Reliability Analysis of Pylon for Yi Sun-sin Bridge

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Keywords: P-M interaction diagram; reinforced concrete column; reliability index; most probable failure point; first-order second-moment reliability method; cubic spline; sensitivity; direct differentiation; modified Newton-Raphson method.

1. Introduction

This paper presents the reliability assessment of the pylon for a real suspension bridge in Korea using the advanced first-order second-moment method(AFOSM)[1]. The P-M interaction diagram (PMID)[2], presenting the strength of the pylon, is defined as a limit state function. Random variables include the load and strength parameters. Because the PMID is nonlinear with respect to the random variables, the iterative procedure is needed to solve the AFOSM . In this paper, double iteration loops based on the modified Newton-Raphson method [3], are adopted to estimate the most probable failure point(MPFP) and reliability index. The sensitivities of the PMID with respect to random variables are obtained by direct differentiation. Cubic spline interpolation [4] is utilized to construct the continuous and differentiable PMID.

2. Formulations of the AFOSM for PMID

The PMID of the pylon under the axial-flexural loading is defined as an implicit function. The geometric and material properties of the cross section represent the strength parameters. Instead of internal forces, the load components such as dead load, live load and wind load are considered as load parameters. The load and strength parameters are considered as random variables. All random variables are assumed as the normal random variables and statistically independent. Nonnormal random variables are transformed to the equivalent normal random variables by the Rackwitz-Fiessler method [5]. The AFOSM is adopted to estimate the MPFP and reliability index. Because the PMID is nonlinear with respect to the random variables, the solution procedure covers iteration schemes. The modified Newton-Raphson method is utilized to solve the problem. Estimation of the MPFP using the AFOSM requires the sensitivity of the PMID with respect to the random variables. To obtain the sensitivities of the PMID, an analytic form of the PMID is needed. An explicit expression of the PMID is not defined, however, because the PMID consists of discrete points of the ultimate strength for the pylon. To construct the differentiable and continuous PMID, the discrete points on the PMID are interpolated by cubic spline method. The direct differentiation with the chain-rule is employed to calculate the sensitivity of the PMID.

3. Pylon section of suspension bridge

The reliability analysis is conducted for the pylon section of Yi Sun-sin Bridge located in the

Jeollanam-do, Korea. The total length of the suspension bridge is 2,260m. The bridge has four traffic lanes with 25.7m width. The pylon of the bridge is H-type with 270m height. The analyzed section is the bottom section. The cross section has 1,132 D32 reinforcement steels.

The reliability analyses are performed for two examples. In the first example, there are four load combinations(LC). Each of load combination is consists of the dead load and the wind load caused by 40m/s design wind speed. The second example shows two load combinations, which include the dead load and normal vehicular load with wind. The analysis results for the transverse wind load of the example 1 are presented in Fig. 1. The reliability indices for LC1 and LC2 of the circled section are 5.34 and 3.39, respectively. The detailed results of all load combinations are presented in the full version.

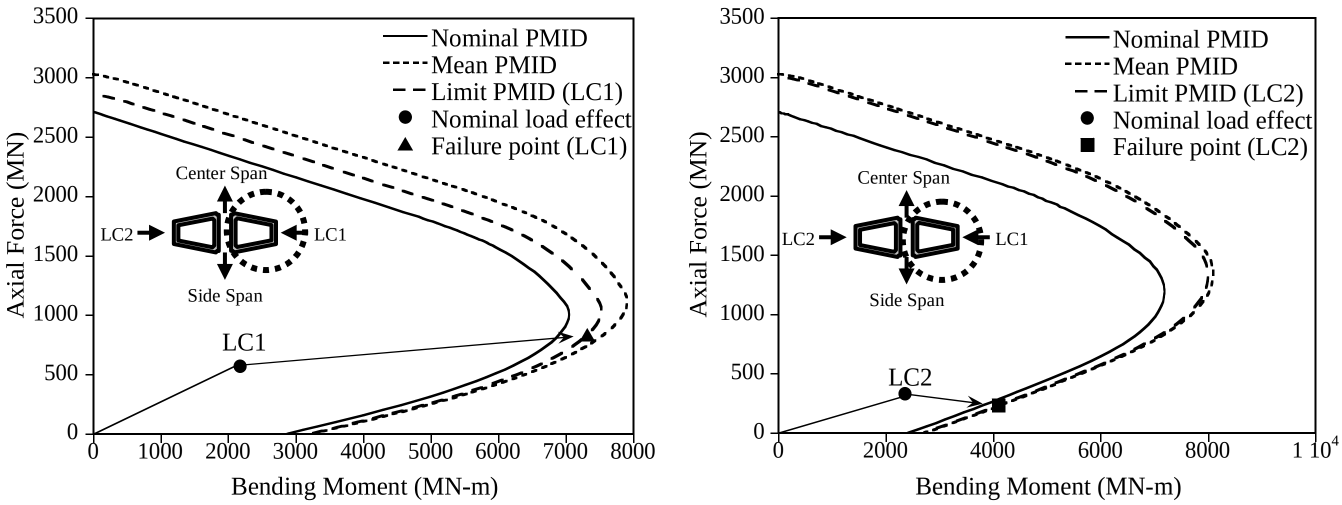

Fig. 1. Failure points and limit PMIDs of transverse wind load

4. Conclusions

The reliability indices of the pylon for Yi Sun-sin Bridge are evaluated using the AFOSM. For the load combinations of the dead and live load, the reliability indices are significantly high compare to the those of the dead and wind load. The lowest reliability index, 3.39, is obtained in case of the transverse wind load that induces tensile axial force in the section. It is concluded that the pylon sections of Yi Sun-sin Bridge provide sufficient structural safety against the wind load.

5. References

[1] Haldar A., Mahadevan S., *Probability, Reliability and Statistical Methods in Engineering Design*, John Wiley & Sons, Inc.; 2000.

[2] Nilson A.H., Darwin D., Dolan C.W., *Design of Concrete Structures*, 4th ed., McGraw-Hill; 2010.

[3] Conte S.D., Boor C.d., *Elementary Numerical Analysis And Algorithmic Approach*, 3th ed., McGraw-Hill; 1981.

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Summary

This paper presents the reliability analysis of the pylon for Yi Sun-sin Bridge. In order to evaluate the reliability index and most probable failure point(MPFP), the advanced first-order second-moment method with the double iteration loops is adopted. Random variables are load and strength parameters which are associated with the load effects and the pylon strength, respectively. The limit state function is defined by the P-M interaction diagram. The sensitivities of the limit state function are calculated by the direct differentiation of P-M interaction diagram. To construct the continuous and differentiable function, the discrete points of P-M interaction diagram are interpolated using cubic spline method. The reliability analyses of the pylon for Yi Sun-sin Bridge are conducted for the design wind load and service wind load.

Keywords: P-M interaction diagram; reinforced concrete column; reliability index; most probable failure point; first-order second-moment reliability method; cubic spline; sensitivity; direct differentiation; modified Newton-Raphson method.

1. Introduction

This paper presents the reliability assessment of the pylon for a real suspension bridge in Korea using the advanced first-order second-moment method(AFOSM)[1]. The P-M interaction diagram (PMID)[2] presenting the strength of the pylon is defined as a limit state function. Random variables consist of load and strength parameters. Instead of internal forces, each individual load components such as dead load, live load and wind load are considered as the load parameters. The geometric and material properties of the cross section compose the strength parameters. Because the PMID is nonlinear with respect to the random variables, the iterative procedure is needed to solve the AFOSM . In this paper, double iteration loops based on the modified Newton-Raphson method [3] are adopted to estimate the MPFP and reliability index. The sensitivities of the PMID with respect to the random variables are obtained by direct differentiation. Cubic spline interpolation [4] is utilized to construct the continuous and differentiable PMID.

2. Formulations of the AFOSM for PMID

The PMID of the pylon is defined as an implicit function,

$$\Phi = \Phi(\mathbf{F}, \mathbf{A}) = 0 \tag{1}$$

where $\mathbf{F} = (P, M)^T$ and $\mathbf{A}$ is the curve parameter vector determined by the geometric and material properties of the cross section of the pylon. The material properties involve the compressive strength of concrete, f_{ck}, the yield strength of the reinforcing bar, f_y, and the Young's modulus of the reinforcing bar, E_s. The geometric properties include the gross area of the cross section, A_g, the area and position of each reinforcing bar. The geometric and material properties compose the strength parameter vector $\mathbf{s}$.

When the external loads are applied on an RC column, $\Phi(\mathbf{F}_q, \mathbf{A}) > 0$ and $\Phi(\mathbf{F}_q, \mathbf{A}) < 0$ represent the safe and failure states of the pylon, respectively. Here, $\mathbf{F}_q$ is the internal force vector that represents the load effects of external load components. Therefore, the limit state of the pylon is defined by the PMID. The relations between the internal forces and the external load components are assumed as linear, $\mathbf{F}_q = (P_q, M_q)^T = \mathbf{Cq}$. Here, $\mathbf{C}$ is the load effect matrix calculated in the structural analysis and $\mathbf{q}$ is the load parameter vector. Each load parameters has nominal value of 1, and its mean becomes the bias factor of the original load component such as dead load, live load and wind load, etc. The coefficient of variation (COV) of the load parameter is that of the original load component.

The random variables which are denoted by **X** consist of the load and strength parameters, $\mathbf{X} = (\mathbf{q}, \mathbf{s})^T$. All random variables are assumed as the normal random variables and statistically independent. Nonnormal random variables are transformed to the equivalent normal random variables by the Rackwitz-Fiessler method [5].

To estimate the MPFP and reliability index, the AFOSM is adopted to solve the following optimization problem:

$$\underset{\overline{\mathbf{X}}}{\mathrm{Min}}\ \beta^2 = \left\|\overline{\mathbf{X}}\right\|_2^2 \text{ subject to } \overline{\Phi}(\overline{\mathbf{X}}) = 0 \tag{2}$$

where $\overline{\mathbf{X}}$, β and $\|\cdot\|$ are the standardized random variables of **X**, the reliability index and the 2-norm of a vector, respectively. Because the minimization problem is nonlinear with respect to the random variables, the solution procedure covers iteration schemes. In this paper, double iteration loops based on the modified Newton-Raphson method, are adopted to estimate the MPFP and reliability index. The outer iteration determines the direction of the MPFP. The inner iteration decides the location of the MPFP on the PMID. Estimation of the MPFP using the AFOSM requires the sensitivity of the PMID with respect to the random variables.

To obtain the sensitivities of the PMID, an analytic form of the PMID is needed. Because the PMID consists of discrete points of the ultimate strength for the pylon, an explicit expression of the PMID is not defined. To construct the differentiable and continuous PMID, the discrete points on the PMID are interpolated by cubic spline method. Assuming *M* as a function of *P*, the *i*-th segment of the PMID for $P_i < P < P_{i+1}$ is formulated as:

$$\Phi_i(P, M, \mathbf{A}_i) = (a_i + b_i(P - P_i) + c_i(P - P_i)^2 + d_i(P - P_i)^3) - M = 0,\ i = 1, \ldots, N_s - 1 \tag{3}$$

where $\mathbf{A}_i = (a_i, b_i, c_i, d_i)$. The union of each spline segments becomes the analytic function of the PMID.

$$\Phi(P, M, \mathbf{A}) = \bigcup_{i=1}^{N_s - 1} \Phi_i(P, M, \mathbf{A}_i) \tag{4}$$

The unknown coefficients of $\mathbf{A}_i$ are functions of the sampling points [4]. The sensitivities of the PMID are obtained by the direct differentiation of Eq. (3) with respect to random variables.

3. Pylon section of suspension bridge

The reliability analysis is conducted for the pylon section of Yi Sun-sin Bridge located in the Jeollanam-do, Korea. The bridge connects Yeosu-si and Gwangyang-si for shortening the transportation distance to Yeosu Industrial Complex. The total length of the suspension bridge is 2,260m. The bridge has four traffic lanes with 25.7m width. Fig. 1 illustrates the general view of the bridge, the geometry of the pylon and the plain view of the cross section at the bottom of the pylon. As shown in Fig.1(b), the pylon is H-type with 270m height. The analyzed section is the bottom section of the pylon in a dotted circle in Fig. 1(b). The cross section has 1,132 D32 reinforcement steels in Fig. 1(c).

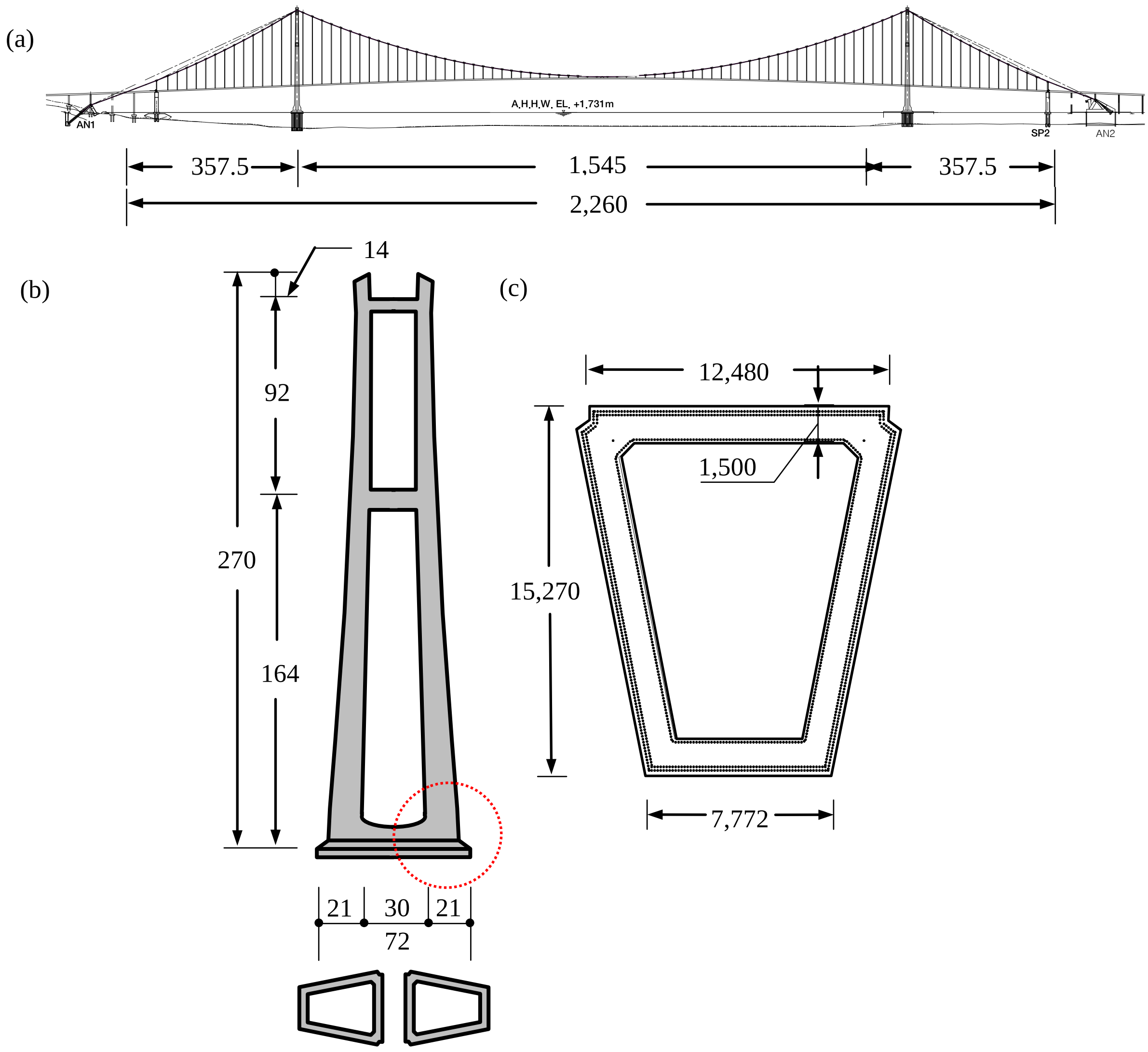

Fig. 1. Yi Sun-sin Bridge: (a) general view (unit: m); and (b) geometry of the pylon (unit: m); and (c) cross section at the bottom of the pylon (unit: mm).

The reliability analyses are performed for two examples. In the first example, there are four load combinations(LC). Each of load combination is consists of the dead load and the wind load caused by 40m/s design wind speed. The second example shows two load combinations, which include the dead load and normal vehicular load with wind.

To explain the results, several terms are defined as follows: the nominal and mean PMID represent the PMID determined by the nominal and mean value of the strength parameters, respectively. The

PMID made by the MPFP of the strength parameters, are defined as the limit PMID. The MPFP of the load parameters compose the failure point. The normalized MPFP presents the MPFP divided by its nominal value.

3.1 Dead and wind load combinations

The reliability analysis is performed for the load combinations under a strong wind condition without the live load. The design wind speed of 40 m/sec generates a uniformly distributed load 9.30 kN/m^2 over the pylon. Table 1 shows the total nominal effects and load effects induced by the load parameters. The statistical properties of random variables are given in Table 2[6-8].

The normalized MPFPs and reliability indices of the pylon are presented in Table 3. The wind load, which has the largest value of normalized MPFP, dominates the failure of the pylon. The reason is because the wind load follows the extreme type distribution and has a large COV compared to the other random variables. As shown in Table 3, the reliability index of LC1 is always higher than that of LC2. In case of transverse wind load, the axial force is large compared to the longitudinal wind load case. For that reason, the difference in the reliability indices between LC1 and LC2 of the transverse direction is larger than that of the longitudinal direction. The graphical expression of the MPFP is shown in Fig 2.

Table 1. Nominal load effects and load effect matrices of example 1

Wind load direction	LC	Nominal load effects		Load effect matrix	
				D	W
Transverse	1	P_q (MN)	572.0	446.4	125.6
		M_q (MN • m)	2174.7	329.8	-2504.5
	2	P_q (MN)	333.9	446.4	-112.5
		M_q (MN • m)	2352.8	329.8	2023.0
Longitudinal	1	P_q (MN)	451.1	446.4	4.7
		M_q (MN • m)	1255.0	0.4	-1255.4
	2	P_q (MN)	441.7	446.4	-4.7
		M_q (MN • m)	1258.6	0.4	1258.2

Table 2. Statistical properties of the random variables for examples

Random variable		Nominal value		Bias factor	COV	Distribution type
		Example 1	Example 2			
Material properties	f_{ck}	41 MPa	40 MPa	1.120	0.042	Lognormal
	f_y	420 MPa	400 MPa	1.140	0.040	Lognormal
	E_s	200 GPa	200 GPa	1.000	0.060	Lognormal
Geometric properties	e_s	0.0 mm	0.0 mm	1.000	-	Normal
	A_s	8.19×10^2 mm^2	7.94×10^2 mm^2	1.000	0.015	Normal
	A_{gt}	3.75×10^5 mm^2	6.98×10^7 mm^2	1.010	0.056	Normal
Load parameters	DC	1.00		1.050	0.100	Normal
	DW	1.00		1.000	0.250	Normal
	L	1.00		1.200	0.180	Lognormal
	W	1.00		0.875	0.200	Extreme-type I

Table 3.Reliability indices and normalized MPFPs for example 1

Wind load direction	LC	Reliability index	Normalized MPFP							
			Material properties			Geometric properties			Load parameters	
			f_{ck}	f_y	E_s	$(e_s)_{avg.}$	$(A_s)_{avg.}$	A_{gt}	D	W
Transverse	1	5.34	1.10	1.13	1.00	0.00	1.00	0.97	1.01	3.04
	2	3.39	1.12	1.13	1.00	0.00	1.00	1.01	0.99	1.85
Longitudinal	1	5.80	1.11	1.12	1.00	0.00	1.00	1.00	0.93	3.36
	2	5.67	1.11	1.12	1.00	0.00	1.00	1.00	0.94	3.26

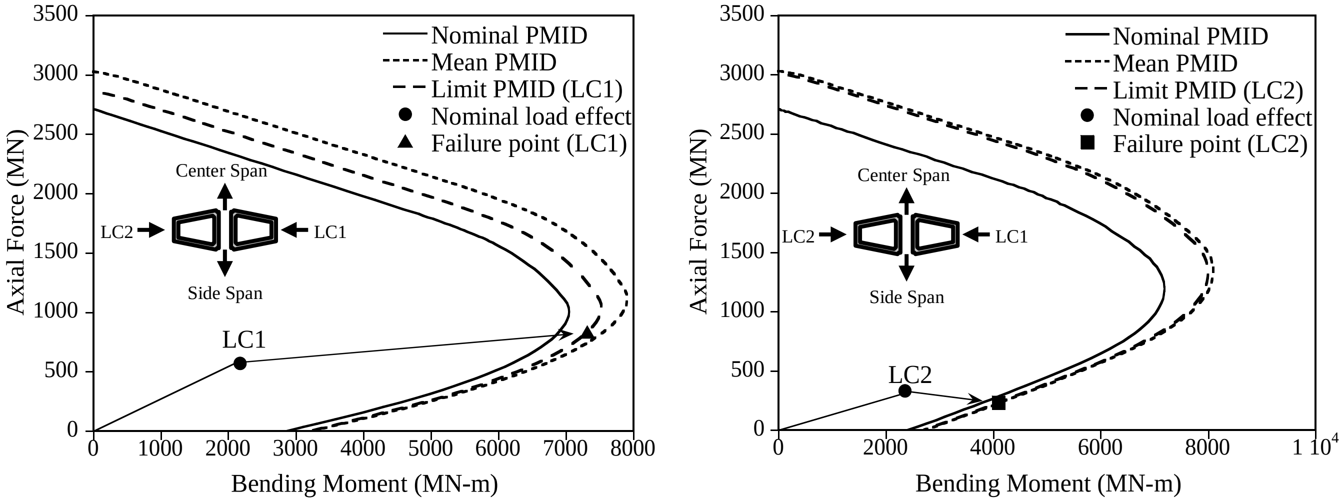

Fig. 2. Failure points and limit PMIDs of transverse wind load

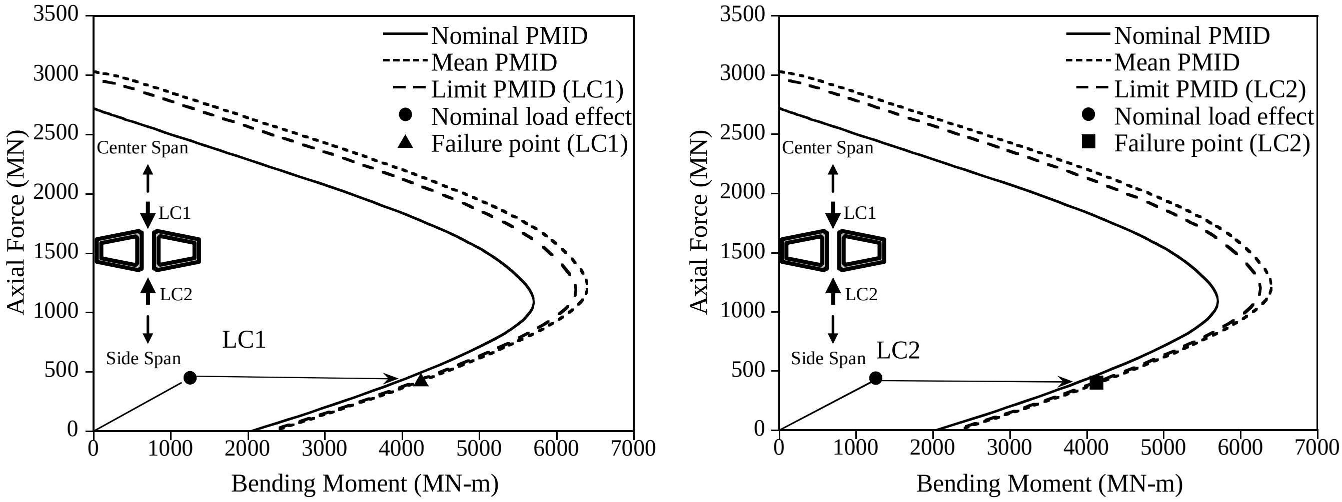

Fig. 3. Failure points and limit PMIDs of longitudinal wind load

3.2 Dead and live load combinations

The reliability indices are estimated for load combinations relating to normal vehicular load with wind. The statistical properties of random variables are already given in Table 2. Table 4 presents the total nominal effects and load effect matrices. Reliability analyses of load combinations in transverse direction exceed computable significant digits. Therefore, the results of reliability analysis are presented only for the longitudinal direction. As shown in Table 4, the load effects induced by wind load are about 1/10 of example 1. It can be expected that the reliability indices of the service wind load are higher than those of the design wind load.

Table 5 shows that the live load dominates the failure of the pylon. And the live load moves very far from the nominal effect to reach the failure state. For LC2, the moment direction of the wind load is opposite to those of the others. This is the reason why the reliability index of LC 2, 13.64, is higher than that of LC1, 11.21. The graphical results are illustrated in Fig. 4.

Table 4. Nominal load effects and load effect matrices of example 2

LC	Nominal load effects		Load effect matrix		
			D	L	W
1	P_q (MN)	476.9	446.4	13.6	16.9
	M_q (MN • m)	634.0	0.4	-514.1	-120.4
2	P_q (MN)	474.2	446.4	10.9	16.9
	M_q (MN • m)	231.6	0.4	351.6	-120.4

Table 5.Reliability indices and normalized MPFPs for example 2

LC	Reliability index	Normalized MPFP								
		Material properties			Geometric properties			Load parameters		
		f_{ck}	f_y	E_s	$(e_s)_{\text{avg.}}$	$(A_s)_{\text{avg.}}$	A_{gt}	D	L	W
1	11.21	1.10	1.09	0.99	0.00	1.00	0.97	0.73	7.87	0.87
2	13.64	1.09	1.08	0.99	0.00	1.00	0.97	0.67	12.09	0.78

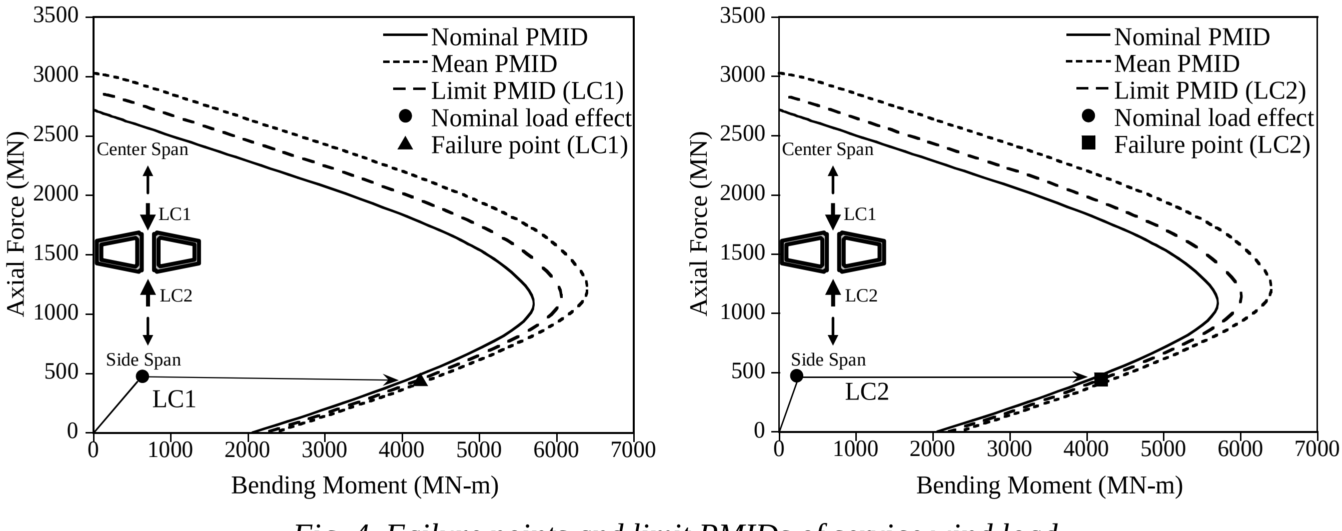

Fig. 4. Failure points and limit PMIDs of service wind load

4. Conclusions

The reliability indices of the pylon for Yi Sun-sin Bridge are evaluated using the AFOSM. For the load combinations of the dead and live load, the reliability indices are significantly high compare to those of the dead and wind load. The lowest reliability index, 3.39, is estimated in case of the transverse wind load that induces tensile axial force in the section. It is concluded that the pylon sections of Yi Sun-sin Bridge provide sufficient structural safety against the wind load.

5. Acknowledgement

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6. References

[1] Haldar A., Mahadevan S., *Probability, Reliability and Statistical Methods in Engineering Design*, John Wiley & Sons, Inc.; 2000.

[2] Nilson A.H., Darwin D., Dolan C.W., *Design of Concrete Structures*, 4th ed., McGraw-Hill; 2010.

[3] Conte S.D., Boor C.d., *Elementary Numerical Analysis And Algorithmic Approach*, 3th ed., McGraw-Hill; 1981.

[4] Kreyszig E., *Advanced Engineering Mathematics*, 9th ed., Wiley; 2006.

[5] Rackwitz R., Fiessler B., "Structural reliability under combined random load sequences", *Computers & Structures*, 1978; **9 (5)**: 489–494.

[6] Nowak A.S., Szerszen M.M., "Calibration of design code for buildings (ACI 318): Part1-Statistical models for resistance", *ACI Structural Journal*, 2003; **100 (3)**: 377-382.

[7] Nowak A.S., *Calibration of LRFD Bridge Design Code, NCHRP Report 368,* Transportation Research Board, Washington, D.C.; 1999.

[8] Nowak A.S., Yamani A.S., Tabsh S.W., "Probabilistic models for resistance of concrete bridge girders", *ACI Structural Journal*, 1994; **91(3)**: 269-276.