The Evaluation Methods of Aerodynamic Impulse Response Functions for the Time-domain Aeroelastic Analysis

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Abstract
The importance of time-domain aeroelastic analysis has been increasingly emphasized in recent decades to consider various nonlinearities of a structural system and/or non-stationary effects of air flows. The critical issue in the time-domain aeroelastic analysis is the evaluation of aerodynamic impulse response function as the aerodynamic forces are basically defined in frequency domain with the terms of flutter derivatives identified in wind-tunnel tests. However, the flutter derivatives generally do not satisfy the causality condition that should be satisfied for the evaluation of aerodynamic impulse response functions. Here, the causality condition states that the impulse response functions vanish for the negative time from the physical point of view. The rational function approach (RFA) has been successfully adopted for the various type of bridge section, but it is reported by several researchers on the potential limitations of the RFA related to its applicability to bluff sections. To overcome this drawback, the penalty function approach (PFA), in which a FEM-based approach is adopted and the causality condition is weakly imposed as a penalty function in the optimization, is proposed. However, the PFA requires a rather complicated formulation and huge computational efforts for large-scale structures. In this study, the proposed method adopts a truncated Fourier series to represent the aerodynamic transfer functions defined with the flutter derivatives, and the exact relation between the real and imaginary parts of aerodynamic transfer functions to satisfy the causality condition is derived. The accuracy and effectiveness of the proposed method is demonstrated by applying to the bluff sections.

Key words: Impulse response function; Transfer function; Fourier Series; Causality condition; Convolution integral; Aeroelastic analysis, Flutter derivative

1. Introduction

The importance of time-domain aeroelastic analysis has been increasingly emphasized in recent decades to consider various nonlinearities of a structural system and/or non-stationary effects of air flows[(1),(2)]. The self-excited aerodynamic forces in the time-domain are expressed by the one-sided convolution integral of deck motion using aerodynamic impulse response functions. Since, however, the aerodynamic forces are basically defined in frequency domain with the terms of flutter derivatives identified in wind-tunnel tests, the critical issue in the time-domain aeroelastic analysis is the evaluation of aerodynamic impulse response function obtained by the inverse Fourier transform of the aerodynamic transfer functions. Here, the aerodynamic impulse response functions should satisfy the causality condition in order to perform a one-sided convolution integral[2]. The causality condition states that the impulse response functions vanish for the negative time from the physical point of view. Because the measured flutter derivatives generally do not satisfy the causality condition, the transfer functions corresponding to the flutter derivatives should be modified to satisfy the causality condition.

The rational function approximation (RFA) has been successfully adopted for the various type of bridge section to modify the aerodynamic transfer function[3]. However it is reported by several researchers on the potential limitations of the RFA related to its applicability to bluff sections[2],[4], as the RFA is based on the solution to the ideally thin section. The limitations of RFA are generated from the fact that the rational functions cannot reasonably approximate intricate aerodynamic transfer functions that are observed for bluff sections. To overcome the limitation of the RFA, Jung et al. proposed the penalty function approach (PFA) in which a FEM-based approach is adopted and the causality condition is weakly imposed as a penalty function in the optimization using the cubic spline interpolation[2]. However, the PFA requires a rather complicated formulation and the huge computational efforts for large-scale structures to perform the convolution integrals.

In this study, the proposed method adopts a truncated Fourier series to modify the aerodynamic transfer functions defined with the flutter derivatives, and the exact relation between the real and imaginary parts of aerodynamic transfer functions to satisfy the causality condition is derived. The coefficients of truncated Fourier series are determined by minimizing the error between measured and modified transfer function. The computational effort to evaluate the aerodynamic forces is considerably reduced as the modified impulse response functions become a series of Dirac delta functions. The validity and effectiveness of the proposed method is demonstrated by applying to the H-type bluff sections.

2. Causality Condition for the Time-domain Aerodynamic Forces

The aerodynamic forces induced by motion of an object in a stationary wind flow are expressed by convolution integrals in the time domain[2],[3]:

$$\begin{aligned} L_{ae}(t) &= \frac{1}{2}\rho U^2 B\left(\int_0^t \Phi_{hh}(t-\tau)\frac{h(\tau)}{B}d\tau + \int_0^t \Phi_{h\alpha}(t-\tau)\alpha(\tau)d\tau\right) \\ M_{ae}(t) &= \frac{1}{2}\rho U^2 B^2\left(\int_0^t \Phi_{\alpha h}(t-\tau)\frac{h(\tau)}{B}d\tau + \int_0^t \Phi_{\alpha\alpha}(t-\tau)\alpha(\tau)d\tau\right) \end{aligned} \tag{1}$$

where $L_{ae}(t)$ and $M_{ae}(t)$ = the aerodynamic lift force and moment, respectively; h and α = the vertical and rotational displacement, respectively; ρ = air density; U = mean cross wind velocity; and B = width of the section. The real function, Φ_{kl} for $k,l = h,\alpha$ is the kl-component of the aerodynamic impulse response function representing the aerodynamic force in the k direction at time t induced by the unit impulse motion of an object in the l direction at $t=0$. The one-sided convolution integrals in Eq. (1) are valid if and only if every component of the impulse response function vanishes identically for the negative time domain[2], that is, $\Phi_{kl} \equiv 0$ for $t<0$, which is referred to as the causality condition. The causality condition represents the physical fact that aerodynamic forces are induced only after an object moves.

The evaluation of the aerodynamic impulse response from the wind tunnel test directly is very formidable, hence, generally, is performed by taking an inverse Fourier transform of the aerodynamic transfer functions.

$$\Phi_{kl}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}(i\phi_{kl}^I + \phi_{kl}^R)e^{i\omega t}d\omega \tag{2}$$

Here, ϕ_{kl} = the kl-component of the aerodynamic transfer function; i = the imaginary unit; and superscript I and R indicate the imaginary and real part of a complex variable, respectively. The aerodynamic transfer function is expressed in terms of flutter derivatives identified in wind tunnel tests[5]:

$$i\phi_{hh}^{I}+\phi_{hh}^{R}=iK^{2}H_{1}^{*}+K^{2}H_{4}^{*},\quad i\phi_{h\alpha}^{I}+\phi_{h\alpha}^{R}=iK^{2}H_{2}^{*}+K^{2}H_{3}^{*}$$
$$i\phi_{\alpha h}^{I}+\phi_{\alpha h}^{R}=iK^{2}A_{1}^{*}+K^{2}A_{4}^{*},\quad i\phi_{\alpha\alpha}^{I}+\phi_{\alpha\alpha}^{R}=iK^{2}A_{2}^{*}+K^{2}A_{3}^{*} \tag{3}$$

where $K=B\omega/U$ = the non-dimensional reduced frequency where ω = the angular frequency of oscillation; and H_m^* and A_m^* ($m=1,2,3,4$) = the flutter derivatives.

As the impulse response function is a real function, ϕ_{kl}^{R} and ϕ_{kl}^{I} are an even and odd function, respectively, in the frequency domain. Then, the impulse response function and the causality condition become as follows[2]:

$$\Phi_{kl}(t)=\frac{1}{\pi}\int_0^\infty(\phi_{kl}^{R}(\omega)\cos\omega t-\phi_{kl}^{I}(\omega)\sin\omega t)d\omega \quad \text{for}\quad t\geq 0 \tag{4}$$

$$\Phi_{kl}(t)=\int_0^\infty(\phi_{kl}^{R}(\omega)\cos\omega t-\phi_{kl}^{I}(\omega)\sin\omega t)d\omega\equiv 0 \quad \text{for}\quad t<0 \tag{5}$$

The causality condition in Eq. (5) implies that the real and imaginary part of the aerodynamic transfer function have a certain relationship. Since, however, this relation is not considered in the extract of flutter derivatives, the aerodynamic transfer functions are modified to evaluate the aerodynamic impulse response function.

3. Enforcement of the Causality Condition

The aerodynamic transfer function is defined up to the maximum reduced frequency, K_{max}, adopted in actual wind-tunnel tests. Since the real and imaginary part of the aerodynamic transfer function are an even and odd function, respectively, the Fourier cosine series and the Fourier sine series are separately adopted for the individual part as follows:

$$\bar{\phi}_{kl}^{R}(K)=a_{kl}^{0}+\sum_{n=1}^{N}a_{kl}^{n}\cos\frac{n\pi}{K_{\max}}K$$
$$\bar{\phi}_{kl}^{I}(K)=b_{kl}^{0}K+\sum_{n=1}^{N}b_{kl}^{n}\sin\frac{n\pi}{K_{\max}}K \tag{6}$$

where $\bar{\phi}_{kl}$ = the *kl* component of the modified transfer function; a_{kl}^{n} and b_{kl}^{n} = unknown coefficients of the Fourier series; and N = the number of terms in the Fourier series. The linear term in the imaginary part of Eq. (6) prevents the oscillations of the Fourier sine series caused by a discontinuity between the Fourier sine series and the measured transfer function at K=K_{max}.

Substituting Eq. (6) into Eq. (4) yields the modified impulse response function.

$$\begin{aligned}\bar{\Phi}_{kl}(t)&=\frac{1}{\pi}\int_0^\infty(\bar{\phi}_{kl}^{R}(K)\cos\omega t-\bar{\phi}_{kl}^{I}(K)\sin\omega t)d\omega\\ &=a_{kl}^{0}\delta(t)+\frac{B}{U}b_{kl}^{0}\dot{\delta}(t)+\sum_{n=1}^{N}\delta\Big(t-\frac{B}{U}\frac{n\pi}{K_{\max}}\Big)\frac{a_{kl}^{n}-b_{kl}^{n}}{2}+\sum_{n=1}^{N}\delta\Big(t+\frac{B}{U}\frac{n\pi}{K_{\max}}\Big)\frac{a_{kl}^{n}+b_{kl}^{n}}{2}\end{aligned} \tag{7}$$

where $\bar{\Phi}_{kl}$ = the *kl*-component of the modified impulse response function. The first three terms in the last equation of Eq. (7) exist for $t\geq 0$, while the last term exist for $t<0$ unless $b_{kl}^{n}=-a_{kl}^{n}$. Therefore, the causality condition for the modified transfer function in Eq. (6) is defined as:

$$b_{kl}^{n}=-a_{kl}^{n} \quad \text{for}\quad n=1,\cdots,N \tag{8}$$

The final modified transfer function and the impulse response function that exactly satisfy the causality condition are obtained by enforcing the causality condition of Eq. (8) on Eq. (6) and Eq. (7)

$$\bar{\phi}_{kl}^{R}=a_{kl}^{0}+\sum_{n=1}^{N}a_{kl}^{n}\cos\frac{n\pi}{K_{\max}}K$$
$$\bar{\phi}_{kl}^{I}=b_{kl}^{0}K-\sum_{n=1}^{N}a_{kl}^{n}\sin\frac{n\pi}{K_{\max}}K \tag{9}$$

$$\bar{\Phi}_{kl}(t)=a_{kl}^{0}\delta(t)+b_{kl}^{0}\frac{B}{U}\dot{\delta}(t)+\sum_{n=1}^{N}a_{kl}^{n}\delta\Big(t-\frac{B}{U}\frac{n\pi}{K_{\max}}\Big) \tag{10}$$

The unknown coefficients in Eq. (9) are easily determined by minimizing the errors between the measured and modified transfer functions, which is the minimization scheme similar to that adopted in the PFA[(2)]. However, the procedure of the minimization in the proposed method is much simpler than that in the PFA by virtue of the exact enforcement of the causality condition.

The aerodynamic forces in Eq. (1) are easily evaluated without numerical integration as the modified impulse response functions are a series of Dirac-delta functions.

$$\begin{aligned} L_{ae}(t) &= \frac{1}{2}\rho U^2 B\Big(a_{hh}^0 \frac{h(t)}{B} + b_{hh}^0 \frac{\dot{h}(t)}{U} + \frac{1}{B}\sum_{n=1}^{N} a_{hh}^n h\big(t - \frac{B}{U}\frac{n\pi}{K_{\max}}\big) \\ &\quad + a_{h\alpha}^0 \alpha(t) + b_{h\alpha}^0 \frac{B}{U}\dot{\alpha}(t) + \sum_{n=1}^{N} a_{h\alpha}^n \alpha\big(t - \frac{B}{U}\frac{n\pi}{K_{\max}}\big)\Big) \\ M_{ae}(t) &= \frac{1}{2}\rho U^2 B^2\Big(a_{\alpha h}^0 \frac{h(t)}{B} + b_{\alpha h}^0 \frac{\dot{h}(t)}{U} + \frac{1}{B}\sum_{n=1}^{N} a_{\alpha h}^n h\big(t - \frac{B}{U}\frac{n\pi}{K_{\max}}\big) \\ &\quad + a_{\alpha\alpha}^0 \alpha(t) + b_{\alpha\alpha}^0 \frac{B}{U}\dot{\alpha}(t) + \sum_{n=1}^{N} a_{\alpha\alpha}^n \alpha\big(t - \frac{B}{U}\frac{n\pi}{K_{\max}}\big)\Big) \end{aligned} \tag{11}$$

To evaluate the aerodynamic forces in Eq. (10), only N past displacements are needed unlike the PFA which requires the complete time histories of the displacements. Therefore, the proposed method greatly improves computational efficiency in the evaluation of the aerodynamic forces.

4. Applications and Verification

For the verification of the proposed method, time-domain aerodynamic analyses are performed for the H-type bluff section. The flutter derivatives of the bluff H-type section are extracted by Kim and King at the Boundary Layer Wind Tunnel Laboratory of the University of Western Ontario in Ontario, Canada[(6)]. Jung et al. demonstrated the limitation of the RFA for this section[(2)]. Therefore, a comparison of the proposed method with the RFA would be meaningless, and the results obtained by the proposed method are compared with those obtained by the PFA.

The equations of motion for an elastically supported system are defined as follow:

$$\begin{aligned} m_h\ddot{h} + c_h\dot{h} + k_h h &= L_{ae}(t) + L_{ex}(t) \\ m_\alpha\ddot{\alpha} + c_\alpha\dot{\alpha} + k_\alpha \alpha &= M_{ae}(t) + M_{ex}(t) \end{aligned} \tag{12}$$

where m_j, c_j and k_j are the mass, damping and stiffness in the direction of $j = h, \alpha$, respectively; L_{ex} and M_{ex} are the external excitation forces in the h and α direction, respectively.

Fig. 1 shows the modified transfer functions for the lift force evaluated using the proposed method and PFA, along with the measured ones. The proposed method and the PFA yield almost the same results, even though some differences are observed in the *hh* components. Since the transfer functions obtained by the proposed method are closer to the measured transfer function than those by the PFA, it is believed that the proposed method represents actual physical phenomena better than the PFA. To ensure the convergence of the proposed method, the number of series terms is varied as 2, 5 and 10. As shown in Fig. 1, the modified transfer functions with 5 terms are closely convergent to those with 10 terms.

The accuracy of the proposed method is examined for the section subjected to harmonic excitation forces.

$$\begin{pmatrix} L_{ex} \\ M_{ex} \end{pmatrix} = \begin{pmatrix} L_0 \\ M_0 \end{pmatrix} \sin \omega_{ex} t \tag{13}$$

where $L_0 = 10\,\mathrm{N/m}$; $M_0 = 1\,\mathrm{N \cdot m/m}$; and $\omega_{ex} = 8\pi\,\mathrm{rad/s}$. Fig. 2 shows the transient responses and steady state responses of the vertical displacement. There is no noticeable difference among the responses by the proposed method, the PFA, and the particular solution of Eq. (12). The results of the transfer functions for the moment and the rotational

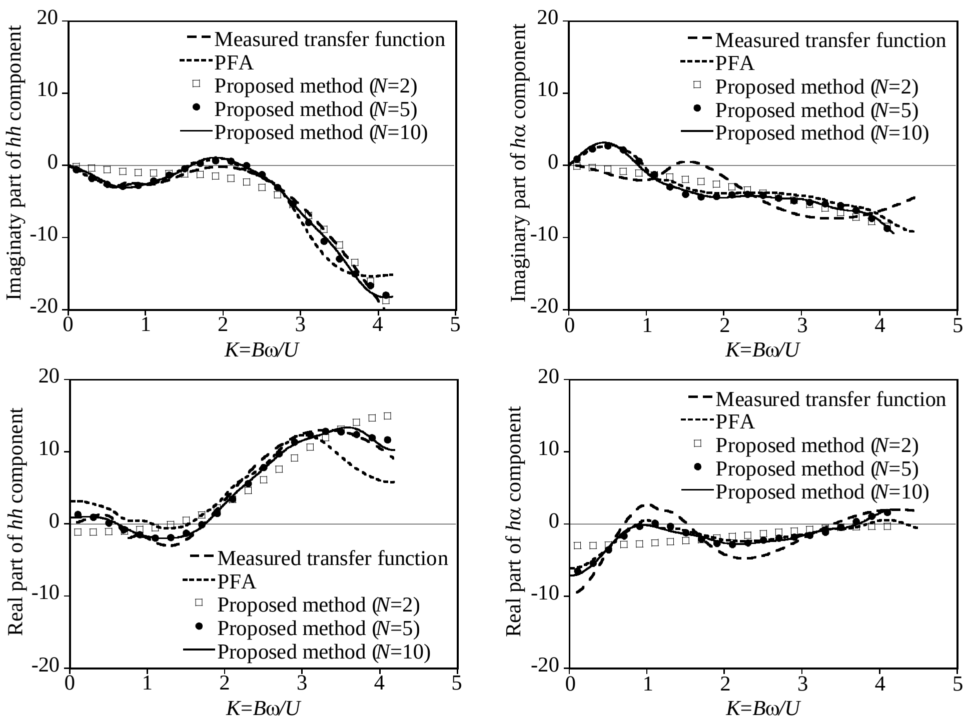

Fig. 1 Transfer functions of the H-type section for the lift force

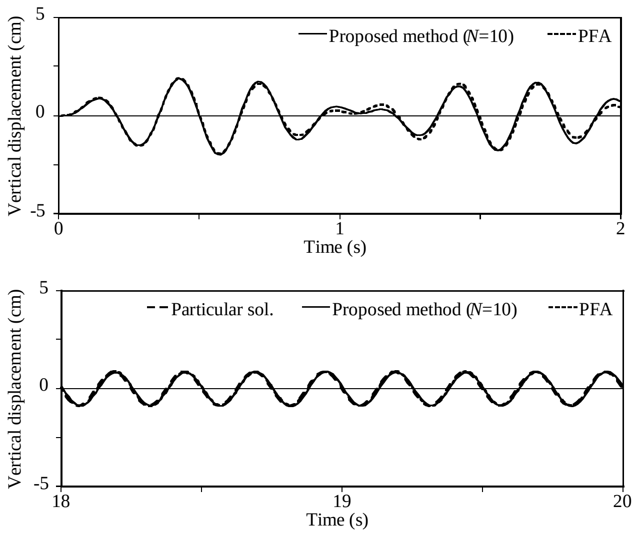

Fig. 2 Forced vibration responses at a wind velocity of 6.0m/s for the H-type section

displacements by the proposed method and the PFA are also almost the same but not presented, here.

To demonstrate the computational efficiency of the proposed method, a time-domain analysis is performed up to 200 s with about 0.01 s of time interval. The computation times for the proposed method and PFA during convolution integration are 1.6 sec for proposed method and 33.5 sec for the PFA. From this result, it seems that the proposed method can be used to perform a time-domain aeroelastic analysis even for a large-scale structure.

5. Conclusion

The causality condition, required to perform one-sided convolution integrals for a time-domain aeroelastic analysis, is strongly enforced by expressing each part of the aerodynamic transfer function with a truncated Fourier series. The aerodynamic forces in the proposed method are expressed only a few terms on the current and past displacements, and the computationaly efficiency is greatly improves compared with the PFA. The validity and effectiveness are demonstrated through the examples for the H-type section.

References

(1) Chen, X., and Kareem, A., Aeroleastic Analysis of Bridges: Effects of Turbulence and Aerodynamic Nonlinearities, ASCE Journal of Engineering Mechanics, Volume 129 (8), 2003, pp. 885-895

(2) Jung, K., Kim, H. K., and Lee, H. S., Evaluation of Impulse Response Functions for Convolution Integrals of Aerodynamic Forces by Optimization with a Penalty Function, ASCE Journal of Engineering Mechanics, Volume 138 (5), 2012, pp. 519-529

(3) Chen X., Matsumoto M., and Kareem A., Aerodynamic Coupling Effects on Flutter and Buffeting of Bridges, ASCE Journal of Engineering Mechanics, Volume 126 (1), 2000, pp. 7-16

(4) Caracoglia, L., and Jones, N. P., Time Domain vs. Frequency Domain Characterization of Aeroelastic Forces for Bridge Deck Sections, Journal of Wind Engineering and Industrial Aerodynamics, Volume 91 (3), 2003, pp. 371-402

(5) Scanlan, R. H., and Tomko, J. J., Airfoil and Bridge Deck Flutter Derivatives, ASCE Journal of Engineering Mechanics Division, Volume 97 (6), 1971, pp. 1717-1737

(6) Kim J. D., and King J. P. C., The Development of Wind Tunnel Test Technique for an Aeroelastic Buffeting Analysis of Long-span Bridges, BLWTL-SS19-2007-DRAFT report submitted to Korea Wind Engineering Research Center, the Boundary Layer Wind Tunnel Laboratory of the University of Western Ontario

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