BUILDING FOR THE 21st CENTURY, Y.C. LOO (Editor)
©EASEC-5 Secretariat, Griffith University Gold Coast Campus, Australia. ISBN 0 86857 671 9

FLUID-STRUCTURE INTERACTION ANALYSIS OF RECTANGULAR LIQUID CONTAINER WITH SUBMERGED OBJECTS

Hae Sung Lee
Full-Time Lecturer, Department of Civil Engineering, Seoul National University, Seoul, Korea

Hyun Moo Koh
Associate Professor, Department of Civil Engineering, Seoul National University, Seoul, Korea

Jaekwan Kim
Assistant Professor, Department of Civil Engineering, Seoul National University, Seoul, Korea

Jang-Ho Park
Ph.D. Student, Department of Civil Engineering, Seoul National University, Seoul, Korea

ABSTRACT: Dynamic response characteristics of a rectangular liquid container are investigated by two and three dimensional coupled boundary element-finite element method. In this hybrid approach, the motion of fluid region is modeled by boundary element method and that of containing structure by finite element method. The developed methods are applied to study the influence of submerged objects on the response of the fluid-structure system and the stability of the submerged objects themselves.

1. INTRODUCTION

Reinforced concrete liquid containers of rectangular plane are usually preferred for the wet storage of nuclear spent fuel assemblies. Despite being massive structures, the effect of their flexibility is known to be significant on the dynamic fluid-structure interaction behavior. To investigate the interaction behavior, a two-dimensional coupled boundary element-finite element method has been proposed[1]. However, three-dimensional characteristics of a rectangular container cannot be adequately described by a simple two-dimensional model. This paper presents a three-dimensional analysis method, and investigates the effect of submerged objects inside the container on the fluid and structural behaviors.

The irrotational motion of the inviscid and incompressible ideal fluid is modeled by three-dimensional boundary element method and the motion of structure by finite element method using plate elements. A singularity-free integral formulation is employed for the implementation of boundary element method. Coupling is performed by imposing compatibility and equilibrium conditions along the interface between fluid and structure. The fluid-structure interaction effects are reflected into the coupled equations of motion as added fluid mass matrix and sloshing stiffness matrix. Free surface sloshing motion and hydrodynamic pressure developed in a flexible rectangular container due to ground motions are computed in time domain and compared with those by two-dimensional analysis. Preliminary three-dimensional analysis results show quite new aspects in the dynamic characteristics of the structure and in the pressure distribution pattern along the height and width of the wall. Also investigated are the stability of free-standing objects inside the container and their influence on the overall response of the containing structure.

2. BOUNDARY ELEMENT MODELING OF FLUID MOTION

The irrotational motion of inviscid, incompressible ideal fluid in a container can be described in terms of velocity potential which satisfies Laplace equation and Green-Lagrange Identity. Then the potential, $\phi(\tilde{x},t)$, at any point $\tilde{\xi}$ on the boundary as a solution of Laplace equation can be given in the following integral equation[1],

$$c(\tilde{\xi})\phi(\tilde{\xi},t) = \int_S \phi(\tilde{x},t)\frac{\partial G}{\partial \tilde{n}}(\tilde{x};\tilde{\xi})ds - \int_S \frac{\partial \phi}{\partial \tilde{n}}(\tilde{x},t)G(\tilde{x};\tilde{\xi})ds \ , \tag{1}$$

where $c(\tilde{\xi})$ is a constant depending only on the geometry of the boundary, $G(\tilde{x};\tilde{\xi})$ Green's function of Laplace equation and ξ the position vector of source point.

Applying standard discretization procedure of boundary element method to Eq(1) the following algebraic equations can be obtained

$$[B]\left\{\frac{\partial \phi}{\partial n}\right\} - [H]\{\phi\} = \{0\} \ , \tag{2}$$

where $[B]$ and $[H]$ are coefficient matrices and $\left\{\frac{\partial \phi}{\partial n}\right\}$ and $\{\phi\}$ denote nodal normal velocity vector and nodal potential vector respectively. The singular integrals appearing in the process can be avoided by using an identity related to constant potential field and transformation into polar coordinate system.

Under the horizontal ground excitations, the linearized dynamic boundary conditions for the model given in Figure 1 may be expressed as[1]

$$\frac{\partial \phi}{\partial t}(\tilde{x},t) + g\eta(x_1,x_2,t) = 0 \quad \text{on free surface,} \tag{3}$$

$$\frac{\partial \phi}{\partial x_3}(\tilde{x},t) = \frac{\partial \eta}{\partial t}(x_1,x_2,t) \quad \text{on free surface,} \tag{4}$$

$$P(\tilde{x},t) = -\rho\frac{\partial \phi}{\partial t}(\tilde{x},t) \quad \text{on the interface,} \tag{5}$$

$$\frac{\partial \phi}{\partial n}(\tilde{x},t) = n \bullet v_p \quad \text{on the interface,} \tag{6}$$

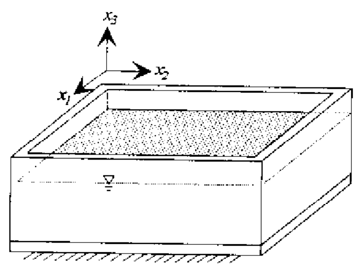

Figure 1 Three-dimensional model and its coordinate system

where $\eta(x_1,x_2,t)$ denotes vertical displacement of free surface from the stationary level, $P(\tilde{x},t)$ hydrodynamic pressure in excess of hydrostatic pressure, g gravitational acceleration, $\tilde{n}=\tilde{n}(\tilde{x})$ exterior normal vector on the boundary, ρ fluid density, and v_p the velocity vector.

Differentiating equation (2) with respect to time and applying boundary conditions, equations for the fluid region can be obtained as follows[1].

$$\mathrm{M}_{\eta p}\{\dot{\mathrm{v}}_p\} + \mathrm{M}_{\eta\eta}\{\ddot{\eta}\} + \mathrm{K}_{\eta\eta}\{\eta\} + \mathrm{M}_{\eta\eta}\{1\}\ddot{u}_{g3} = \{0\}, \tag{7}$$

$$\mathrm{P} = \mathrm{P}_1\{\ddot{\mathrm{u}}_c\} - \mathrm{P}_2\{\ddot{\eta}\} - \mathrm{P}_3\{\eta\} + \mathrm{P}_1\{\ddot{\mathrm{u}}_g\} + \mathrm{P}_2\{1\}\ddot{u}_{g3}, \tag{8}$$

where $\{\ddot{u}_g\}$ indicates horizontal ground acceleration vector.

3. COUPLING PROCEDURE

The equation governing the motion of containing structure can be derived by finite element method using plate element to be

$$\mathrm{M}^s\{\ddot{\mathrm{u}}(t)\} + \mathrm{K}^s\{\mathrm{u}(t)\} = -\mathrm{M}^s[\mathrm{r}^s]\{\ddot{\mathrm{u}}_g(t)\} + \mathrm{f}(t), \tag{9}$$

where M^s and K^s denote mass and stiffness matrices, respectively, $\{\mathrm{u}(t)\}$ relative nodal displacement vector, $\{\ddot{\mathrm{u}}_g(t)\}$ ground acceleration vector, $\mathrm{f}(t)$ external nodal force vector and $[\mathrm{r}^s]$ earthquake influence coefficient matrix.

Along the interface between the wall and fluid, compatibility and equilibrium conditions should be satisfied. Fluid particle acceleration normal to the interface boundary is set equal to the acceleration of the structure boundary in the same direction under the assumption of small displacement. In order to impose equilibrium condition at the coupled nodes, the fluid pressure is converted into equivalent nodal forces. The governing equation of the coupled system including the free surface sloshing motion. can be derived by imposing equilibrium and compatibility condition in the following form [1]

$$\begin{bmatrix} \mathrm{M}^s_{oo} & \mathrm{M}^s_{op} & 0 \\ \mathrm{M}^s_{po} & \mathrm{M}^s_{pp} - \mathrm{M}_{pp} & \mathrm{M}_{p\eta} \\ 0 & \mathrm{M}_{\eta p} & \mathrm{M}_{\eta\eta} \end{bmatrix} \begin{Bmatrix} \ddot{\mathrm{u}}_o \\ \ddot{\mathrm{u}}_p \\ \ddot{\eta} \end{Bmatrix} + \begin{bmatrix} \mathrm{K}^s_{oo} & \mathrm{K}^s_{op} & 0 \\ \mathrm{K}^s_{po} & \mathrm{K}^s_{pp} & \mathrm{K}_{p\eta} \\ 0 & 0 & \mathrm{K}_{\eta\eta} \end{bmatrix} \begin{Bmatrix} \mathrm{u}_o \\ \mathrm{u}_p \\ \eta \end{Bmatrix} = -\begin{bmatrix} \mathrm{M}^s_{oo} & \mathrm{M}^s_{op} & 0 \\ \mathrm{M}^s_{po} & \mathrm{M}^s_{pp} + \mathrm{M}_{pp} & \mathrm{M}_{p\eta} \\ 0 & \mathrm{M}_{\eta p} & \mathrm{M}_{\eta\eta} \end{bmatrix} [\mathrm{r}]\{\ddot{\mathrm{u}}_g\}, \tag{10}$$

where $\{\mathrm{u}_o\}$ and $\{\ddot{\mathrm{u}}_o\}$ are nodal displacement and acceleration vectors defined at the nodes of structure not in contact with the fluid region and $\{\mathrm{u}_c\}$ and $\{\ddot{\mathrm{u}}_c\}$ are those in contact with the fluid region.

4. NUMERICAL EXAMPLES

Dynamic responses of the fluid-structure system described in Figure 2 are analyzed by two and three dimensional coupled boundary element-finite element methods. For the present analysis, the model is assumed to be excited into x_1 direction only. The N-S component of the ground acceleration recorded at El Centro during the 1940 Imperial Valley Earthquake is used as input motion with peak ground acceleration adjusted to 0.2g.

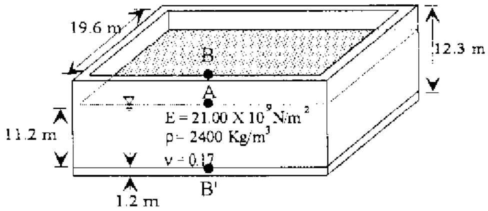

Figure 2 Dimensions and material properties of rectangular container model

Given in Figure 3 are time histories of the free surface sloshing motion at observation point A in Figure 2. It is recognized that the overall trends are quite similar with minor difference in their detailed shapes. Displacement time histories at the top of the structure (observation point B in Figure 2) are provided in Figure 4. The natural frequency of three-dimensional model seems to be slightly higher than that of two dimensional model because of increased constraint effect in three dimensional model. But there are not found any notable difference in general trend and magnitude between two dimensional and three dimensional analysis. However the magnitude and distribution pattern of the developed hydrodynamic pressure are significantly different between two dimensional and three dimensional analysis as will be shown next. Given in Figure 5 and 6 are hydrodynamic pressure distribution at the moment when the base shear reaches its peak value. In Figure 5, pressure distribution curves along the height of the wall(BB' in Figure 2) are compared. It shows that amplification of the pressure in three dimensional model is much more pronounced than in two dimensional model. But this kind of amplification may not occur uniformly along the width of the wall as can be seen in the spatial distribution pattern of the pressure over the wall surface(Figure 6).

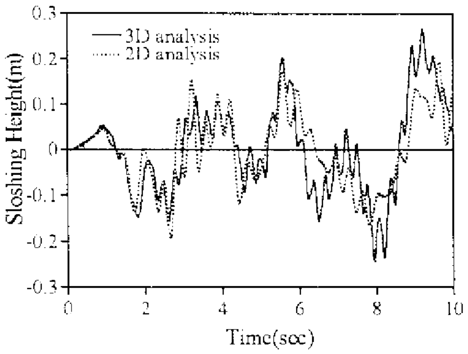

Figure 3 Histories of free surface sloshing motion at the intersection point

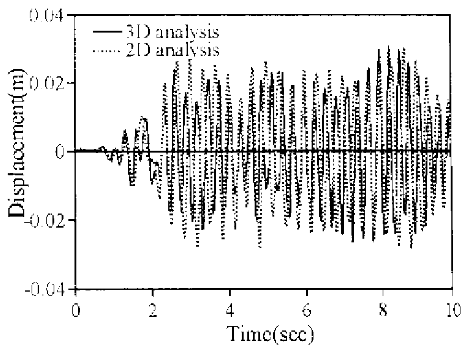

Figure 4 Histories of displacement at the top of the wall

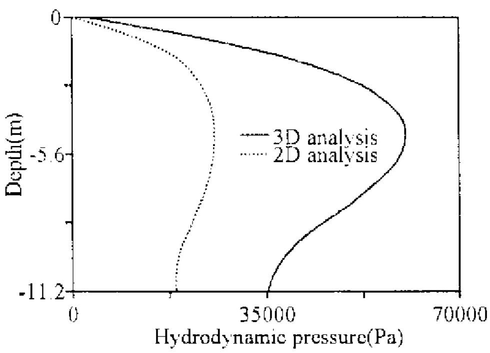

Figure 5 Hydrodynamic pressure distribution along the wall

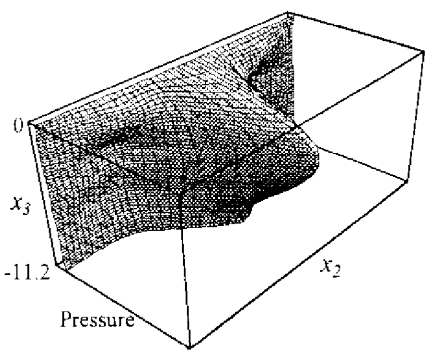

Figure 6 Spacial distribution of hydrodynamic pressure

It is of interest to investigate the effect of submerged objects on the response of fluid-structure system and the stability of the objects themselves under the ground excitation. The complete study of this problem may require three dimensional analysis. But because of limited preparation time for this paper, only two dimensional analysis results for the model shown in Figure 7 will be presented. Since the submerged objects under consideration are quite heavy and stiff, they are assumed to be rigid and fixed to the base slab for the analysis purpose.

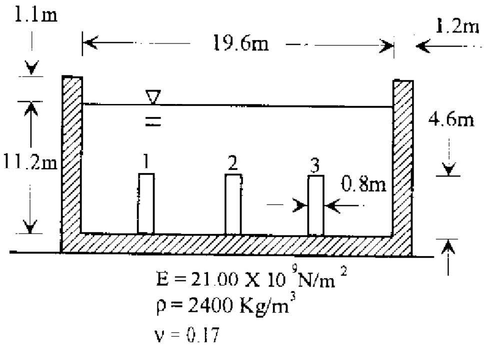

Figure 7 Dimensions and material properties of rectangular container with objects

The profiles of fluid surface are compared in Figure 8. Acceleration time histories at the top of the structure(point B) are given in Figure 9 and comparison of pressure distribution along the height of the wall is made in Figure 10. Though the pressure distribution is not affected significantly due to the presence of submerged objects, free surface sloshing motion seems to be very sensitive to the submerged objects.

In order to study stability of the objects pressure distribution around the outer object is calculated at the moment when the base shear reach its peak value and plotted in Figure 11. From the pressure distribution, the overturning moment developed exceeds the restoring moment by a factor 4.5. But this estimation is based on two dimensional model therefore it will be premature to discuss the stability of the object definitely.

5. CONCLUSIONS

A three-dimensional coupled boundary element-finite element method is developed and applied to investigated dynamic response of rectangular liquid container with or without submerged objects.

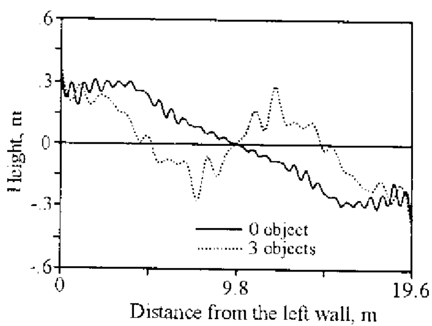

Figure 8 Profiles of fluid surface

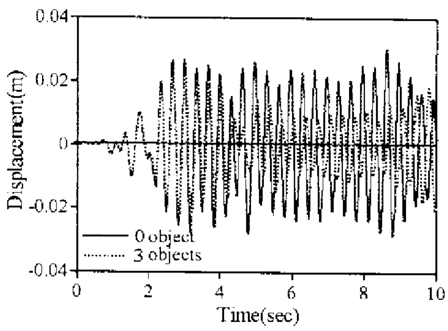

Figure 9 Histories of displacement at the top of the wall

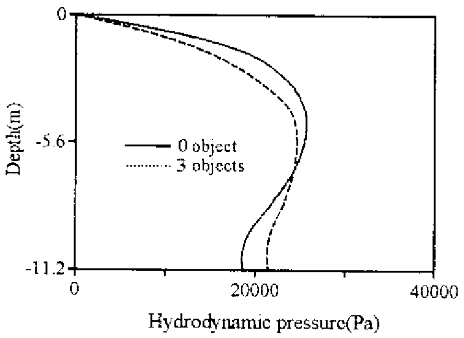

Figure 10 Hydrodynamic pressure distribution along the height of wall

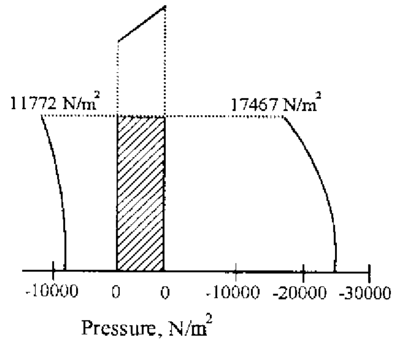

Fig 11 Hydrodynamic pressure distribution along the third object

The natural frequency of the coupled system is predicted to be higher in three-dimensional model than in two-dimensional model. The pressure distribution developed in three-dimensional model is found to be significantly different than that anticipated by two-dimensional analysis. It is also found that free surface sloshing motion is strongly influenced by the presence of submerged objects but pressure distribution to be a lesser degree.

REFERENCES

1. Park, J.-H., Koh, H.M., Kim, J., "Dynamic Analysis of Liquid Storage Rectangular Container by a Coupled Boundary Element-Finite Element Method," *Proc. of EASEC-4*, Seoul, Korea, pp. 1815-1820, 1993.
2. Rizzo, F. J., Shippy, D. J., and Rezayat, M., "A Boundary Integral Equation Method For Radiation And Scattering of Elastic Waves In Three Dimensions," *Internaitonal Journal For Numerical Methods In Engineering*, vol. 21, pp.115-129.
3. Currie, I. G., *Fundamental Mechanics of Fluids*, McGraw-Hill Inc, 1974.
4. Cook, Robert D., David S. Malkus and Michael E. Plesha, *Concepts and Application of Finite Element Method*, John Wiley & Sons Inc., 1989.
5. Rammerstorfer, Franz G., Scharf, Kunt and Fisher, Franz D., "Storage tanks under earthquake loading," *Applied Mechanics Reviews*, ASME, Vol. 43, pp. 261-282, 1990.
6. Housner, George W., "Dynamics Pressure On Accelerated Fluid Container," *Bulletin of the Seismological Society of America*, vol. 47, pp.15-35, 1957.