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A System Identification Scheme for Damage Detection in Elasto-Plastic Materials

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ABSTRACT

The paper presents a system identification scheme to detect damage in elasto-plastic solids. The proposed algorithm is based on the minimization of the least squared errors between the measured displacement field and calculated displacement field by finite element model. The constitutive relation of an elasto-plastic material is linearized by the consistent tangent moduli. The first-order sensitivity of displacement is calculated by the direct differentiation of the variational statement of the equation of motion. The recursive quadratic programming technique with Fletcher's active set algorithm is adopted for optimization.

INTRODUCTION

System identification techniques have been widely used to detect damage in structures. Most of system identification techniques available at the present time are based on the elastic behaviors of structures. However, damage such as cracks usually leads to very high stress concentration, which causes elasto-plastic response of structures. In this case, system identification technique based on elastic behaviors of structures may not yield meaningful information on locations of damage and behaviors of structures. This paper presents a system identification scheme in elasto-plastic materials for detecting damage and evaluating stress distribution in structures.

The proposed algorithm is based on the minimization of the least squared errors between the measured displacement field and calculated displacement field by finite element model. The recursive quadratic programming technique is adopted for the minimization procedure. To linearize the variational statement of an equilibrium equation, the consistent tangent moduli is used (Simo and Taylor, 1985). The first-order sensitivity of displacement is calculated by the direct differentiation of the variational statement of the equation of motion. Since the second-order sensitivity of displacement in elasto-plastic problems is very difficult to calculate, the Gauss-Newton hessian is employed.

The proposed method is applied to detect damage in a pipe subject to internal pressure. The structural damage is given by the mathematical sharp crack. Numerically simulated displacement with artificially generated errors is used as measured displacement. The proposed method yields very accurate information on the location of cracks and elasto-plastic behaviors of the structures near the cracks. The advantages of the proposed method over the conventional elastic system identification scheme are demonstrated.

OBJECT FUNCTION AND OPTIMIZATION

The parameter estimation scheme presented in this study is based on the minimization of least squared errors between measured displacement $\bar{\mathbf{u}}$ and calculated displacement by finite element model $\mathbf{u}$ at observation points as follows.

$$\text{Minmize } \Pi = \frac{1}{2}\sum_{i=1}^{nlc}(\mathbf{u}_i(\mathbf{M}) - \bar{\mathbf{u}}_i)^2 \quad \text{subject to } \mathbf{R}(\mathbf{M}) \leq 0 \tag{1}$$

where nlc, $\mathbf{M}$ and $\mathbf{R}$ are the number of loading cases, the plastic material properties such as initial yield stress and hardening constant and constraints, respectively. The Lagrangian function of the constrained optimization problem (1) becomes

$$\Pi_L = \frac{1}{2}\sum_{i=1}^{nlc}(\mathbf{u}_i(\mathbf{M}) - \bar{\mathbf{u}}_i)^2 + \lambda \cdot \mathbf{R}(\mathbf{M}) \tag{2}$$

where λ is a vector of the Lagrange multipliers.

To optimize (2), the recursive quadratic programming (RQP) technique with the Fletcher's active set algorithm is employed (Luenberger, 1989). In case that all the constraints defined in (1) are linear, the hessian of (2) becomes

$$\mathbf{H} = \sum_{i=1}^{nlc}\nabla_M \mathbf{u}_i \cdot \nabla_M \mathbf{u}_i + \sum_{i=1}^{nlc}\nabla_M^2 \mathbf{u}_i \cdot (\mathbf{u}_i - \bar{\mathbf{u}}_i) \tag{3}$$

where ∇_M is the gradient operator with respect to $\mathbf{M}$. Since it is very difficult to evaluate the second-order sensitivity of displacement in elasto-plastic problems, the Gauss-Newton hessian in which the second term of (3) is neglected is used in the optimization procedure.

INCREMENTAL FORMULATION OF VARIATIONAL STATEMENT OF EQUATION OF MOTION IN ELASTO-PLASTIC PROBLEMS

The behavior of elasto-plastic material is characterized by a yield surface, a hardening function, and a hardening parameter. For the isotropic hardening, the yield surface is defined as

$$\Psi = \phi(\sigma_{ij}) - \kappa(h) = 0 \tag{4}$$

where ϕ, κ and h are a yield function, a hardening function, and the hardening parameter, respectively. The rate of hardening parameter and the rate of plastic strain for the associated flow is defined as

$$\dot{h} = \dot{\lambda} g \tag{5}$$

$$\dot{\varepsilon}_{ij} = \dot{\lambda} \frac{\partial \phi}{\partial \sigma_{ij}} \tag{6}$$

where $\dot{\lambda}$ and g is a unknown Lagrange multiplier for plastic constraint and a known function of stress, respectively.

The consistent tangent moduli which define incremental stress-strain relationship in an elasto-plastic material is obtained by integrating (5) and (6) with the generalized midpoint rule and applying the truncated Taylor expansion of the resulting equations (Simo and Taylor, 1985).

$$\Delta\sigma_{ij} = [\Xi_{ijkl} - \frac{\Xi_{ijmn} \dfrac{\partial \underline{\phi}}{\partial \sigma_{mn}} \left(\dfrac{\partial \underline{\phi}}{\partial \sigma_{pq}} - \alpha \underline{\underline{\kappa'}} \underline{\lambda} \dfrac{\partial \underline{g}}{\partial \sigma_{pq}} \right) \Xi_{pqkl}}{\left(\dfrac{\partial \underline{\phi}}{\partial \sigma_{mn}} - \alpha \kappa' \underline{\lambda} \dfrac{\partial g}{\partial \sigma_{mn}} \right) \Xi_{mnpq} \dfrac{\partial \underline{\phi}}{\partial \sigma_{pq}} + \underline{\underline{\kappa'}} \phi}] \Delta\varepsilon_{kl} = D^{ep}_{ijkl} \Delta\varepsilon_{kl} \tag{7}$$

where repeated subscripts represent summation, and D^{ep}_{ijkl} and α are the consistent moduli and a specified integration parameter for the generalized midpoint rule within the range from 0 to 1, respectively, and

$$\Xi_{ijkl} = \left(C^{-1}_{ijkl} + \alpha \underline{\lambda} \frac{\partial^2 \underline{\phi}}{\partial \sigma_{ij} \partial \sigma_{kl}} \right)^{-1}$$

$$\underline{(\ )} = {}^{t+\Delta t}(\ )$$

$$\underline{\underline{(\ )}} = {}^{t+\alpha\Delta t}(\ )$$

The incremental form of variational statement of the equation of motion for the current analysis step is given as

$$\int_V \frac{\partial \delta u_i}{\partial X_j} D^{ep}_{ijkl} \frac{\partial \Delta u_k}{\partial X_l} dV = \int_V \delta u_i b_i dV + \int_A \delta u_i \overline{T}_i dA - \int_V \frac{\partial \delta u_i}{\partial X_j} {}^t\sigma_{ij} dV \tag{8}$$

where δu_i, Δu_i, b_i, $\overline{T}_i$ and ${}^t\sigma_{ij}$ are the virtual displacement, increment of displacement, body force, surface traction and stress and the previous analysis step, respectively.

SENSITIVITY OF DISPLACEMENT

The first-order sensitivity of displacement can be obtained by the direct differentiation of the converged variational statement of the equation of motion (8) for the current analysis step. In case that the body force and the prescribed surface traction are independent of **M**, the direct differentiation of (8) with respect to **M** yields the following expression.

$$\int_V \frac{\partial \delta u_i}{\partial X_j} D^{ep}_{ijkl} \frac{\partial \Delta u_{k,m}}{\partial X_l} dV = - \int_V \frac{\partial \delta u_i}{\partial X_j} D^{ep}_{ijkl,m} \frac{\partial \Delta u_{k,m}}{\partial X_l} dV - \int_V \frac{\partial \delta u_i}{\partial X_j}{}' \sigma_{ij,m} dV \tag{9}$$

where a comma in subscript denotes the differentiation with respect **M**. Since all field variables such as displacement, stress and strain are known after (8) is converged for current analysis step, the first-order sensitivity of displacement is easily obtained by applying the usual finite element discretization to (9). The complete derivation of sensitivity analysis for elasto-plastic materials is presented by Vidal and Haber (1993).

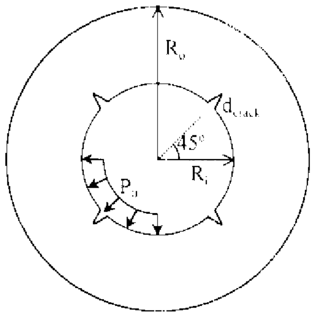
NUMERICAL EXAMPLE

A numerical simulation study is carried out to detect internal longitudinal cracks and to predict the distribution of effective stresses in a pipe under internal pressure when part of the pipe section is in the plastic zone. The Mises yield criterion is employed. Hardening phenomenon is not considered in this example. The geometry of the pipe is shown in Fig. 1(a), and the quarter of the pipe section shown in Fig.1(b) is modeled with 120 8-node isoparametric elements to simulate measured displacements. In the model, the crack locates in the middle of the quarter circle due to the geometric symmetry. In the wall of 100*mm* thick, we assume that the plastic zone is developed over 20*mm* thick from the inner surface and each crack is extended 16.67*mm* into the wall. We also assume that we measure the static displacements only at the equally spaced 21 nodes on the outer surface.

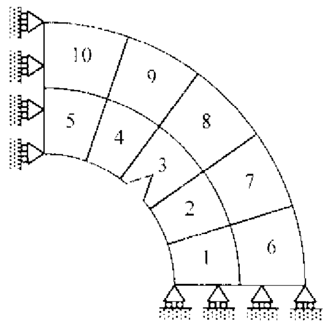
For the identification, we model the system with the same finite element mesh for the direct analysis but without crack. We collect the elements by 10 different groups depending on the location so that we estimate the group parameters rather than element parameters. In the current elasto-plastic identification, we estimate the initial yield stresses without changing the elastic modulus in each element, and then compute stresses in the structure with the estimated initial yield stress for each group of elements.

Table 1 compares the damage detection results by the elasto-plastic identification with noise-free measured data and also those by the elastic identification. From the table, we can observe that the initial yield stresses decrease in the groups near group 3 containing the actual crack but those in group 1 and 5 in the same inner layer increase instead. Also, those of the groups in the outer layer do not change because the elements in the outer layer have been in the elastic zone through the identification process. However, by the elastic identification, we can observe that the elastic moduli decrease in all the groups in the inner layer but those in the outer layer hit the upper bound of the constraint.

Fig. 2 shows the behavior of the proposed algorithm with respect to noise in measured displacements. The identification errors are computed from Monte-Carlo simulation with 50 different sets of measured data artificially generated with random noise. From the behavior of the identification, we can easily determine the robustness of the algorithm.



(a) Geometry



(b) Material group

Fig. 1 Pipe with cracks under internal pressure

From the figure, we can observe that group 2 and 4 adjacent to group 3 containing the actual crack show higher identification errors than the others.

The computed effective stresses through the wall thickness are drawn at the location of 1.9 degree rotated from the actual location of the crack in Fig. 3(a) and at 10.9 degree rotated from the location of the crack in Fig. 3(b), respectively. In each degree, the effective stresses computed from the elasto-plastic identification are compared with those from the elastic identification. Also, the distribution of effective stresses identified with noisy measured displacements are compared when applying the elasto-plastic identification. From the figure, we can observe that even with 20% noise in measurements the identified stress distributions almost coincide with those from noise-free data. From the figures, we can also observe that the results obtained from the elasto-plastic identification are much

Table 1 Estimated material properties

Group	Plastic (KPa)	Elastic (MPa)	Group	Plastic (KPa)	Elastic (MPa)
1	26.9	22.0	6	24.0	20.1
2	23.7	18.3	7	24.0	21.9
3	19.2	9.0	8	24.0	22.0
4	23.7	18.3	9	24.0	21.9
5	26.9	22.0	10	24.0	20.1

Baseline value for plastic identification : 24 KPa
Baseline value for elastic identification : 21 MPa

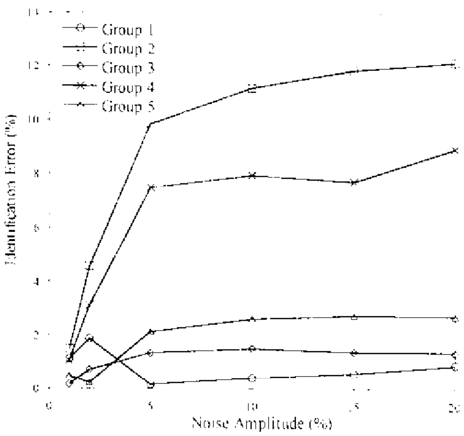
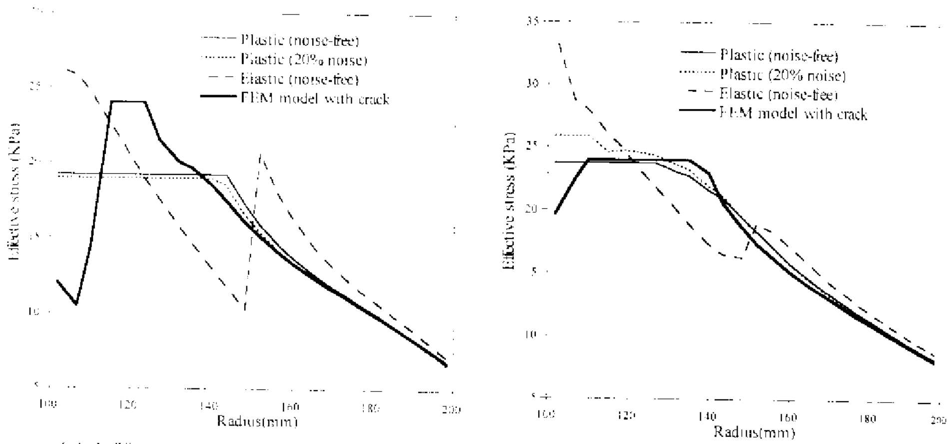


Fig. 2 Variation of identification errors



(a) 1.9° rotated from the actual crack

(b) 10.9° rotated from the actual crack

Fig. 3 Effective stress through the thickness

closer to the numerically obtained stresses through the thickness, even though some difference can be observed around the crack. However, when the elastic identification is applied, we can observe that there are unrealistic stress jumps between the inner and outer layer and the computed stresses near the inner surface are much higher than the stressed by the FEM model with crack. The unrealistic stress jump is inevitable by the elastic identification because only the elastic moduli in the groups are subjected to change.

CONCLUSIONS

The proposed system identification scheme for elasto-plastic materials yield very accurate information on the location of cracks and elasto-plastic behaviors of the structure near the crack. The advantages of the proposed method over the conventional elastic system identification scheme could be well demonstrated through the comparison of the stress distributions in the simulated examples. The robustness of the proposed algorithm with respect to noise in measurements could be also well observed through the simulated examples.

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