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Structural Damage Detection Algorithm from Measured Acceleration

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ABSTRACT: This paper introduces an algorithm for the parameter estimation scheme based on system identification (SI) in time domain. Acceleration data measured by dynamic tests are used as the measured responses. The least squared errors of the difference between calculated acceleration and measured acceleration are adopted as an error function. Damping parameters as well as stiffness properties of a structure are considered as system parameters. The Rayleigh damping is adopted for the SI. A regularization technique is applied to alleviate the ill-posed characteristics of SI problems. A new regularization function suitable to the time domain is proposed. The regularization factor is determined by the geometric mean scheme (GMS). First order of sensitivity of acceleration is obtained by direct differentiation of the equation of motion. The validity of the proposed method is demonstrated by numerical examples.

KEYWORDS: parameter estimation, system identification, time domain, regularization, geometric mean scheme, least square error, measured acceleration, inverse problem

INTRODUCTION

The modal analysis approaches have been widely adopted to identify structural properties using measured acceleration. The modal analysis approaches suffer from drawbacks caused by insensitiveness of modal data to changes of structural properties. In addition, the damping properties of structures cannot be estimated by the modal analysis. To overcome the drawbacks of the modal analysis approaches, this paper presents a system identification scheme to determine structural properties such as stiffness and damping parameters of structures using measured acceleration data. The proposed algorithm is based on the minimization of an error function with respect to the structural parameters. The error function is defined as the time integral of the least squared errors between the measured acceleration and the calculated acceleration by a mathematical model.

A system identification problem is a type of inverse problems, which are usually ill-posed. An ill-posed problem is characterized by the non-uniqueness and instability of solutions. The regularization technique has been employed to overcome the ill-posedness of inverse heat transfer problems and inverse elasticity problems. In the regularization technique, a predefined regularization function is added to the error function to impose constraints on the admissible solutions of a given inverse problem. This paper introduces a new regularization function that is defined as the L_2 norm of the time derivative of system parameters. To determine the regularization factor, which has crucial effect on the solution of the SI scheme, the geometric mean scheme is adopted.

The validity and effectiveness of the proposed method are demonstrated with several numerical examples. The numerically generated data with noises are utilized as measured acceleration. Detailed discussions on the numerical behaviors of the proposed method are presented.

PARAMETER ESTIMATION SCHEME IN TIME DOMAIN

The discretized equation of motion of a given structure is obtained by the finite element method as follows.

$$\mathbf{M}\mathbf{a} + \mathbf{C}(\mathbf{x})\mathbf{v} + \mathbf{K}(\mathbf{x})\mathbf{u} = \mathbf{P}(t) \tag{1}$$

where $\mathbf{x}$ and $\mathbf{P}$ are a system parameter vector and a load vector, and $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ represent the mass, damping and stiffness matrix of the structure, respectively. $\mathbf{a}$, $\mathbf{v}$ and $\mathbf{u}$ are the acceleration, velocity and displacement of the structure, respectively. Newmark β-method is used to integrate the equation of motion (1).

It is assumed for damage detection that accelerations of a given structure are measured from a dynamic test at some discrete observation points, and that the stiffness properties and damping properties during the test do not change. The unknown system parameters of a structure including stiffness and damping properties are identified through minimizing least squared errors between computed and measured acceleration.

$$\Pi_E(t) = \underset{\mathbf{x}}{\text{Min}} \; \frac{1}{2}\int_0^t \left\| \tilde{\mathbf{a}}(\mathbf{x}) - \bar{\mathbf{a}} \right\|^2 dt \;\; \text{subject to} \;\; \mathbf{R}(\mathbf{x}) \le 0 \tag{2}$$

where $\tilde{\mathbf{a}}$, $\bar{\mathbf{a}}$ and $\mathbf{R}$ are the calculated acceleration and the measured acceleration at observation points and constraint vector, respectively, with $\|\cdot\|$ representing the Euclidean norm of a vector. Linear constraints are used to set physically significant upper and lower bounds of the system parameters. The minimization problem defined in Eq. (2) is a constrained nonlinear optimization problem because the acceleration vector $\tilde{\mathbf{a}}$ is a nonlinear implicit function of the system parameters $\mathbf{x}$.

The parameter estimation defined by a minimization problem as Eq. (2) is a type of ill-posed inverse problems. Ill-posed problems suffer from three instabilities: nonexistence of solution, non-uniqueness of solution and discontinuity of solutions when measured data are polluted by noises. Because of the instabilities, the optimization problem given in Eq. (2) may yield meaningless solutions or diverge in optimization process. Attempts have been made to overcome instabilities of inverse problems merely by imposing upper and lower limits on the system parameters. However, it has been demonstrated by several researchers that the constraints on the system parameters are not sufficient to guarantee physically meaningful and numerically stable solutions of inverse problems [1].

The regularization technique proposed by Tikhonov is considered as a more rigorous way to overcome the ill-posedness of inverse problems. In the regularization technique, the original object function is modified by adding a positive definite regularization function [2,3]. Various regularization functions are used for different types of inverse problems. The following regularization function is adopted for the parameter estimation in time domain.

$$\Pi_R(t) = \frac{\lambda}{2}\int_0^t \left\| \frac{d\mathbf{x}}{dt} \right\|^2 dt \tag{3}$$

where λ is the regularization factor. By adding the regularization function to the error function, the regularized parameter estimation scheme is defined as follows.

$$\underset{\mathbf{x}}{\text{Min}} \;\; \Pi(t) = \frac{1}{2}\int_0^t \left\| \tilde{\mathbf{a}}(\mathbf{x}) - \bar{\mathbf{a}} \right\|^2 dt + \frac{\lambda}{2}\int_0^t \left\| \frac{d\mathbf{x}}{dt} \right\|^2 dt \;\; \text{subject to} \;\; \mathbf{R}(\mathbf{x}) \le 0 \tag{4}$$

The regularization function defined in Eq. (3) represents the variance of system parameters in time. Since the system parameters are assumed to be invariant in time, the regularization function vanishes in case the SI yields exact solution. However, the ill-posedness of a inverse problem and noises in measurements generally lead to severe oscillations of the solution of Eq. (2) in time. The regularization function added in Eq. (4) becomes smaller as the rates of changes of the system parameters decrease, and thus prevents the system parameters from arbitrary changes in time during optimization.

The regularization effect in parameter estimation process is determined by the regularization factor. The regularization effect vanishes for a small regularization factor while the regularization function has a dominant effect over the error function during the optimization process for a large regularization factor. In either case, the optimization problem is unable to estimate correct system parameters due to instabilities or excessive regularization effects on the system parameters. Therefore, selection of a proper regularization factor is very crucial to obtain meaningful solutions of system identification problems. The geometric mean scheme proposed (GMS) by Park is adopted in this study to determine the optimal regularization factor [4]. In the GMS, the optimal regularization factor is defined as the geometric mean between the maximum singular value and the minimum singular value of the Gauss-Newton hessian matrix of the discretized error function given in Eq. (2).

$$\lambda_{opt} = \sqrt{S_{\max} \cdot S_{\min}} \tag{5}$$

where λ_{opt}, $S_{\max}$, $S_{\min}$ denote regularization factor, maximum singular value and minimum singular value which is greater than zero, respectively. The singular values of any given matrix can be obtained by using the singular value decomposition [5]. The sensitivity of the computed acceleration required in the optimization process is obtained by the direct differentiation of the equation of motion (1).

DAMPING MODEL

It is a difficult task to model damping properties of real structures. In fact, existing damping models cannot describe actual damping characteristics exactly, and are approximations of real damping phenomena to some extents. Since the damping has an important effect on dynamic responses of a structure, the damping properties should be considered properly in the parameter estimation scheme. In most of previous studies on the parameter estimation, the damping properties of a structure are assumed as known properties, and only stiffness properties are identified. However, the damping properties are not known a priori and should be included in system parameters in the SI.

Among various classical damping models, the modal damping and the Rayleigh damping are the most frequently adopted model. In the modal damping, a damping matrix is constructed by using generalized modal masses and mode shapes [6].

$$\mathbf{C} = \mathbf{M}\left(\sum_{n=1}^{N} \frac{2\zeta_n \omega_n}{M_n} \mathrm{f}_n \mathrm{f}_n^T\right)\mathbf{M} \tag{6}$$

where N, M_n, ζ_n, f_n and ω_n denote the number of the degrees of freedom (DOF), n-th generalized modal mass, modal damping ratio for n-th mode, the n-th mode shape and n-th mode frequency, respectively. In Rayleigh damping, a damping matrix is represented by a linear combination of the mass matrix and stiffness matrix.

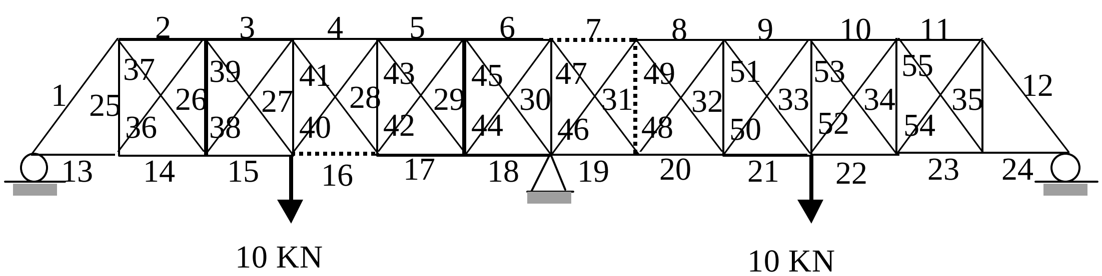
$$\mathbf{C} = a_0\mathbf{M} + a_1\mathbf{K} \tag{7}$$

The damping coefficients of the Rayleigh damping can be determined when any two modal damping ratios and the corresponding modal frequencies are specified.

In case the modal damping is employed in the parameter estimation, the number of the system parameters associated with the damping is equal to that of the total number of DOFs, which increases the total number of unknowns in the optimization problem given in Eq. (4). Since neither modal damping nor Rayleigh damping can describe actual damping exactly, and the modal damping requires more unknowns than the Rayleigh damping in the parameter estimation, this study employs the Rayleigh damping for the SI. The Rayleigh damping yields a linear fit to the exact damping of a structure.

EXAMPLE

The validity of the proposed time domain SI is examined through a simulation study with a two-span continuous truss shown in Figure 1. Typical material properties of steel (Young' s modulus = 210 GPa, Specific mass = 7.85Kg/m^3) are used for all members. The cross sectional area of each member (top member, bottom member, vertical member and diagonal member) is given in Figure 1. The natural frequencies of the truss range from 6.6 Hz to 114.7 Hz.



Member	Area(cm^2)	Member	Area(cm^2)
Top	112.5	Vertical	62.5
Bottom	93.6	Diagonal	75.0

Fig. 1 2-span continuous truss

Damage of the truss is simulated with 40%, 50% and 34 % reductions in the sectional areas of member 7, 16 and 31, respectively. The damaged members are depicted by dotted lines in Figure 1. It is assumed that accelerations are measured from a free vibration induced by a sudden release of applied loads of 10KN shown in Figure 1. The measured accelerations are generated by the finite element model used in SI. The measurement errors are simulated by adding 8% random noise generated from a uniform probability function to acceleration calculated by the finite element model. The observation points are located at 12 bottom nodes of the truss. Both x- and y- component of acceleration are measured in the time period from 0 sec to 0.2 sec with the interval of 1/200 sec. The modal damping is employed for the calculation of measured acceleration while Rayleigh damping is adopted for the SI. The modal damping ratios for the calculation of measured acceleration are shown in Figure 4.

In case either the regularization scheme or damping estimation is not included in the SI, the optimization procedure does not converge or converges to meaningless solutions. Therefore, only the results with the regularization scheme and damping estimation are presented here. Figure 2 illustrates the variation of the identified stiffness properties of the damaged members with time. Although rather large measurement noise of 8% is presented, the proposed method is able to identify accurately the severity of damage of each damaged member. The identified stiffness properties of all members at the final time step are shown in Figure 3. Since stiffness properties of the damaged members reduce prominently compared with the oscillation magnitude of the other members, the damaged members are clearly assured. Figure 4 shows the exact modal damping ratios used for the calculation of measured accelerations together with identified modal damping ratios by the Rayleigh damping. The initial modal damping ratio calculated by the assumed Rayleigh damping coefficient is also drawn in the same figure. The identified Rayleigh damping well approximates the real modal damping.

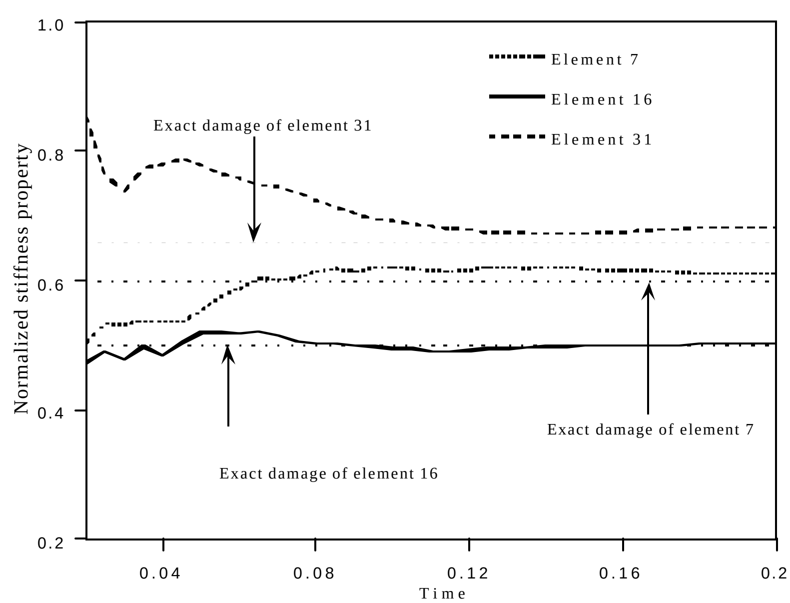


Fig. 2 Variation of estimated stiffness properties of damaged members with time

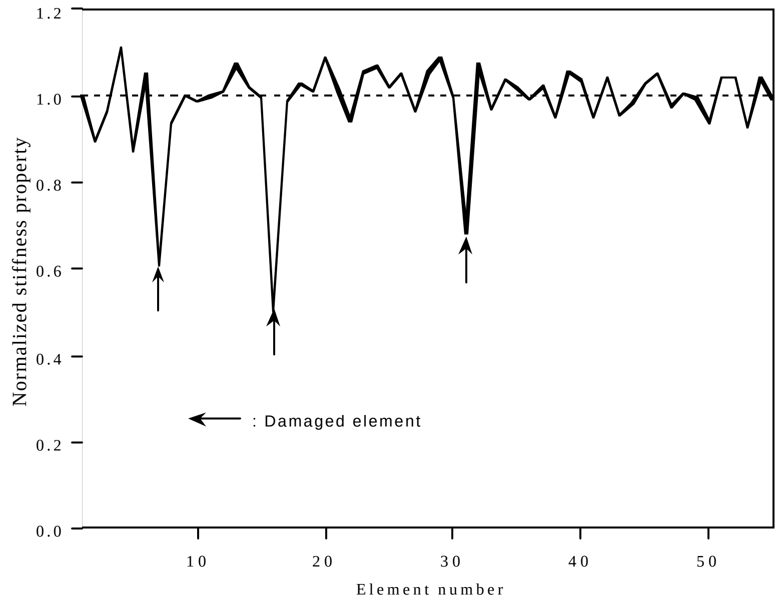


Fig. 3 Estimated stiffness properties at the final time step

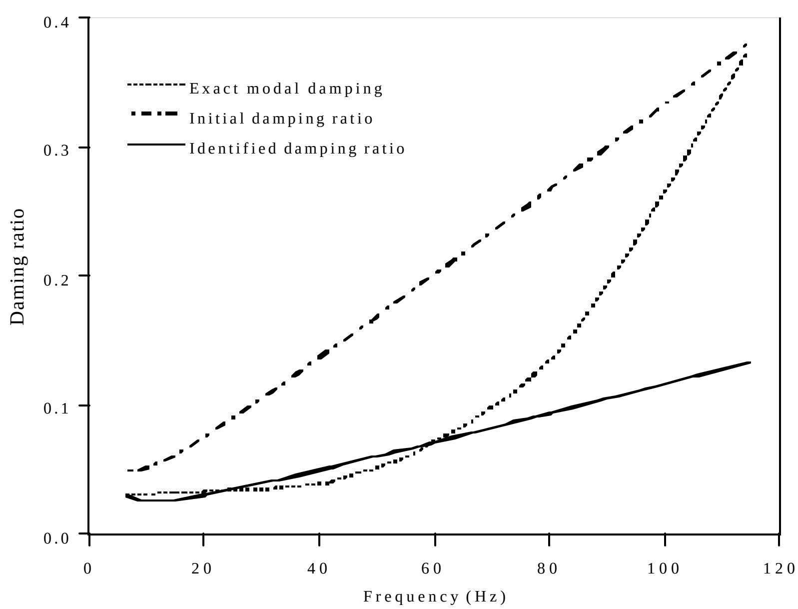


Fig. 4 Estimated damping ratio

CONCLUSION

A time domain SI using measured acceleration data is proposed. The least squared errors of the difference between calculated acceleration and measured acceleration is adopted as an error function. The Tikhonov regularization technique is employed to alleviate the ill-posedness of the inverse problem in SI. The GMS is utilized to determine the optimal regularization factor. The Rayleigh damping is used to estimate the damping characteristics of a structure. The system parameters include the damping coefficients of the Rayleigh damping as well as the stiffness parameters of a structure.

In most previous study, the damping characteristics of a structure are assumed as known values. It is confirmed that the damping characteristics should be adjusted properly according to measured acceleration data. Although it is not possible to form the exact damping matrix of a structure, it is very important to approximate the damping matrix to the real damping matrix as close as possible. The proposed method can estimate the stiffness properties accurately even though the damping characteristics are approximated by Rayleigh damping. The final solution converges to the exact solution even for noise-polluted data. It is believed the proposed method provides a very powerful engineering tool to identify dynamic characteristics of structures and to detect damage in structures based on measured acceleration.

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