Structural Damage Detection Using Modal Data with Regularization Technique

J. S. Kang, I. H. Yeo, H. S. Lee, S. B. Shin

Abstract : This paper proposes an improved damage detection and assessment algorithm based on the system identification. In this algorithm, the regularization technique is introduced to overcome ill-posedness of the inverse problem in the conventional algorithm. Frobenius norm for the change of the stiffness matrix of a structure is used as the regularization function. VRFS is employed to determine a regularization factor. Although measured information suffers from sparseness and noise, reliable damage detection and assessment can be carried out by this algorithm. In this algorithm, measuring responses by both static and dynamic test can be used, however current paper introduces only the case where the modal data are used as the measuring responses. The sensitivity of the normalized mode shape vector by an arbitrary matrix is proposed. The validity of the proposed algorithm is demonstrated by a numerical example.

Key word : ill-posedness, regularization, VRFS, damage detection, mode shape

Introduction

Structural damage often causes a loss of stiffness in one or more elements of a structure that affects its dynamic behavior such as modal frequencies and mode shapes. Many methods have been developed to detect the location and severity of damage based on these changes. Adams [Adams, 1978] developed a method to locate damage in which the ratio of the frequency change in any two modes was proved to be a function of the damage location only. Lim and Kashangaki [Lim and Kashangaki, 1994] developed the damage detection algorithm by computing Euclidian norm between measured mode shape and best achievable eigenvectors.

Kaouk and Zimmerman [Kaouk and Zimmerman, 1994] proposed computationally attractive algorithms to determine the location and extent of structural damage assuming damage results in a localized change in stiffness properties, and the extent of damage was determined using a minimum rank updating theory. Cobb and Liebst [Cobb and Liebst, 1997] presented damage identification algorithm with cost-function minimization based on an assigned partial eigenstructure algorithm in which the physical properties of the structural elements are treated as control variables, which are chosen to achieve the measured partial eigenstructure.

In this study, system identification (SI) based on the minimization of least square errors between measured mode shape vector and calculated mode shape vector is employed. Since least square error function defined in this study is nonlinear with respect to system parameters, minimization of least square error function is nonlinear optimization problem. Material properties of the structural members, such as flexural stiffness or axial stiffness are assumed as the system parameters.

To solve nonlinear optimization problem, the recursive quadratic programming (RQP) and the Fletcher active set strategy are employed [Banan and Hjemlstad, 1993]. In RQP, sensitivity of calculated mode shape vector with respect to the system parameters is required. Current algorithms to calculate sensitivity of mode shape vector, such as the modal method, the modified modal method, and the Nelson's method [Nelson, 1976] are valid only when the mode shape vector is normalized with respect to mass matrix. Unless fully measured mode shape vector is available due to economic or physical restriction, normalization of both measured mode shape vector and calculated mode shape vector cannot be done. In this study, algorithm to calculate sensitivity of mode shape vector which is normalized with respect to an arbitrary matrix is developed.

Because of economic and physical restriction, we can not get enough information generally. It is necessary to reduce the number of unknown system parameters instead of increasing the number of measured degrees of freedom. Parameter grouping technique [Hjelmstad *et al.*, 1990] is employed to reduce the number of system parameters efficiently.

It is known that SI is typically ill-posed inverse problem which suffers from severe numerical instabilities, such as non-existence, non-uniqueness, and discontinuity of solution. Regularization technique proposed by Tikhonov[Groetsch, 1984;Bui, 1994] is adopted to overcome such numerical instabilities. As a regularization function, Frobenius norm which is difference between an initial stiffness matrix and an estimated stiffness matrix is used. A regularization factor plays the most important role for estimation of both numerically and physically meaningful solution[Groetsch 1984; Bui, 1994]. VRFS proposed by Lee et al[Lee et al., 1999] is used to determine an appropriate regularization factor.

When the system parameters estimated from SI are compared with those of baseline value which represents the undamaged state, structural members of which the estimated system parameters reduce significantly are often considered as damaged ones. The severity of the damage is represented as the ratio between the estimated system parameters and the baseline value.

However, when the measurement data are polluted with noise, it is very difficult to distinguish whether the damage is caused either by real damage or by noise in measurement. Since measurement noise is inevitable in the real situation, the estimated system parameters from SI may be easily meaningless in the damage detection and assessment. To overcome the drawbacks, data perturbation scheme proposed by Hjelmstad and Shin [Hjemstad and Shin, 1997] and statistical approach proposed by Yeo [Yeo, 1999] are incorporated with SI for damage detection and assessment.

To demonstrate the validity of the proposed method, one numerical example is presented.

Output error estimator using modal data

In this study damage is defined as the reduction of a system parameters from its baseline value which is assumed as a priori knowledge. System parameters are estimated by the output error estimator. In the output error estimator static responses and/or dynamic responses can be used a posteriori information. The modal data of a structure can be evaluated easily through various dynamic tests. In this study the parameter estimation algorithm using modal data is proposed. The output error for the modal response of a structure is defined as follows :

$$\mathbf{e}_i(\mathbf{x}) = \boldsymbol{\phi}_i - \hat{\boldsymbol{\phi}}_i \qquad i = 1, \ldots, nmd \tag{1}$$

where $\mathbf{x}$, $\boldsymbol{\phi}_i$, $\hat{\boldsymbol{\phi}}_i$, and *nmd* are system parameter vector, calculated mode shape vector of *i*-th mode, measured mode shape vector of *i*-th mode, and the number of the measured modes, respectively.

The unknown parameters are obtained by the minimization of an objective function formulated by the output error vector defined by Eq. (1). Therefore, the parameter estimator becomes a constrained nonlinear optimization problem like Eq. (2)

$$\underset{\mathbf{x}}{Minimize} \quad \Pi(\mathbf{x}) = \frac{1}{2}\sum_{i=1}^{nmd} \alpha_i \left\| \boldsymbol{\phi}_i - \hat{\boldsymbol{\phi}}_i \right\|^2 \qquad \text{subject to} \qquad \mathbf{R}(\mathbf{x}) \geq 0 \tag{2}$$

where $\mathbf{R}(\mathbf{x})$ is the upper or lower constraints of system parameters, and α_i is the weight factor for the *i*-th mode. To solve the constrained nonlinear optimization problem expressed by Eq. (2), the recursive quadratic programming(RQP) and the Fletcher active set strategy are employed [Bannan and Hjemlstad, 1993].

On RQP we need the sensitivity of the error function for system parameters. The sensitivity is shown in Eq. (3)

$$\Pi_{,x} = \sum_{i=1}^{nmd} \alpha_i \left\| \boldsymbol{\phi}_i - \hat{\boldsymbol{\phi}}_i \right\| \cdot \boldsymbol{\phi}_{i,x} \tag{3}$$

where the subscript $(\,),_x$ denotes the partial derivative with respect to a system parameter, x. In case where the mode shape vector is normalized by mass matrix of the structural system, several methods to calculate the sensitivity are proposed, such as the modal method, the modified modal method, and the Nelson's method [Nelson, 1976]. When the mode shape vector is normalized by mass matrix, the sensitivity matrix of the vector is as follows :

$$\boldsymbol{\phi}_{j,x} = -\sum_{i \neq j}^{nmd} \frac{\boldsymbol{\phi}_i^T \mathbf{K},_x \boldsymbol{\phi}_j}{(\lambda_i - \lambda_j)\boldsymbol{\phi}_i^T \mathbf{M} \boldsymbol{\phi}_i} \boldsymbol{\phi}_i \qquad (\ i \neq j\ ) \tag{4}$$

where λ, **M**, and $\mathbf{K},_x$ denote eigen value, the mass matrix, and the sensitivity matrix of the stiffness matrix of a structural system with respect to a system parameter, respectively. However, in the system identification where the partially measured mode shape is used to determine design variables we can not use the sensitivity of the normalized mode shape by the mass matrix.

If we assume that $\bar{\boldsymbol{\phi}}$ is normalized by any matrix **C**, then we can express its sensitivity $\bar{\boldsymbol{\phi}},_x$ as follows :

$$\bar{\boldsymbol{\phi}}_{j,x} = \frac{1}{\boldsymbol{\phi}_i^T \mathbf{C} \boldsymbol{\phi}_i} \left(\boldsymbol{\phi}_{j,x} \sqrt{\boldsymbol{\phi}_i^T \mathbf{C} \boldsymbol{\phi}_i} - \boldsymbol{\phi}_j \frac{\boldsymbol{\phi}_i^T \mathbf{C} \boldsymbol{\phi}_{i,x}}{\sqrt{\boldsymbol{\phi}_i^T \mathbf{C} \boldsymbol{\phi}_i}} \right) \tag{5}$$

If we substitute m_c for $\sqrt{\boldsymbol{\phi}_i^T \mathbf{C} \boldsymbol{\phi}_i}$, the Eq. (5) can be represented as the following equation.

$$\bar{\boldsymbol{\phi}}_{i,x} = \frac{1}{m_c} \boldsymbol{\phi}_{i,x} - \frac{1}{m_c^3} \left(\boldsymbol{\phi}_i^T \mathbf{C} \boldsymbol{\phi}_{i,x} \right) \boldsymbol{\phi}_i \tag{6}$$

Parameter grouping scheme

The first problem in parameter estimation is that the quantity of the known information(No. of measured dofs) is insufficient when compared with the number of design variables. Because of economic and physical restriction, we can not get enough information generally. Although the amount of the known information is sufficient, to regard all parameters as variables is not economic because of the time consumed in evaluation. Furthermore, it may yield unstable solutions in cases when the measuring error is included in the measured data. Therefore it is necessary to reduce the number of unknown system parameters instead of increasing the number of measured degrees of freedom. We can reduce the number of parameters using grouping similar system parameters together without changing the finite element model itself. The grouping scheme may be useful for civil structures because most of them are composed of only a few different types of member properties.

When the parameter grouping scheme is applied, the number of total unknown system parameters n_p can be computed by Eq. (7)

$$n_p = \sum_{i=1}^{n_g} nelp_i \qquad (7)$$

where n_g and $nelp_i$ denote the number of groups and the number of constitutive parameter types in the *i*-th group, respectively.

A similar idea reducing the number of unknowns by decomposition is the substructure approach [Lim, 1990; Natke, 1989]. The substructure technique is popular in the analysis of very large finite element systems. Because of some drawbacks of substructure modeling, Hjelmstad [Hjelmstad *et al.*, 1990] concluded that the parameter grouping scheme makes more physical sense.

Simulation of measured data

The behavior of a parameter estimation procedure depends on the accuracy of the mathematical model and the measured data. Because the modeling error can be reflected as the change in constitutive system parameters like the effect of the damage, to build a correct mathematical model is not the main issue in this study. Therefore, the modeling error is excluded completely in this study. Thus, the assumed mathematical model is an exact representation of the “real” structure. The only measured error is considered as noise. Whenever some aspect of a given problem has a random nature, the solution of the problem is a random variable. In a parameter estimation problem, it can be regarded that the measurements have a random noise, therefore the estimated system parameters are random variables, too. Shin [Shin, 1994] employed Monte Carlo simulation as a useful statistical method that considers sparseness and noise in measured information. Monte Carlo simulation uses a random sequence of numbers to change the values of the particular random aspects of the problem to construct a sample of the solution population. The statistics of the same population are easily computed and they provide an estimate of the statistical properties of the random solution. As the size of the sample population increases, the reliability of the estimated statistics also increases. Monte Carlo simulation is used to generate measured modal data, not in real measurement but in numerical method.

In this study, the system parameters of a finite element model are estimated by solving a constrained nonlinear optimization problem expressed in Eq. (2). For a given finite element model, the unknown variables $\mathbf{x}$ are functions only of the response vector $\hat{\boldsymbol{\phi}}$ measured at a few degrees of freedom. These response values are polluted with measuring error. In the simulation environment the noisy response $\hat{\boldsymbol{\phi}}$ is generated by adding a measurement error to the computed response $\hat{\boldsymbol{\phi}}_0$ at the selected degrees of freedom of the given finite element model. The simulated measured mode vector is expressed as follows:

$$\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}_0 \{1 + \bar{\boldsymbol{\varepsilon}} \Re(-1,1)\} \tag{8}$$

where $\bar{\boldsymbol{\varepsilon}}$ is the known percentage fraction of the calculated displacement. True measurement errors may lie somewhere between the bounds of the absolute and the proportional errors. Generally, both error types produce similar trends in the variations of identification errors. For the current study, only the proportional error type has been applied, but the results with absolute errors are expected not to be much different.

Regularization technique

The parameter estimation with the conventional output error estimator is typically ill-posed inverse problem. Ill-posed problems suffer from three instabilities: nonexistence of solution, non-uniqueness of solution and/or discontinuity of solution when measured data is polluted by noise [Bui, 1994]. This means that the results of the optimization problem may be meaningless solutions or diverge in the optimization process. So far, several authors have attempted to overcome an instability problem, merely by imposing upper and lower limits on the constitutive system parameters in the conventional output error estimator. However, [Neuman, 1979] and [Hjelmstad, 1996] have shown that this measure alone is not sufficient to guarantee a meaningful solution.

In this study, regularization technique is employed to overcome such instabilities of the conventional output error estimator. Regularization technique has been applied successfully to various inverse problems[Becks and Murio, 1984; Lee et al., 1999; Neuman and Yakowitz, 1979; schnur and Zabaras, 1990]. In the regularization technique, the possible solution space of the minimization problem can be reduced by adding a continuous regularization function to the object function of Eq.(2). Various regularization functions are used for different types of inverse problems. The following regularization function is adopted for the current identification of a structure.

$$\Pi_R = \frac{\beta}{2} \left\| \mathbf{K}(\mathbf{x}) - \mathbf{K}(\mathbf{x}_0) \right\|_F^2 \tag{9}$$

where β and $\mathbf{x}_0$ denote the regularization factor and the system parameters representing baseline properties of a structure, respectively, and $\|\cdot\|_F$ is the Frobenius norm of a matrix. By adding the regularization function to the error function, the regularized system identification problem is defined as follows :

$$\underset{\mathbf{x}}{\text{Minimize}} \; : \quad \Pi = \frac{1}{2}\sum_{i=1}^{nmd} \alpha_i \left\| \boldsymbol{\phi}_i(\mathbf{x}) - \hat{\boldsymbol{\phi}}_i \right\|^2 + \frac{\beta}{2}\left\| \mathbf{K}(\mathbf{x}) - \mathbf{K}(\mathbf{x}_0) \right\|_F^2 \quad \text{subject to } \mathbf{R}(\mathbf{x}) \le 0 \tag{10}$$

We call the error estimator defined in Eq. (10) regularized output error estimator (ROEE). The regularization function enhances the uniqueness and continuity of the system identification problems by preventing the system parameters from arbitrary changes since the regularization function has a minimum value at the baseline properties of the system parameters. Since the solution space of the system identification problem is restricted by the regularization function during optimization, the upper bounds of system parameters do not have to be set near the baseline properties of the system parameters. As the minimization problem defined by the ROEE searches for a set of system parameters that are nearest to the baseline values among infinitely many possible solutions, the identification results can be statistically distributed around the baseline properties.

The regularization effect in parameter estimation process is determined by the regularization factor. The regularization effect vanishes for a small regularization factor while the regularization function has a dominant effect over the error function during the optimization process for a large regularization factor. In either case, the optimization problem Eq. (10) is unable to estimate optimal system parameters due to instabilities or too dominant regularization effects on the system parameters. Therefore, selection of a proper regularization factor is very crucial to obtain meaningful solutions of system identification problems.

Some rigorous methods to find an optimal regularization factor have been proposed for linear inverse problems, in which the error function given by Eq. (10) is expressed as a quadratic form of the system parameters. These methods include the L-curve method [Hansen, 1992], the cross validation method [Golub et al, 1978] and a statistical approach based on the Bayesian theory [Maniatty, 1994]. Only one optimal regularization factor has to be defined in quadratic optimization problems since the solutions of quadratic optimization problems are obtained directly without iterative procedure. However, as solution of nonlinear optimization problems are obtained by employing an iterative procedure, in which quadratic subproblems are solved successively, an optimal regularization factor should be defined in each iteration step. Recently, Lee proposed an effective method, referred to as the variable regularization factor scheme (VRFS)[Lee et al., 1999], to determine the regularization factor in nonlinear inverse problems. In the VRFS, the regularization factor is defined so that the error function is always lager than the regularization function during the optimization process.

$$\sum_{i=1}^{nmd} \left\| \phi_i(\mathbf{x}^k) - \hat{\phi}_i \right\|^2 \geq \beta^k \left\| \mathbf{K}(\mathbf{x}^k) - \mathbf{K}(\mathbf{x}_0) \right\|_F^2 \tag{11}$$

where the superscript k denotes the iteration count for optimization process. In case the solution of the k-th iteration yields error function smaller than the regularization function the regularization factor is reduced by multiplying a predefined reduction factor, γ, ranging from 0 to 1 at the next iteration.

$$\beta^{k+1} = \gamma \beta^k \tag{12}$$

Lee *et al.* show that the magnitude of the reduction factor has little effect on the solution of the identification problems Eq. (10) for moderate reduction factor. The optimization of Eq. (10) is performed by the recursive quadratic programming, in which quadratic subproblems are solved successively [Banan et al., 1994].

By using the ROEE, the existence of the unique solution of the optimization problem is guaranteed without the upper constraints of the constitutive system parameters. Therefore it is not necessary to establish upper limits which restrict convergence space of the constitutive system parameters. This effect of the regularized estimator may improve damage detectability on damage detection algorithm and it will provide a good damage index. Merely, it needs to restrict lower limit of the constitutive system parameters so that the Hessian matrix of the object function may not become singular. Lower limits are selected as 0.01 in this study.

Measured data perturbation scheme

If a sufficient number of measured data sets are available for the same measurement condition, the effect of measurement noise on identification results may be reduced by averaging the measured data. In real situations, however, only limited sets of noisy measurement data are available, and noise characteristics such as its distribution and average in measurements are not known. Therefore, it is usually difficult to determine whether the changes of the system parameters are caused by measurement noise or by actual damage.

The data perturbation method, which has been proposed by Hjelmstad and Shin [Hjelmstad and Shin, 1997] for a numerical simulation study, is employed. In the data perturbation method, a series of the system identification is performed with generated data sets around a given set of measured displacements by perturbing the given data with a small magnitude. As a result, the identified system parameters are interpreted statistically with their distributions.

To obtain samples of a system parameter for its statistical distribution, the measured data perturbation iteration is performed with the following perturbed measurement data.

$$(\bar{u}_j)_i^k = (\bar{u}_j)_i (1 + \eta_j^k) \tag{13}$$

where $(\bar{u}_j)_i^k$ and η_j^k are a perturbed displacement for load case i and a random number, respectively, for the j-th component of the measured displacement at the k-th iteration. The random number is generated independently for each component of measured displacement and each load case from a uniform probability density function between ± maximum perturbation amplitude, $\Im_{\max}$. In the present study, the maximum perturbation amplitude is taken as an average distance from the unperturbed, measured displacements to the nearest compatible displacements for all load cases [Yeo, 1999].

$$\Im_{\max} = \frac{1}{nlc} \sum_{i=1}^{nlc} \frac{\left\| \tilde{\mathbf{u}}_i(\mathbf{x}_s) - \bar{\mathbf{u}}_i \right\|^2}{\left\| \bar{\mathbf{u}}_i \right\|^2} \tag{14}$$

where x_s is the solution of the minimization problem Eq. (10) for unperturbed measured displacements. As noise in measurements becomes smaller, the maximum perturbation magnitude decreases, and more reliable estimation of the system parameters is possible. The degree of continuity of the ROEE around the given measurement data can be represented by the standard deviation of the system parameters obtained by the Monte-Carlo simulation.

Although the number of perturbations is implemented, the perturbed compatible displacement vector set is affected by the initial measured displacement vector. Therefore, it must be noted that estimation of perfectly accurate system parameters with noisy measured data is impossible.

Damage severity

Material properties of the current structure are estimated by m times Monte Carlo trials, n times measured data perturbation iterations, and the parameter grouping scheme. After mean and standard deviation are evaluated using the measured data perturbation scheme, it is the next mission to determine which member is damaged. Because baseline value of the system parameter is a prior knowledge, it is possible to define damage by comparing the estimated mean value of a member with its baseline value. When any estimated system parameter value is less than its baseline value, it is regarded that the system parameter encounters damage. According to the statistical interpretation proposed by [Yeo, 1999] we can calculate the damage severity as follows :

$$S_D = \frac{x_0 - \bar{x}}{x_0} \times 100(\%) \qquad (15)$$

where x_0 and $\bar{x}$ are the baseline value and the estimated mean value of a design system parameter.

Numerical Example

The example structure is 2-story frame structrue as shown in Fig. 1. Since the axial rigidity can be neglected in a frame structure, the only flexural rigidity EI of each member is selected as the system parameter. The damage is assumed that the rotational rigidity of support 11 is lost perfectly by any severe load. It is assumed that the damage is reflected as the reduction of the flexural stiffness of the member connected to the support.

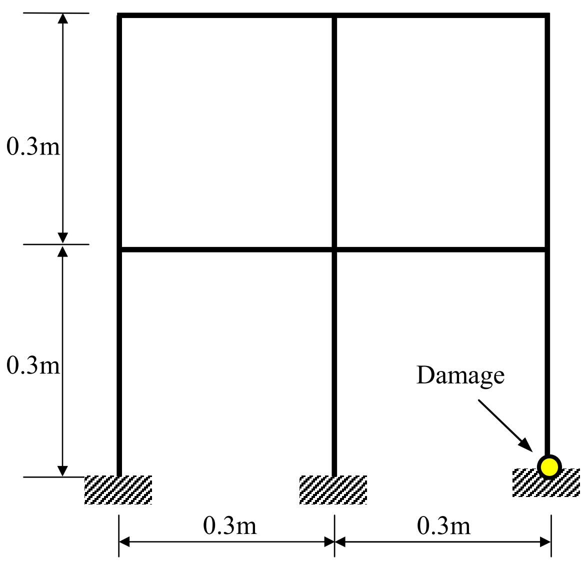

Fig. 1 geometry and boundary condition

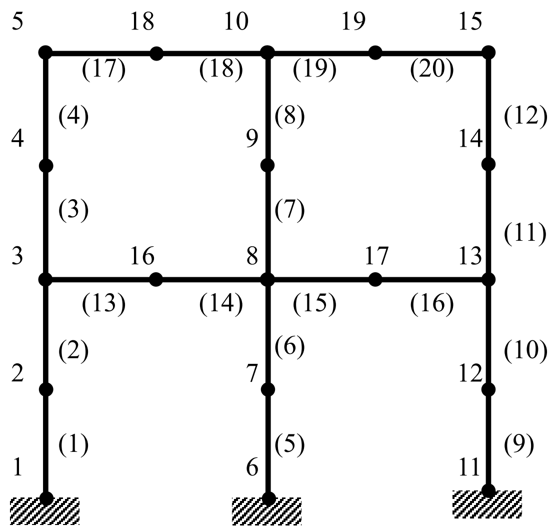

Fig. 2 FEM modeling

Fig. 1 and Fig. 2 show the geometry condition and boundary conditions and the finite element model of the structre, respectively. It is assumed that the structure is made of steel. Therefore Young's modulus of each element is assumed uniformly 206 Gpa. A cross-sectionion of each element is 0.02 m × 0.02 m rectangle. Moment of inertia from the neutral axis is $1.33\times10^{-8}\,m^4$. 5% proportional random noise for each mode shape is added to the noise-free mode shape vector in Monte Carlo simulation. The baseline values of all the system parameters are 2746.67 $N{\cdot}m^2$, and which is used as the initial values for design parameters. A posteriori information used as the measurement is modal displacement including horizontal, vertical, and rotational displacement at nodes 3,5,13 and 15. The total number of degrees of freedom of the structure is 48, and the number of measured degrees of freedom is 12. First three modes are used in the objective function of Eq. (10).

Fig. 3 represents the estimation result as the normalized values of system parameters. The figrure shows that system parameters of several members is reduced than their baseline values. However only the two members (9,10) are assessed as damaged members using the statistical approach of Yeo [1999]. Their damage severities are shown in Fig. 4. The two members are just ones conneted with the hinged support. Therefore the loss of the rotational rigidity of the support is refelected as the reduction of flexural stiffness the member which is connected to the support.

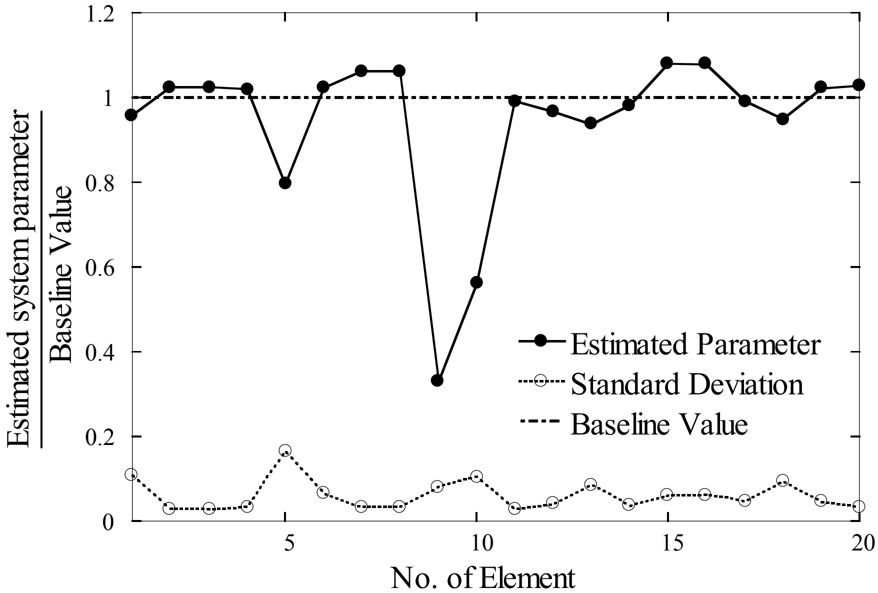

Fig. 3 estimated average system parameters and standard deviation

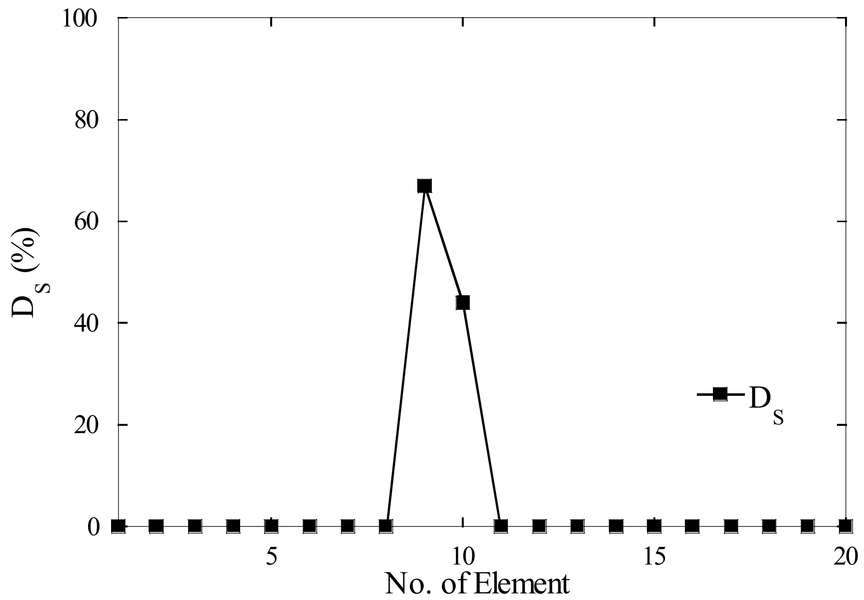

Fig. 4 Damage severity

Concluding Remarks

The damage detection and assessment algorithm using mode shape vector is proposed. The method to calculate the sensitivity matrix of the mode shape vector, which is normalized by an arbitrary matrix, is presented. In this algorithm statistical approach is used to determine damaged members and the damage severity. Measured data perturbation scheme is used to statistics of the estimated system parameters. The validity of the proposed algorithm is demonstrated by a numerical example. In this example a damage is modeled that the rotational rigidity is lost by plastic hinge at a support of a frame. The deterioration of the frame is assessed as the reduction of flexural rigidity of the member connected to the support, successfully.

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