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# A Parameter Estimation Algorithm by Energy Formulation from Static Response

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## ABSTRACT

This paper presents an energy error estimator for the system identification of structures. The energy error estimator is defined as the strain energy of a structure induced by displacement discrepancy between calculated displacement by the finite element model and measured displacement. Advantages of the energy error estimator over the conventional output error estimator are demonstrated by Monte Carlo simulation. The recursive quadratic programming with the Fletcher's active set strategy is used to solve a nonlinear optimization problem.

## INTRODUCTION

System identification techniques are widely used in many structural problems such as damage detection. Most of system identification techniques are based on the minimization of a predefined, scalar error estimator with respect to system parameters to be estimated. Since the system parameters are estimated in the optimization process of an error estimator, accuracy and stability of the system identification scheme heavily depend on the error estimator.

The output error estimator is one of the most widely used estimators. The output error estimator(Banan *et al.*, 1994) is defined as the least-squared error between the measured displacement field and the calculated displacement field by finite element model at observation points. In this study, an energy error estimator is proposed for the system identification of structures. The energy error estimator is defined as the strain energy of a structure induced by displacement discrepancy between calculated displacement using the finite element model and the measured displacement of a structure. Since the number of observation points are usually less than the number of degrees of freedom in a structure, displacements at unmeasured degrees of freedom are estimated by the equilibrium equation of the finite element model considering the measured displacement as prescribed displacement. The recursive quadratic programming technique is adopted for the minimization procedure(Luenberger, 1989).

Noise is always presented in measurements. Sophisticated measuring skills and instruments may reduce the noise level, but noise can't be eliminated completely. Monte Carlo simulation is adopted to investigate the behaviors of the proposed estimator in the presence of noise in measurement, and to demonstrate advantages of the proposed estimator over the output error estimator.

## ENERGY ERROR ESTIMATOR

The proposed energy error estimator for a linear structural system is defined as

$$\Pi(\mathbf{X}) = \frac{1}{2}\sum_{k=1}^{ncl}\alpha^k \int_v (\boldsymbol{\varepsilon}^k - \bar{\boldsymbol{\varepsilon}}^k) : \mathbf{D}(\mathbf{X}) : (\boldsymbol{\varepsilon}^k - \bar{\boldsymbol{\varepsilon}}^k)dv \tag{1}$$

where $\mathbf{X}$, $ncl$, $\alpha^k$, $\mathbf{D}$, $\varepsilon^k$, and $\bar{\varepsilon}^k$ are system parameters, the number of load cases, the weighting value associated with the $k$-th load case, the elasticity tensor, strain tensor calculated by mathematical model and the measured strain tensor of a structure, respectively. Since it is impossible to measure strain at all points of a structure, finite element discretization is employed to express (1) in terms of nodal displacements of the discretized model of a structure as follows.

$$\Pi(\mathbf{X}) = \frac{1}{2}\sum_{k=1}^{ncl}\alpha_k(\mathbf{u}^k - \bar{\mathbf{u}}^k)\cdot\mathbf{K}\cdot(\mathbf{u}^k - \bar{\mathbf{u}}^k) = \sum_{k=1}^{ncl}\alpha_k\Pi^k \tag{2}$$

where $\mathbf{K}$, $\mathbf{u}^k$, and $\bar{\mathbf{u}}^k$ are the stiffness matrix, the calculated nodal displacement, and the measured nodal displacement, respectively. It is assumed that the stiffness matrix is linear with respect to system parameters, and external loads applied to a structure are independent of the system parameters. The equilibrium equation of finite element model for the $k$-th load case is given by the stiffness equation.

$$\mathbf{K}\cdot\mathbf{u}^k = \mathbf{f}^k \tag{3}$$

where ( $\mathbf{f}^k$ ) is a force vector for $k$-th load case.

To evaluate (2), displacement has to be completely measured at all nodes. However, since the number of measured degrees of freedom is usually less than the total number of degrees of freedom of a structure, $\bar{\mathbf{u}}$ contains unmeasured degrees of freedom in real situations. The unmeasured components of $\bar{\mathbf{u}}$ are estimated by use of the equilibrium equation of the finite element model. Suppose that no error is contained in measurement, the following equilibrium equation for $k$-th load case is satisfied exactly for actual system parameters.

$$\begin{bmatrix} \mathbf{K}_{mm} & \mathbf{K}_{mu} \\ \mathbf{K}_{um} & \mathbf{K}_{uu} \end{bmatrix} \begin{pmatrix} \bar{\mathbf{u}}_m^k \\ \bar{\mathbf{u}}_u^k \end{pmatrix} = \begin{pmatrix} \mathbf{f}_m^k \\ \mathbf{f}_u^k \end{pmatrix} \tag{4}$$

where $\bar{\mathbf{u}}_m^k$, $\bar{\mathbf{u}}_u^k$, $\mathbf{f}_m^k$ and $\mathbf{f}_u^k$ are the measured part of $\bar{\mathbf{u}}^k$, the unmeasured part of $\bar{\mathbf{u}}^k$, the equivalent nodal forces for the $k$-th load case at measured degrees of freedom, and the equivalent nodal forces applied at unmeasured degrees of freedom, respectively. Since $\bar{\mathbf{u}}_m^k$ is independent of system parameters to be identified, and constant through the identification procedure, $\bar{\mathbf{u}}_m^k$ can be considered as prescribed displacement in the identification procedure. The unmeasured part of $\bar{\mathbf{u}}^k$ that satisfies the equilibrium at the unmeasured degrees of freedom can be obtained by solving the lower part of (4).

$$\bar{\mathbf{u}}_u^k = \mathbf{K}_{uu}^{-1} \cdot (\mathbf{f}_u^k - \mathbf{K}_{um} \cdot \bar{\mathbf{u}}_m^k) \tag{5}$$

Since the actual system parameters are unknown, the system parameters obtained at the previous iteration step of the identification procedure are used to evaluate stiffness matrices in (5). Note that the displacement estimated with (5) are not a actual displacements at the unmeasured degrees of freedom but a possible set of displacement that satisfies equilibrium of a structure for the assumed system parameters.

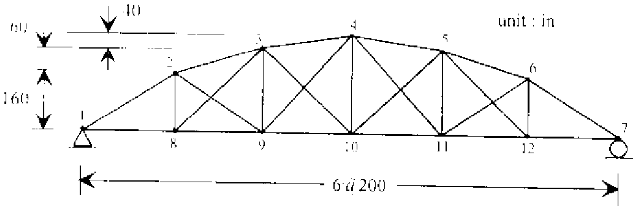
The recursive quadratic programming technique is employed to minimize (2). Direct differentiation of (3) and the lower part of (4) evaluate the first- and the second-order sensitivities of displacement with respect to system parameters which are required in the minimization procedure, respectively. The sensitivities of the measured displacement are zero since the measured displacement is independent of system parameters. The gradient and the hessian of (2) for the $k$-th load case with respect to system parameters are given as follows.

$$\begin{aligned} \nabla_i \Pi^k &= \frac{1}{2}\mathbf{u}^k \cdot \mathbf{K} \cdot \mathbf{u}_{,i}^k - \mathbf{u}^k \cdot \mathbf{K} \cdot \bar{\mathbf{u}}_{,i}^k + \bar{\mathbf{u}}^k \cdot \mathbf{K} \cdot \bar{\mathbf{u}}_{,i}^k + \frac{1}{2}\bar{\mathbf{u}}^k \cdot \mathbf{K}_{,i} \cdot \bar{\mathbf{u}}^k \\ \nabla_{ij}^2 \Pi^k &= \frac{1}{2}\mathbf{u}^k \cdot \mathbf{K} \cdot \mathbf{u}_{,ij}^k - \mathbf{u}^k \cdot \mathbf{K} \cdot \bar{\mathbf{u}}_{,ij}^k + \bar{\mathbf{u}}^k \cdot \mathbf{K} \cdot \bar{\mathbf{u}}_{,ij}^k + \bar{\mathbf{u}}^k \cdot \mathbf{K}_{,j} \cdot \bar{\mathbf{u}}_{,i}^k + \bar{\mathbf{u}}_{,j}^k \cdot \mathbf{K} \cdot \bar{\mathbf{u}}_{,i}^k + \bar{\mathbf{u}}^k \cdot \mathbf{K}_{,i} \cdot \bar{\mathbf{u}}_{,j}^k \end{aligned} \tag{6}$$

where the commas in subscripts denote differentiation with respect to the system parameters.

## NUMERICAL EXAMPLE

The energy error estimator is applied to identify cross sectional areas of a bowstring truss. The truss consists of 25 members with four different cross sectional areas. The geometric configuration and actual cross sectional areas of the members of the planar bowstring truss are shown in Fig. 1. Eight displacements are measured at 6 observation nodes under 3 different static load cases. All of load cases are independent. The measured displacements and the locations of loads for each load case are presented in Fig. 2(a). The arrows in Fig 2 a) present the measured component of displacements at the observation nodes.




| Member | Area (in²) |
|---|---|
| Top | 18 |
| Bottom | 15 |
| Vertical | 10 |
| Diagonal | 12 |

Fig. 1 Geometric configuration and sectional areas of the bowstring truss

The following constraint for each system parameter is introduced in the optimization procedure.

$$0.01 \le A_i \le 5.0A_i^t \tag{7}$$

Here, $A_i^t$ is the actual value of the $i$-th system parameter. The random errors involved in the measurement are assumed to be proportional error. To simulate errors in the measurements statistically, noisy displacements are generated by adding random errors to the computed displacements of the finite element model corresponding to the observation nodes as follows.

$$(\tilde{u}_i)_m^k = (u_i)_m^k(1+\lambda\xi_i) \qquad i = 1,\cdots,mdof \tag{8}$$

where $\lambda$, $\xi_i$, and $mdof$ are the magnitude of the noise, random variable ranging from −1 to 1 and the number of measured degrees of freedom, respectively. Monte Carlo simulation is performed with the measured displacement generated by (8). A sufficient sample size is required to establish statistically meaningful estimates of system parameters in Monte Carlo simulation. In this example, the sample size of 400 trials is used

Fig. 3 and Fig. 4 show the values of the estimated mean areas of the top and bottom members, and the vertical and the diagonal members for error magnitude of 10 % by output error estimator and the energy error estimator, respectively. The estimated mean areas of the four members for the output error estimator and the energy error estimator reach steady values after 100 trials. The estimated mean areas of the top and the bottom members show similar behaviors in the mean values and the consistencies for both estimators. For the vertical and diagonal members, the mean values by the energy error estimator are more close to the actual value than by the output error estimator. Also, the amplitude of the fluctuation by the energy error estimator is smaller than by the output error estimator.

The standard deviations of identification results based on the two estimators are presented in Fig. 5 and Fig. 6 for the top and the bottom members, and the vertical and the diagonal members, respectively. For the top and the bottom members, both estimators yield very similar results. Since displacement of a truss is mainly governed by stiffness of the top and bottom members, errors in measurements have little influence on the identification results of these members, which leads to relatively small standard deviation by both estimators.

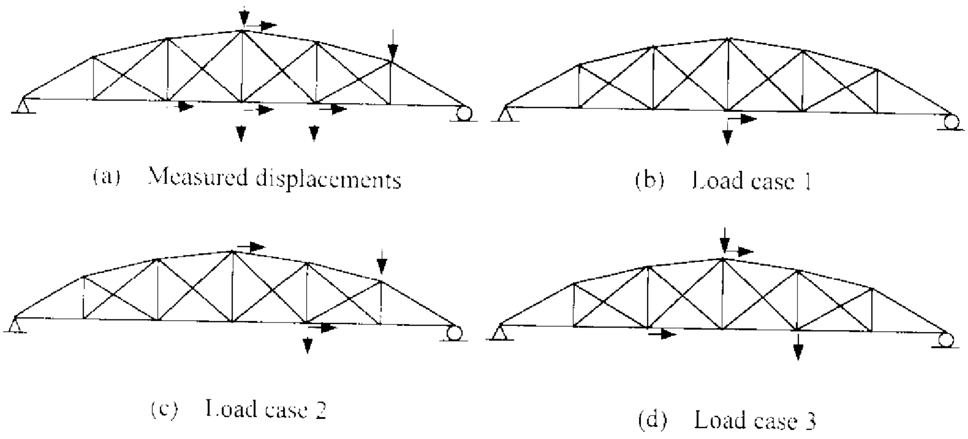



Fig. 2 Measured displacements and load cases

For the vertical and diagonal members, the energy error estimator yields much smaller standard deviation of identification results than the output error estimator as the magnitude of error in measurement becomes large. The standard deviation using the energy error estimator is bounded even for large error. This reveals that the results of the energy error estimator are more reliable than those of the output error estimator, since the consistency of the estimated value is more important than the estimated values in system identification (Shin, 1994).

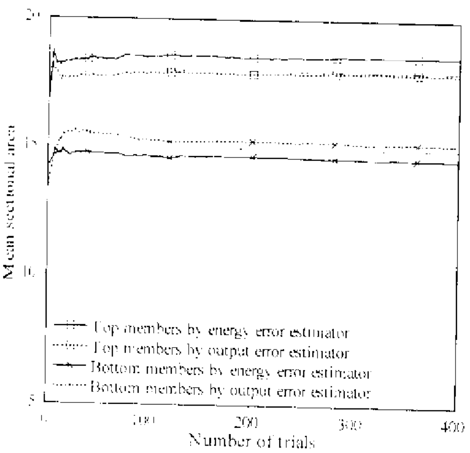



Fig. 3 Mean average values for top and bottom members

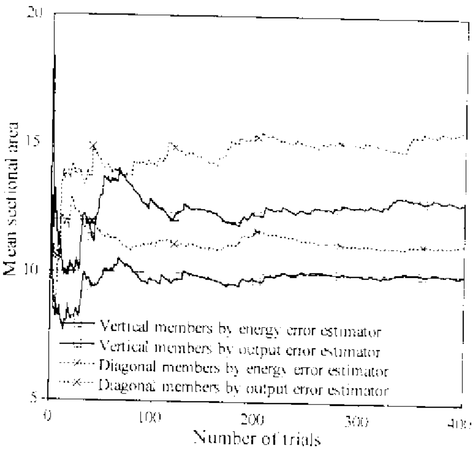



Fig. 4 Mean average values for vertical and diagonal members

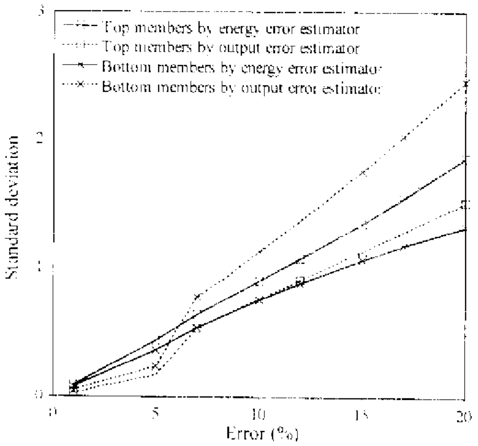



Fig. 5 Standard deviation of top and bottom elements

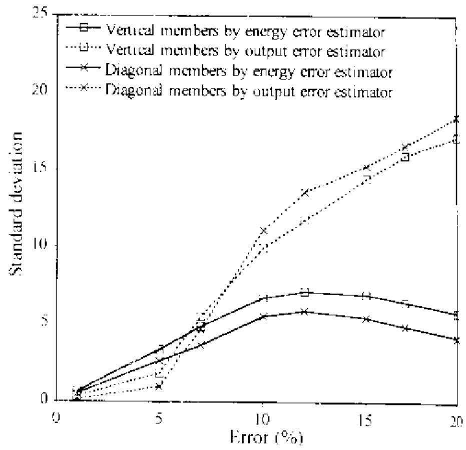



Fig. 6 Standard deviation of vertical and diagonal elements

## CONCLUSIONS

An energy error estimator for system identification of a structure is proposed. The energy error estimator is defined as the strain energy of a structure induced by displacement error between the calculated displacement by finite element model and measured displacement. Displacement at unmeasured degrees of freedom is estimated by the equilibrium equation of the finite element model.

The proposed estimator is applied to identify cross sectional areas of a bowstring truss. Monte Carlo simulation is performed for the generated noisy displacement data. The energy estimator yields more accurate and stable results than the conventional output error estimator. The energy estimator exhibits outstanding behaviors in insensitive members to displacement, which is a significant characteristic of the proposed estimator. It is believed that the energy error estimator could be a better alternative to the output error estimator for various types of identification problems of structures.

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