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Determination of Geometry of Material Interface of an Inclusion in a Finite Body by Boundary Parameterization

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ABSTRACT

A new system identification scheme based on the boundary element method is proposed to determine geometric shape and elastic material properties of an inclusion in a finite body. To deal with the shape variation of an inclusion, the boundary parameterization technique is applied to the boundary element method. The proposed algorithm is based on the minimization of a predefined error function that is the least squared errors between the measured displacement field and calculated displacement field by the boundary element model. The spatial regularization term that represents the length of the boundary curve of the material interface is added to the error function to overcome nonuniqueness of the solution.

INTRODUCTION

Various shape identification schemes have been proposed to find geometric shape and material properties of an inclusion in a finite body in detecting flaws or damage in solids. Tanaka and Yamagiwa (1989) present an inverse analysis algorithm to identify the shape of internal defect by the boundary element method using eigenfrequency data. Schnur and Zabaras (1992) determine the location and size of a circular inclusion in a finite body by the finite element method with static response. In these previous works, however, the geometric shape of an inclusion is assumed as a circle *a priori*.

Kim *et al* (1997) proposed a system identification scheme for the arbitrary shapes of inclusions by the finite element method based on the domain parameterization technique. However, since the domain parameterization is applied to the finite element method, this scheme suffers from severe mesh distortion in case that the final shape of an inclusion is far from the assumed one.

This paper presents a system identification technique based on the boundary element method and the boundary parameterization technique to determine geometric shape and elastic material properties, *i.e.*, Young's modulus and Poisson ratio, of an inclusion in a finite body without any assumption on the shape and location of an inclusion. The proposed algorithm is based on the minimization of the least squared errors between the measured displacement field and calculated displacement field by the boundary element model. The spatial regularization term that represents the length of the boundary curve of the material interface is added to the error function to overcome the nonuniqueness of the solution.

The boundary parameterization technique is utilized to manipulate the shape variation of an inclusion during optimization procedure. The sensitivity of displacement with respect to system parameters is obtained by the direct differentiation of the boundary integral equation.

The proposed algorithm is applied to problems with two different shapes of inclusions in a finite body. The numerically simulated displacement data are used as the measured data.

ERROR MINIMIZATION AND SPATIAL REGULARIZATION

Fig. 1 illustrates a two-dimensional domain V that consists of two subdomains V_1 and V_2. The boundary of the domain and the boundary of an inclusion are denoted by S_o and S_I, respectively. The elastic material properties of subdomain V_1 (matrix) are known while the elastic material properties and the shape of subdomain V_2 (inclusion) are unknown. Prescribed traction is applied on the boundary S_o. Displacements are measured at some discrete observation points on S_o that are denoted by solid circles in Fig. 1.

The system parameters – the material properties and geometric shape of an inclusion – are estimated through the minimization of least squared errors between measured displacement $\bar{\mathbf{u}}$ and calculated displacement by the boundary element model $\mathbf{u}$ at the observation points as follows.

$$\text{Minmize } \Pi = \frac{1}{2}\sum_{i=1}^{nlc}(\mathbf{u}_i(\mathbf{M},f) - \bar{\mathbf{u}}_i)^2 \quad \text{subject to } \mathbf{C}(\mathbf{M},f) \leq 0 \tag{1}$$

where nlc, $\mathbf{M}, f$ and $\mathbf{C}$ are the number of load cases, the elastic material properties of an inclusion, the boundary curve of inclusion and constraints, respectively.

In the boundary element method, the boundaries of a domain are discretized by boundary elements that are defined by a set of nodes on the boundaries and their connectivities. Since the shape of the inclusion is unknown, the coordinates of the nodes on the material interface curve which are referred to as the control nodes are also unknown. In this study, it is assumed that connectivity of the control nodes is given. With this assumption,

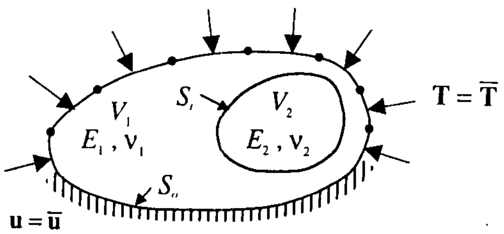


Fig. 1 Definition of a geometric parameter estimation problem

the discretized boundary curve is completely defined by the unknown coordinates of the control nodes, which are denoted by $\mathbf{X}^c$ hereafter.

Because any set of the control nodes can be a solution of (1) as long as they are on the boundary curve of the inclusion, there exist infinitely many solutions of (1). The non-uniqueness of the solution of (1) results in numerical difficulties such as slow convergence, severe mesh distortion and mesh overlap in the optimization procedure. To stabilize the optimization procedure and accelerate convergence rate, a spatial regularization function which represents the length of the boundary of the inclusion is introduced. The spatial regularization term is defined as

$$R(\mathbf{X}^c) = \alpha\frac{1}{2}\int_{S_I}(dx^2 + dy^2)dS \approx \alpha\frac{1}{2}\sum_{k=1}^{nc}(\mathbf{X}_k^c - \mathbf{X}_{k-1}^c)^2 \tag{2}$$

where α and nc are the regularization parameter and the number of control nodes respectively. The regularization parameter α is selected to be small enough so that the regularization function does not have dominant effect on optimization procedure, but large enough to stabilize the optimization procedure.

The discretized form of the Lagrangian function of (1) with the regularization term becomes

$$\Pi_I = \sum_{i=1}^{nlc}(\mathbf{u}_i(\mathbf{D}) - \bar{\mathbf{u}}_i)^2 + R(\mathbf{X}^c) + \boldsymbol{\lambda}\cdot\mathbf{C}(\mathbf{D}) \tag{3}$$

where $\mathbf{D} = (\mathbf{M}, \mathbf{X}^c)$ denotes design variables, and $\boldsymbol{\lambda}$ is a vector of the Lagrange multipliers. The modified Levenberg-Marquardt method with Fletcher's active set algorithm (Luenberger, 1989) is adopted to optimize (3).

BOUNDARY PARAMETERIZATION AND SENSITIVITY

In the boundary parameterization technique, a fixed reference configuration, Ω, is introduced in addition to the material configuration, V, which varies with optimization iteration as shown in Fig. 2. A material particle on the boundary in the material configuration associated with a fixed reference coordinate varies as the shape of the inclusion changes. The boundary curve of the material configuration is defined by the coordinates of the control nodes and the curve parameter of the boundary curves in the reference configuration as follows.

$$\mathbf{X} = \Psi(\xi, \mathbf{X}^c) \tag{4}$$

where ξ is the curve parameter of the boundary curve in the reference configuration. The differential area of boundary curves of the material and reference configurations are related by

$$dS = \tilde{J}d\Gamma \tag{5}$$

where $\tilde{J}$ and Γ are an area metric and the boundary of the reference configuration, respectively.

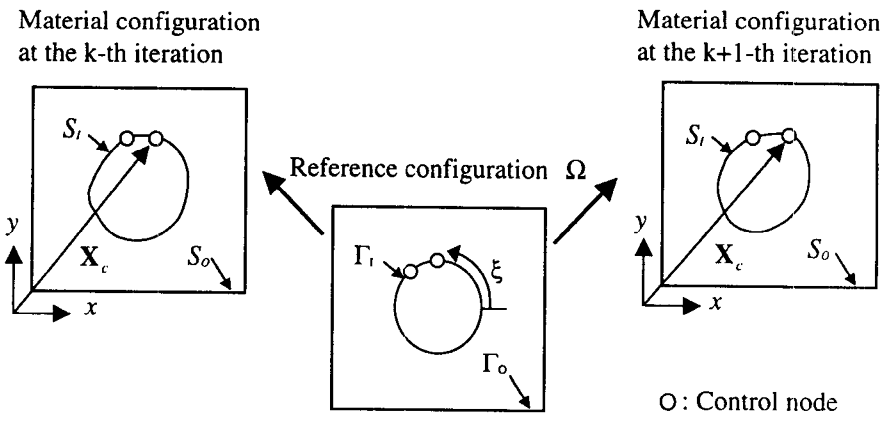


Fig. 2 Boundary variation and mapping

The boundary integral equation of an elastostatic problem in the absence of the body force is given as

$$c^i_{lk}u^i_k + \int_S p^*_{lk}u_k dS = \int_S u^*_{lk}p_k dS \tag{6}$$

where p^*_{lk} and u^*_{lk} are k components of traction and displacement due to a unit point load in the l direction in an infinite domain, respectively. Since all the geometry related terms in (6) can be expressed in terms of the curve parameter and the coordinates of the control nodes by (4) and (5), the boundary integral equation corresponding (6) is expressed in terms of the reference geometry.

$$c^i_{lk}u^i_k + \int_\Gamma p^*_{lk}u_k\tilde{J}d\Gamma = \int_\Gamma u^*_{lk}p_k\tilde{J}d\Gamma \tag{7}$$

where, c^i_{lk}, p^*_{lk} and u^*_{lk} in (8) are defined in the reference coordinate system.

The sensitivities of displacements with respect to the system parameters are obtained by direct differentiation of equation (7).

$$\begin{aligned} &c^i_{lk,m}u^i_k + c^i_{lk}u^i_{k,m} + \int_\Gamma p^*_{lk,m}u_k\tilde{J}d\Gamma + \int_\Gamma p^*_{lk}u_{k,m}\tilde{J}d\Gamma + \int_\Gamma p^*_{lk}u_k\tilde{J}_{,m}d\Gamma - \\ &\int_\Gamma u^*_{lk,m}p_k\tilde{J}d\Gamma - \int_\Gamma u^*_{lk}p_{k,m}\tilde{J}d\Gamma - \int_\Gamma u^*_{lk}p_k\tilde{J}_{,m}d\Gamma = 0 \end{aligned} \tag{8}$$

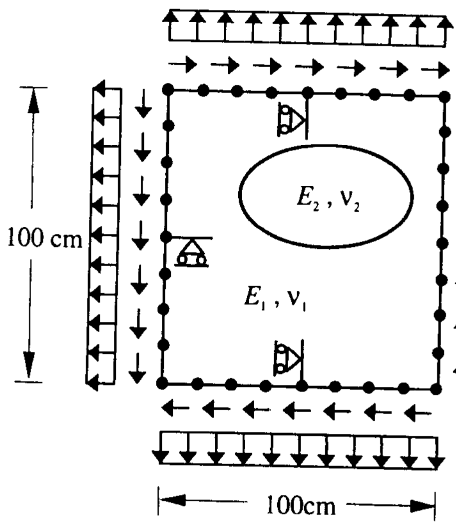
Here, the comma in subscripts denotes the differentiation with respect to the design variables. The differentiation of c^i_{lk} with respect to the system parameters is evaluated by the rigid body consideration that is generally used in the boundary element method.

NUMERICAL EXAMPLES

The proposed method is applied to determine the shapes and the material properties of inclusions in a 1m by 1m square plate in a plane stress state. Two different shapes of inclusions are tested; 1) 28cm rhombus 2) 40cm×20cm ellipse. For the example 1, the center of the rhombus coincides with that of the plate. The center of ellipse is off-centered from that of the plate by 20 cm by 25 cm for the example 2. It is assumed that the elastic material properties of the matrix material are known. The geometry, material properties and boundary conditions for the examples are illustrated in Fig. 3. The material properties for the inclusions and the matrix are representative of steel and naval brass, respectively. Three different types of traction are applied independently on the boundaries of the matrix, and the displacements are measured at equally spaced 32 nodes on the boundary as depicted by solid circles in Fig. 3. for each load case. For the measured displacement, numerically simulated data by the boundary element model with exact shapes of the inclusions are used.

Fig. 4(a) shows the initial mesh layout in which the shape of the inclusion is assumed as a circle of radius 30 cm for all examples. The boundary element mesh contains 48 elements (16 elements on the boundary of the inclusion) and 96 nodes. The displacement field is modeled by quadratic shape functions, while linear shape functions are used for geometric mapping. Open circles in Fig. 4(a) indicate the initial positions of the control nodes. The initial values of material properties for the inclusion material are 150 GPa and 0.32. To keep the inclusions within domain, geometric constraints are imposed on each control node.

The estimated material properties of the inclusions and the final mesh layouts for the examples are shown in Fig. 4(b) and Fig. 4(c). The final positions of the control nodes and the exact shapes of the inclusions are depicted by open circles and thick solid lines, respectively.



•Load Case

Case 1: q_x = 1.8GPa

Case 2: q_x = 1.8GPa

Case 3: τ_n = 1.8GPa

•Material property

1) Matrix - Steel

$E_1 = 210$ GPa

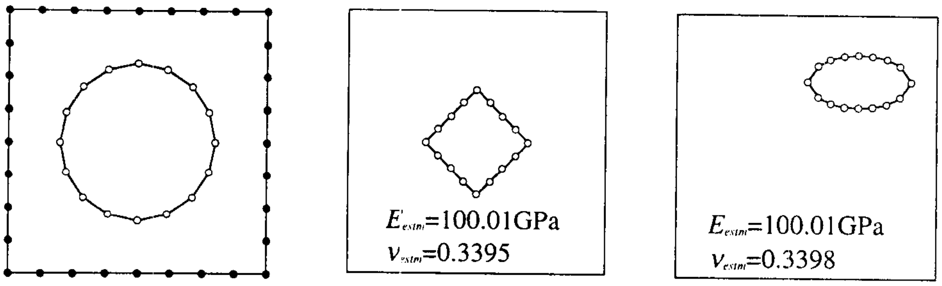
$\nu_1 = 0.3$

2) Inclusion - Naval brass

$E_2 = 100$ GPa

$\nu_2 = 0.34$

Fig. 3 Geometry, material properties and boundary conditions for the examples



a) Initial mesh layout

b) Final mesh layout for example 1

c) Final mesh layout for example 2

Fig. 4 Final mesh layouts and Initial mesh layout for the examples

The proposed method identifies the shapes and the material properties of inclusions very accurately. The numbers of iterations required to converge to accuracy of 10^{-10} order in the object function are 12 and 34 for the example 1 and 2, respectively.

CONCLUSION

The geometric shape and elastic material properties of an inclusion in a finite body are identified without any assumption on the shape and location of an inclusion from static response of a structure. The proposed method utilizes the boundary parameterization technique to manipulate the variation of a domain. The unknown boundary of the inclusion is discretized by control nodes and their connectivity. To stabilize the optimization procedure, the spatial regularization function is introduced. The sensitivity of displacement is calculated by the direct differentiation method.

The validity of proposed method is demonstrated by two examples. The geomerties and the material properties of the inclusions estimated by the proposed method show excellent agreement with the exact solutions. The proposed system identification scheme could provide a stable and accurate numerical scheme in detecting flaws and damage in solids.

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