatrix}\begin{pmatrix} F(h) \\ F(\alpha) \end{pmatrix} = \begin{bmatrix} \phi_{hh} & \phi_{h\alpha} \\ \phi_{\alpha h} & \phi_{\alpha\alpha} \end{bmatrix}\begin{pmatrix} F(h) \\ F(\alpha) \end{pmatrix}$$

where, *F* : Fourier transform

2. Causality Requirement

Linearized Aerodynamic System in Time-domain

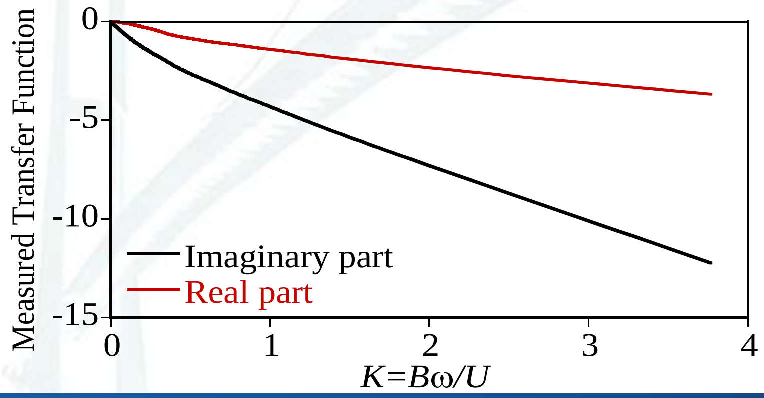
- Using a Convolution Integral Theorem

- By taking the inverse Fourier transform

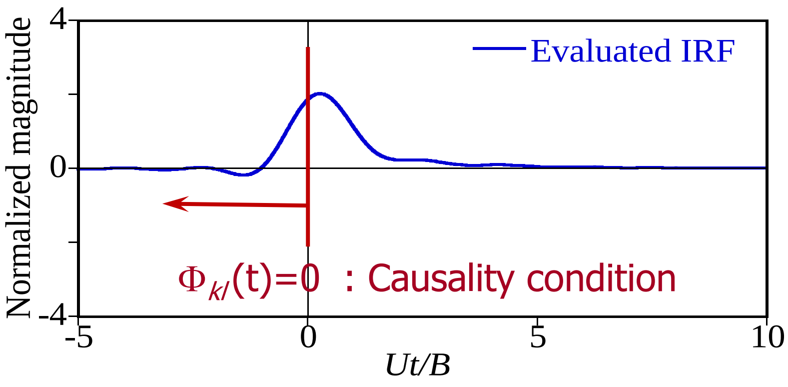
$$\begin{pmatrix} L_{ae}(t) \\ M_{ae}(t) \end{pmatrix} = \begin{pmatrix} \int_0^{\infty} \Phi_{hh}(t-\tau)h(\tau)d\tau + \int_0^{\infty} \Phi_{h\alpha}(t-\tau)\alpha(\tau)d\tau \\ \int_0^{\infty} \Phi_{\alpha h}(t-\tau)h(\tau)d\tau + \int_0^{\infty} \Phi_{\alpha\alpha}(t-\tau)\alpha(\tau)d\tau \end{pmatrix}$$

- Aerodynamic Impulse Response Functions

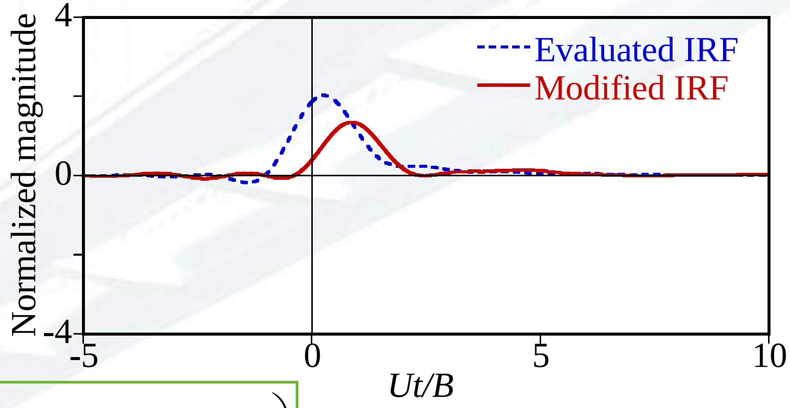
$$\Phi_{kl}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \phi_{kl}(K)e^{i\omega t}d\omega \quad \text{for} \quad k,l = h,\alpha$$



Inverse FT



2. Causality Requirement



Causality Condition

- Definition

$$\boxed{\Phi(t) = 0 \ \text{ for } t < 0}$$

- Mathematical meaning

- One-sided convolution Integral

$$\begin{pmatrix} L_{ae}(t) \\ M_{ae}(t) \end{pmatrix} = \begin{pmatrix} \int_0^t \Phi_{hh}(t-\tau)h(\tau)d\tau + \int_0^t \Phi_{h\alpha}(t-\tau)\alpha(\tau)d\tau \\ \int_0^t \Phi_{\alpha h}(t-\tau)h(\tau)d\tau + \int_0^t \Phi_{\alpha\alpha}(t-\tau)\alpha(\tau)d\tau \end{pmatrix}$$

- Relationship between the real and imaginary part of TF

$$\Phi_{kl}(t) = \int_0^\infty (\phi_{kl}^R(K)\cos\omega t - \phi_{kl}^I(K)\sin\omega t)d\omega = 0 \quad \text{for} \quad t < 0$$

- Physical meaning

- No aerodynamic force is generated before the deck moves

2. Causality Requirement

◆ Previous works

- RFA (Rational Function Approximation)

: RFA is based on the solution to the ideally thin section.

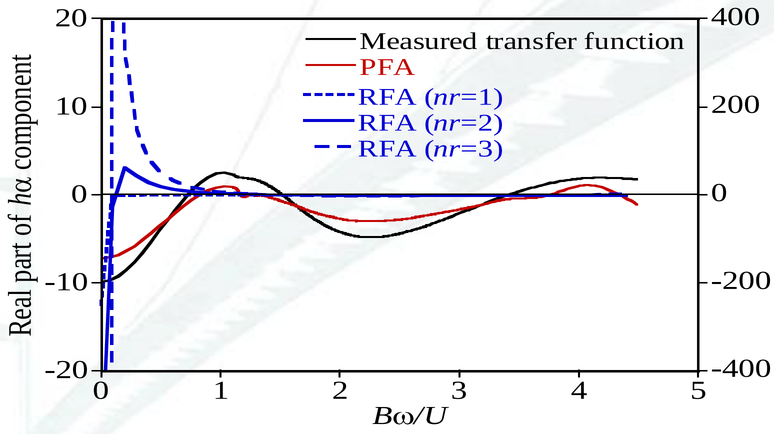
: RFA has a limitation to its applicability to bluff sections.

- PFA (Penalty Function Approach), Jung et al. 2012

: PFA is applicable to the bluff sections

: Causality condition is weakly imposed as a penalty function through the optimization.

-Limitation of RFA, Jung et al. 2012



- Aerodynamic transfer function for H-type section
- RFA is drawn against the right vertical axis.
- RFA yields unreliable results

➡ Propose the efficient method that exactly satisfy the causality condition !!

* Jung, K., Kim, H. K., and Lee, H. S. (2012). "Evaluation of impulse response functions for convolution integrals of aerodynamic forces by optimization with a penalty function." *J. Eng. Mech.*, 138(5), 519-529.

2. Causality Requirement

◆ Comparison between Previous Work and Proposed Method

	RFA	PFA	Proposed
Approximation method	Rayleigh-Ritz type approximation	Finite element method	Rayleigh-Ritz type approximation
Basis function	Rational function	Cubic spline	Sine, Cosine function
Causality condition	Strongly enforcement	Weakly enforcement	Strongly enforcement
Applicability	Limitation of application to the bluff section	Applicable to the bluff section	Applicable to the bluff section

3. Truncated Fourier Series Approximation

◆ Approximation using Truncated Fourier Series

- Fourier series representations of the transfer functions

$$\overline{\phi}_{kl}^{R}(K) = a_{kl}^{0} + \sum_{n=1}^{N} a_{kl}^{n} \cos \frac{n\pi}{K_{\max}} K$$

$$\overline{\phi}_{kl}^{I}(K) = b_{kl}^{0} K + \sum_{n=1}^{N} b_{kl}^{n} \sin \frac{n\pi}{K_{\max}} K$$

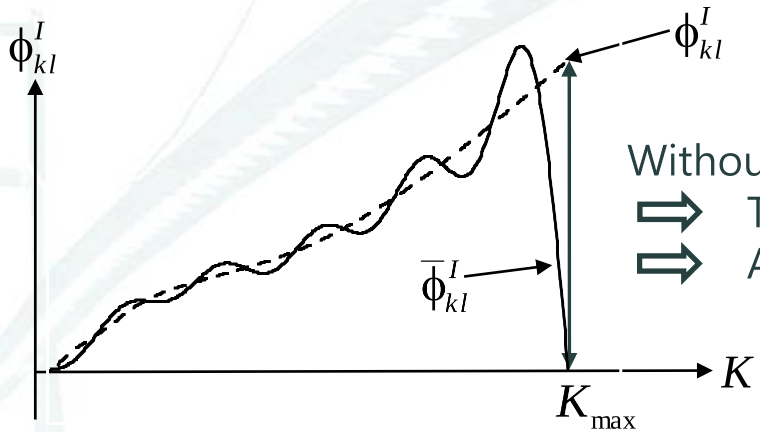
$\overline{\phi}$: modified transfer function

a_{kl}^{n}, b_{kl}^{n} : coefficients of the Fourier series

N : the number of terms in the Fourier series

$K_{\max}$: maximum reduced frequency

- Need of the linear term($b_{kl}^{0}K$) in the imaginary part



Without the linear term, the discontinuity arises at $K_{\max}$.

⇨ The discontinuity causes Gibbs phenomenon.

⇨ A large number of terms should be included.

3. Truncated Fourier Series Approximation

◆ Enforcement of the Causality Condition

- IRF corresponding to the modified transfer function

: IRF becomes a series of Dirac-delta functions.

$$\overline{\Phi}_{kl}(t)=\frac{1}{\pi}\int_0^\infty(\overline{\phi}_{kl}^R(K)\cos\omega t-\overline{\phi}_{kl}^I(K)\sin\omega t)d\omega$$

$$=\frac{U}{B}\frac{1}{\pi}\int_0^\infty[\overline{\phi}_{kl}^R(K)\cos(K\frac{U}{B}t)-\overline{\phi}_{kl}^I(K)\sin(K\frac{U}{B}t)]dK$$

$$=\underbrace{a_{kl}^0\delta(t)+\frac{B}{U}b_{kl}^0\dot{\delta}(t)+\sum_{n=1}^N\delta(t-\frac{B}{U}\frac{n\pi}{K_{\max}})\frac{a_{kl}^n-b_{kl}^n}{2}}_{\boxed{t\geq 0}}+\underbrace{\sum_{n=1}^N\delta(t+\frac{B}{U}\frac{n\pi}{K_{\max}})\frac{a_{kl}^n+b_{kl}^n}{2}}_{\boxed{t<0}}$$

- Causality condition for the modified transfer function

$$\boxed{\overline{\Phi}_{kl}(t)=0\ \text{ for } t<0}\ \Rightarrow\ \boxed{b_{kl}^n=-a_{kl}^n\quad \text{for } n=1,\cdots,N}$$

3. Truncated Fourier Series Approximation

◆ Final Aerodynamic Forces

$$L_{ae}(t) = \frac{1}{2}\rho U^2 B\left(\int_0^t \overline{\Phi}_{hh}(t-\tau)\frac{h(\tau)}{B}d\tau + \int_0^t \overline{\Phi}_{h\alpha}(t-\tau)\alpha(\tau)d\tau\right)$$

$$M_{ae}(t) = \frac{1}{2}\rho U^2 B^2\left(\int_0^t \overline{\Phi}_{\alpha h}(t-\tau)\frac{h(\tau)}{B}d\tau + \int_0^t \overline{\Phi}_{\alpha\alpha}(t-\tau)\alpha(\tau)d\tau\right)$$

$$L_{ae}(t) = \frac{1}{2}\rho U^2 B(a_{hh}^0 \frac{h(t)}{B} + b_{hh}^0 \frac{\dot{h}(t)}{U} + \frac{1}{B}\underline{\sum_{n=1}^{N} a_{hh}^n h(t - \frac{B}{U}\frac{n\pi}{K_{\max}})}$$
$$+ a_{h\alpha}^0 \alpha(t) + b_{h\alpha}^0 \frac{B}{U}\dot{\alpha}(t) + \underline{\sum_{n=1}^{N} a_{h\alpha}^n \alpha(t - \frac{B}{U}\frac{n\pi}{K_{\max}})})$$

$$M_{ae}(t) = \frac{1}{2}\rho U^2 B^2(a_{\alpha h}^0 \frac{h(t)}{B} + b_{\alpha h}^0 \frac{\dot{h}(t)}{U} + \frac{1}{B}\underline{\sum_{n=1}^{N} a_{\alpha h}^n h(t - \frac{B}{U}\frac{n\pi}{K_{\max}})}$$
$$+ a_{\alpha\alpha}^0 \alpha(t) + b_{\alpha\alpha}^0 \frac{B}{U}\dot{\alpha}(t) + \underline{\sum_{n=1}^{N} a_{\alpha\alpha}^n \alpha(t - \frac{B}{U}\frac{n\pi}{K_{\max}})})$$

To perform the convolution integrals, only *N* past displacements are required.

3. Truncated Fourier Series Approximation

◆ Determination of Coefficients of the Fourier Series

- The unknown coefficients are determined by minimizing the errors between the measured and modified transfer functions.

$$\underset{\mathbf{a}_{kl}}{\mathrm{Min}}\ \Pi_{kl}(\mathbf{a}_{kl}) = \frac{1}{2}\frac{w_{kl}}{\left\|\phi_{kl}^{R}\right\|_{L_2}^{2}}\int_0^{K_{\max}}(\bar{\phi}_{kl}^{R}(\mathbf{a}_{kl})-\phi_{kl}^{R})^2\,dK+\frac{1}{2}\frac{(1-w_{kl})}{\left\|\phi_{kl}^{I}\right\|_{L_2}^{2}}\int_0^{K_{\max}}(\bar{\phi}_{kl}^{I}(\mathbf{a}_{kl})-\phi_{kl}^{I})^2\,dK$$

w_{kl} : weighting factor, $\left\|\cdot\right\|_{L_2}$: L_2-norm,

$\phi_{kl}^{R,I}$: measured transfer functions interpolated by cubic spline

$\mathbf{a}_{kl} = (a_{kl}^{0}\quad b_{kl}^{0}\quad a_{kl}^{1}\quad \cdots \quad a_{kl}^{N})^T$: **(N+2) unknowns**

$\dfrac{\partial \Pi_{kl}}{\partial \mathbf{a}_{kl}} = 0$: **(N+2) equations**

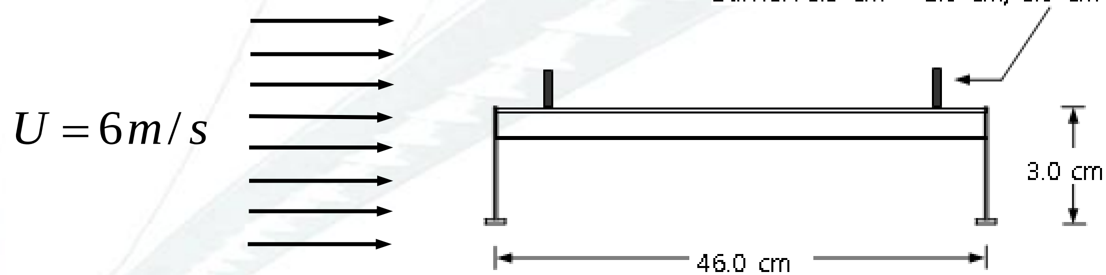
Condition of the Problem

- Jung el al.(2012) demonstrated the limitation of RFA for this H-type section
- Governing equation

$$
\boxed{\begin{aligned} m_h\ddot{h} + c_h\dot{h} + k_h h &= L_{ae}(t) + L_{ex}(t) \\ m_\alpha\ddot{\alpha} + c_\alpha\dot{\alpha} + k_\alpha\alpha &= M_{ae}(t) + M_{ex}(t) \end{aligned}}
$$

$$
\begin{aligned}
&m_h = 3.640\ \text{kg/m}, && m_\alpha = 0.102\,\text{kg}\cdot\text{m}^2/\text{m} \\
&c_h = 1.003\ \text{kg/s/m}, && c_\alpha = 0.022\ \text{kg}\cdot\text{m}^2/\text{s/m} \\
&k_h = 1332.6\ \text{N/m/m}, && k_\alpha = 106.2\ \text{N}\cdot\text{m/m} \\
&f_h = 3.05\ \text{Hz}, && f_\alpha = 5.15\ \text{Hz}
\end{aligned}
$$

- Dimension of the H-type bluff section, Kim and King (2007)



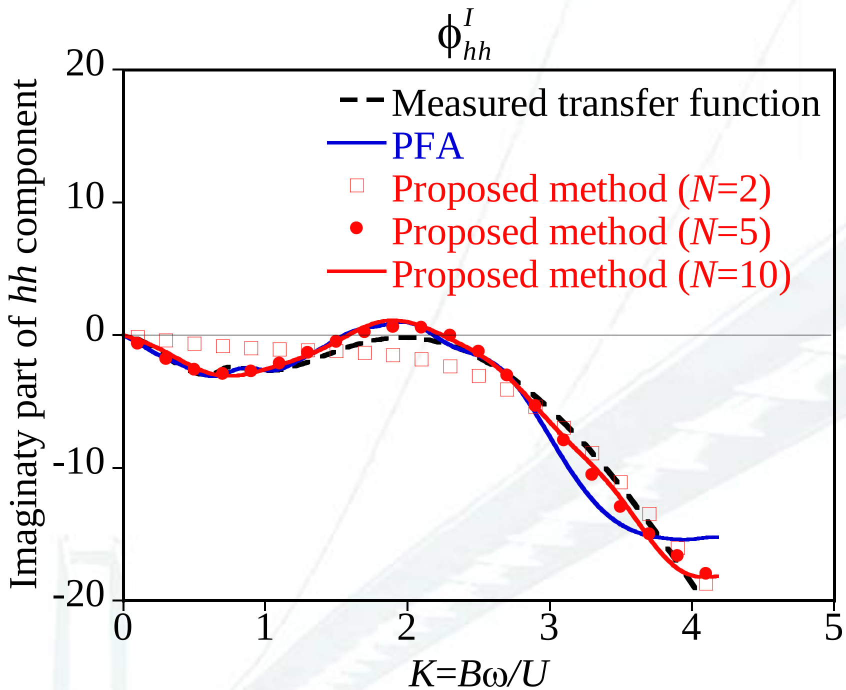
$$\rho = 1.25\ \text{kg/m}^3$$

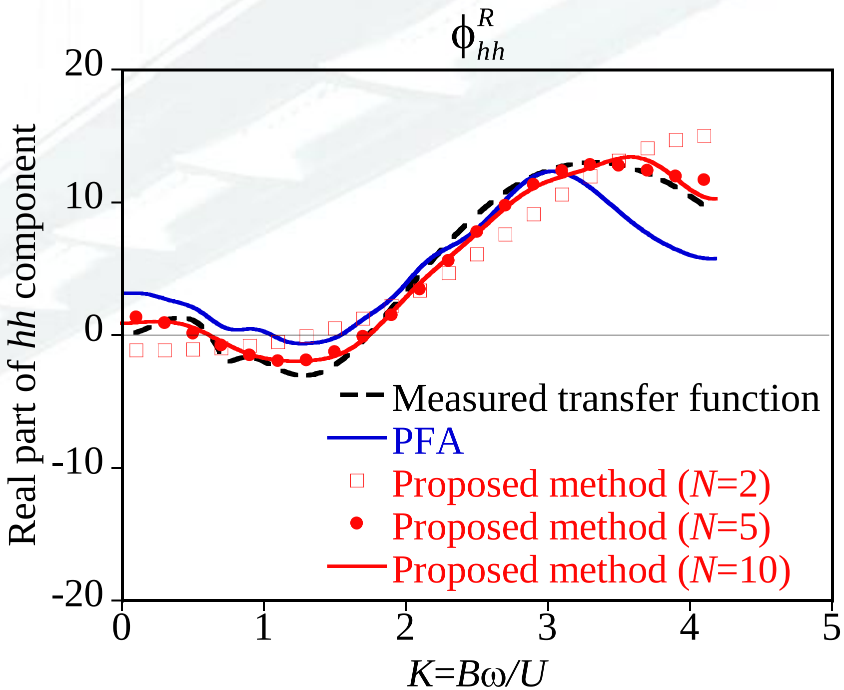
- External forces

: To compare the steady-state responses with those obtained by measured transfer function.

$$
\boxed{\begin{pmatrix} L_{ex} \\ M_{ex} \end{pmatrix} = \begin{pmatrix} 10\ \text{N/m} \\ 1\ \text{N}\cdot\text{m/m} \end{pmatrix} \sin\omega_{ex}t, \quad \omega_{ex} = 8\pi\ \text{rad/s}}
$$

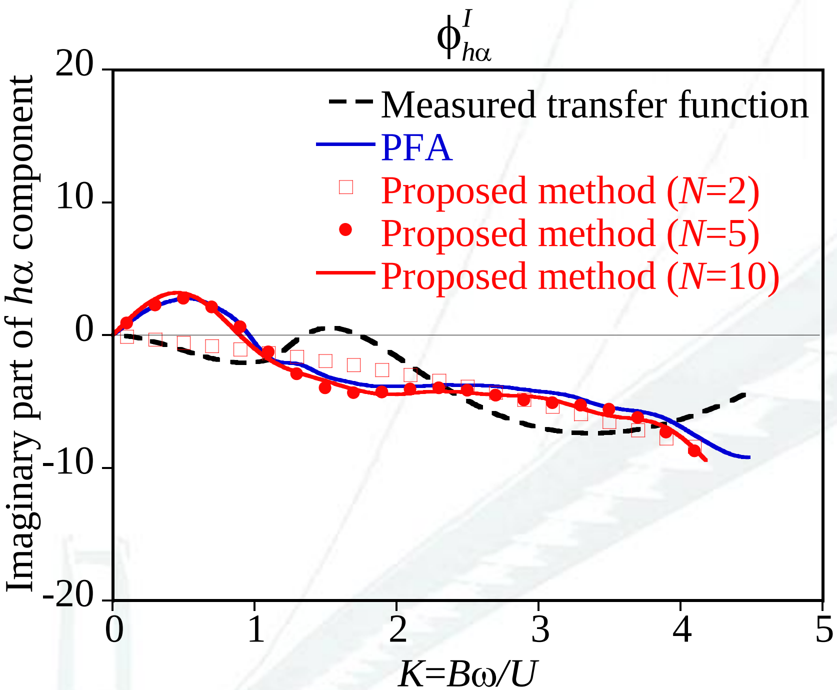
◆ *hh* Components of the Transfer Functions for the Lift Force

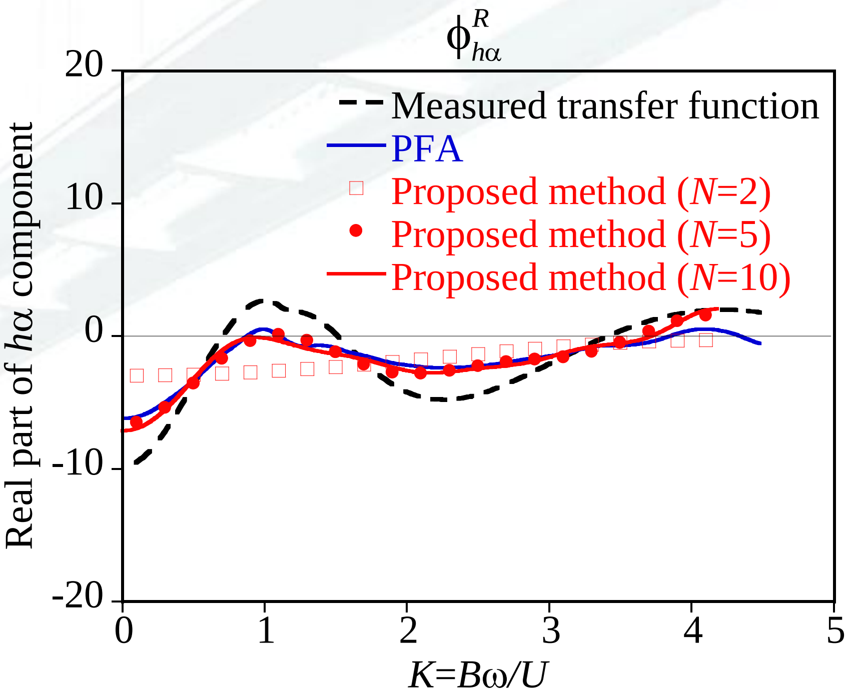




- The PFA and Proposed method yields almost the same results.
- Even, the Proposed method yields closer solution to the 'Measured' than the PFA.
- For the proposed method, only 5 terms of Fourier series are enough convergent.

◆ $h\alpha$ Components of the Transfer Functions for the Lift Force

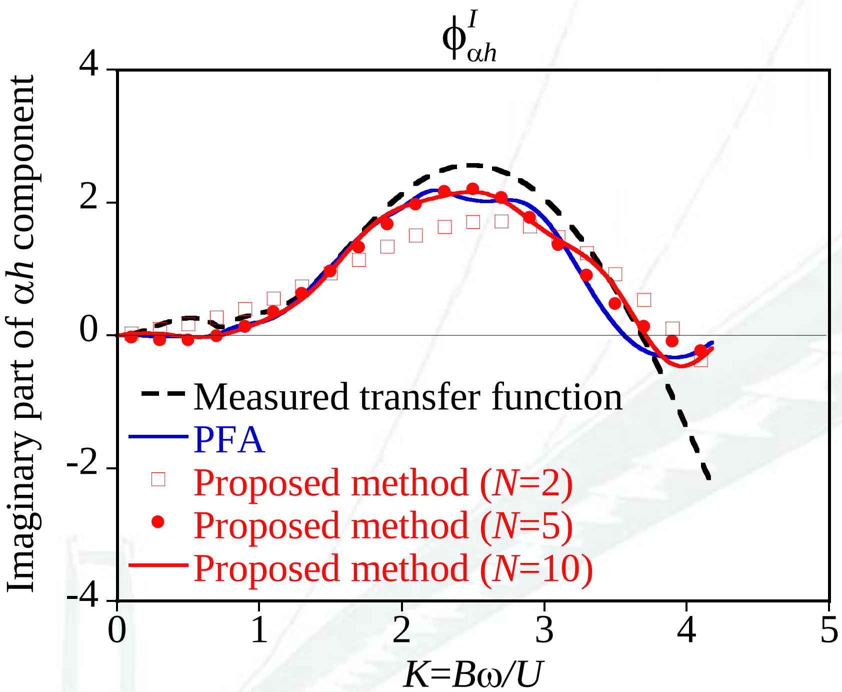


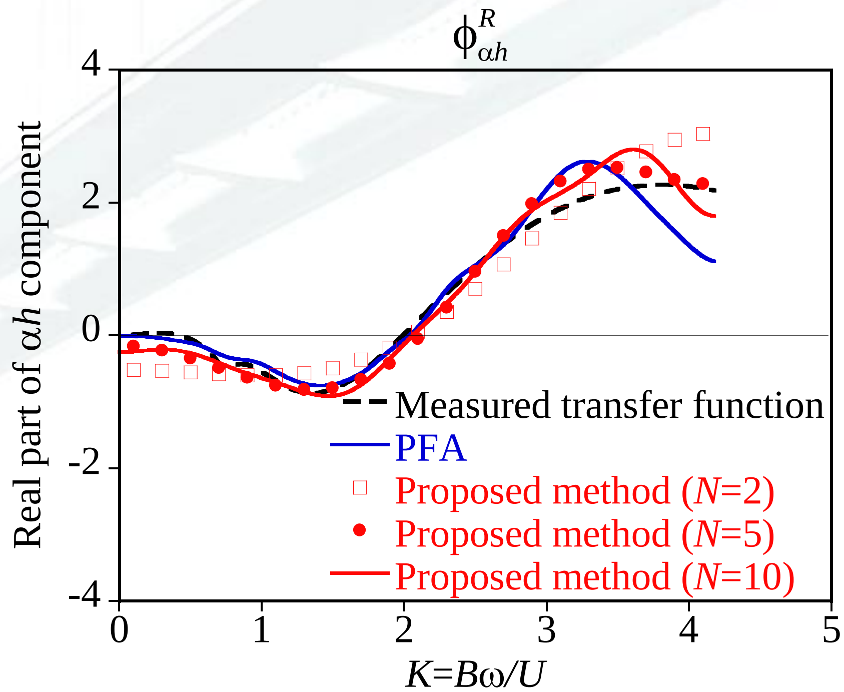


- The PFA and Proposed method yields almost the same results.
- The differences with 'Measured' indicates the degrees of violation of the causality condition in the 'Measured'.

4. Application and Verification
- H-type section

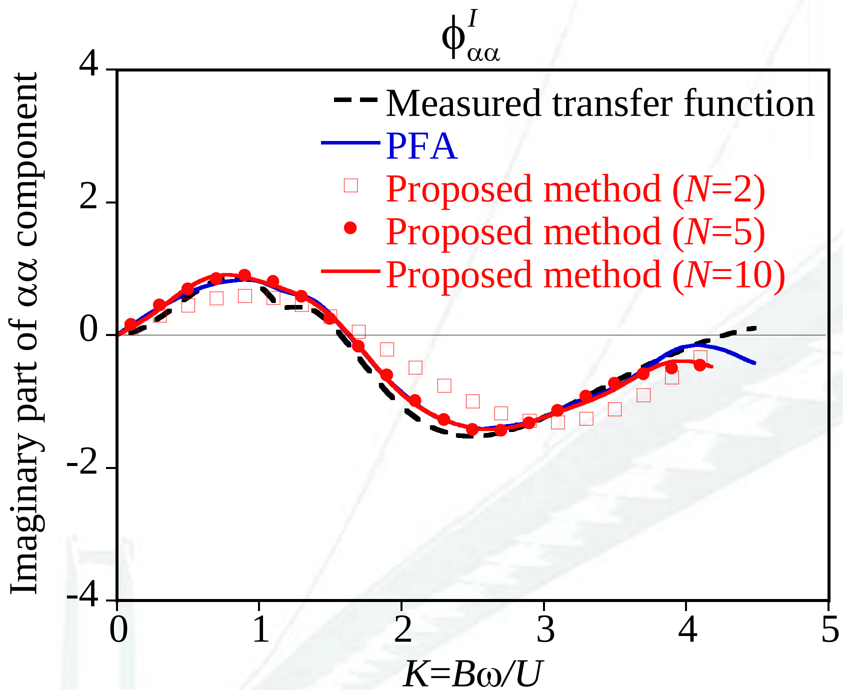
◆ αh **Components of the Transfer Functions for the Moment**

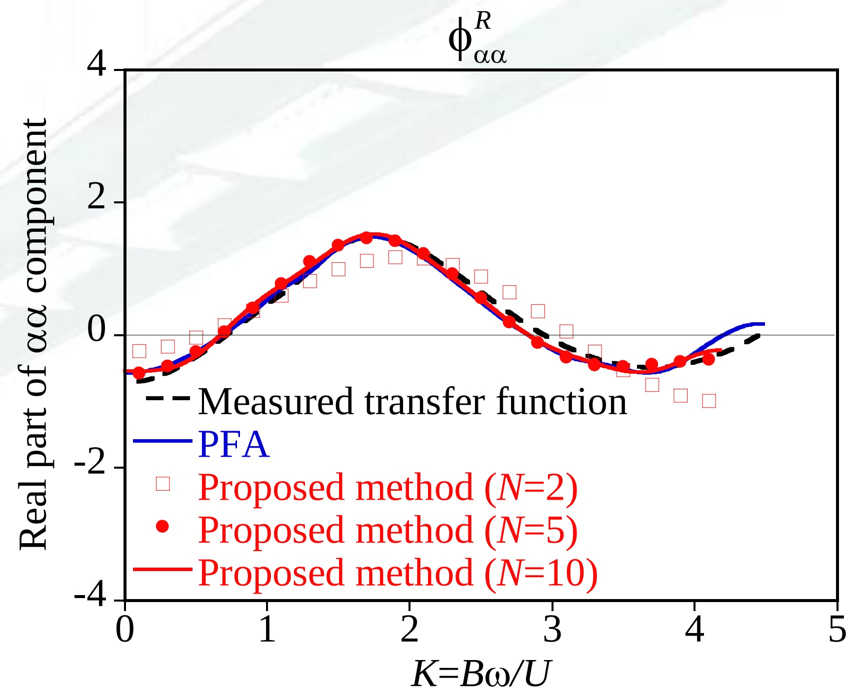




-The results for the moment can be similarly interpreted with those for the lift force

◆ αα Components of the Transfer Functions for the Moment

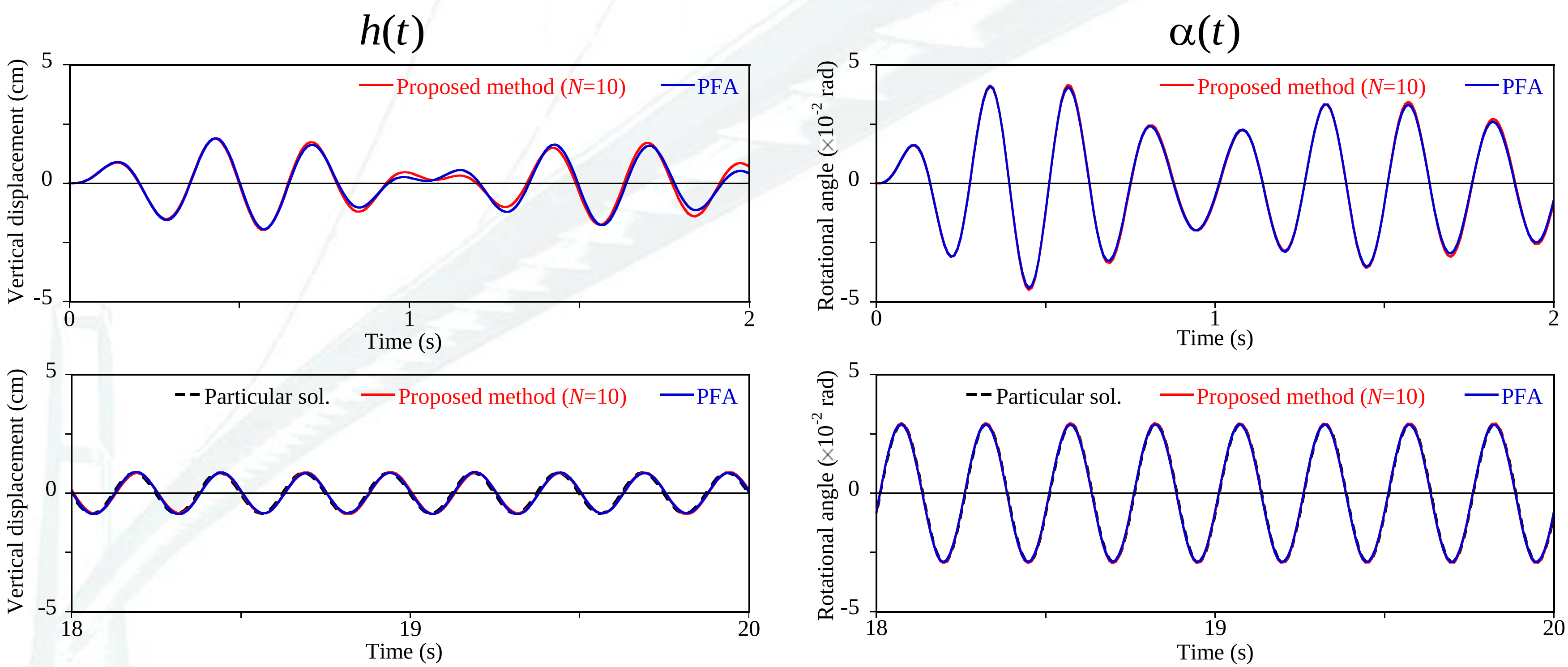




-The results for the moment can be similarly interpreted with those for the lift force

4. Application and Verification
- H-type section

- **Responses**
  - Using Newmark-beta method with time interval of 0.0096 sec
  - The PFA, proposed method and particular solution have no noticeable difference.



4. Application and Verification
- H-type section

◆ Comparison of Computation Time

- Computer: 2.4GHz single core
- Program: MATLAB R2001b
- Analysis time: 200 sec
- Time interval: 0.00958 sec

	PFA	Proposed method (*N*=10)
Computation time	42.7 sec	→ 2.6 sec

Reduction of 94 %

→ Proposed method can be used to perform a time-domain aeroelastic analysis even for a large-scale structure efficiently without any loss of accuracy.

5. Summary

- **Causality Requirement**

- **Propose the Exact and Efficient Method using Truncated Fourier Series**
  - Causality condition is strongly imposed.
  - IRF becomes Dirac-delta functions (Reduction of the computation)

- **Application and Verification through the example of H-type bluff section**

◆ ACKNOWLEDGEMENT

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Thank you !