KEERC-MAE Joint Seminar on
Risk Mitigation for Regions of Moderate Seismicity
University of Illinois at Urbana-Champaign, August 5-8, 2001

Structural Damage Assessment from Modal Data using a System Identification Algorithm with a Regularization Technique

Joo Sung KANG[1], Inho YEO[2] and Hae Sung LEE[3]

[1] Graduate Student, Seoul National University
School of Civil, Urban and Geosystem Eng., Shillim-dong, Kwanak-gu, Seoul 151-742, Korea
email: jskang94@plaza1.snu.ac.kr

[2] Senior Researcher, Highway Management Corporation
Research Institute, Ichon 467-860, Korea
email: ihyeo@himan.co.kr

[3] Associate Professor, Seoul National University
School of Civil, Urban and Geosystem Eng., Shillim-dong, Kwanak-gu, Seoul 151-742, Korea
email: chslee@plaza.snu.ac.kr

ABSTRACT: The parameter estimation scheme for System Identification (SI) has been developed by many researchers for more than three decades. It is achieved by comparing the measured responses of a real structure and the calculated responses of a mathematical model. Generally the dynamic test is applied to a large structure for diagnosis of the system. Modal parameter estimation is formulated as a minimization problem the discrepancy between the real measurement data and the predictions of a mathematical model of the structure. It is usually difficult to measure at all of the degrees of freedom and to collect data from all of the modes. Furthermore, the measurement data always include a considerable measuring errors. Inverse problem for parameter estimation is severe ill-posedness problem because of the sparseness of measurement data and the measuring error. Tikhonov norm for the change of the stiffness is used as the regularization function. Geometric Mean Scheme (GMS) is employed to determine a regularization factor.

KEYWORDS: System Identification, modal data, ill-posedness, regularization, GMS

INTRODUCTION

Structural damage often causes a loss of stiffness in one or more elements of a structure that affects its modal responses such as modal frequencies and mode shapes. Many methods have been developed to detect the location and severity of damage based on these changes. In this study, system identification (SI) based on the minimization of least square errors between measured mode shape vector and calculated mode shape vector is employed.

To solve nonlinear optimization problem, the recursive quadratic programming (RQP) and the Fletcher active set strategy are employed [1]. In RQP, sensitivity of calculated mode shape vector with respect to the system parameters is required. Current proposed algorithms to calculate sensitivity of mode shape vector, such as the modal method, the modified modal method, and the Nelson's method [6] are valid only when the mode shape vector is normalized with respect to mass matrix. Unless fully measured mode shape vector is available due to economic or physical restriction, normalization of measured mode shape vector cannot be obtained. So an algorithm to calculate sensitivity of the mode shape vector which is normalized with respect to an arbitrary matrix is developed.

It is known that SI is typically ill-posed inverse problem which suffers from severe numerical instabilities, such as non-existence, non-uniqueness, and discontinuity of solution. A regularization technique [2,3] is adopted to overcome such numerical instabilities. As a regularization function, Tikhonov norm which is difference between a baseline stiffness property and an assumed stiffness property is used. A regularization factor plays the most important role for estimation of both numerically and physically meaningful solution [2,3]. GMS proposed by Park [8] is used to determine an appropriate regularization factor.

In real situation, measurement data suffer from the measurement noises. When the measurement data are polluted with noise, it is very difficult to distinguish whether the damage is caused either by real damage or by noise in measurement data. Since measurement noise is inevitable in the real situation, the estimated system parameters from SI may be easily meaningless in the damage detection and assessment. To overcome this drawbacks, data perturbation scheme proposed by Hjelmstad and Shin [5] and statistical approach proposed by Yeo [9] are incorporated with SI for damage detection and assessment.

PARAMETER ESTIMATION

In this study damage is defined as the reduction of a system parameters from its baseline value which is assumed as a priori information. System parameters are estimated by the output error estimator using modal data such as Eq. (1).

$$\underset{\mathbf{x}}{Minimize} \quad \Pi(\mathbf{x}) = \frac{1}{2}\sum_{i=1}^{nmd}\left\|\boldsymbol{\phi}_i - \hat{\boldsymbol{\phi}}_i\right\|^2 \qquad \text{subject to} \qquad \mathbf{R}(\mathbf{x}) \geq 0 \tag{1}$$

where $\mathbf{x}$, $\boldsymbol{\phi}_i$, $\hat{\boldsymbol{\phi}}_i$, *nmd,* $\mathbf{R}(\mathbf{x})$ are system parameter vector, calculated mode shape vector of *i*-th mode, measured mode shape vector of *i*-th mode, the total number of the measured modes and constraints of system parameters, respectively.

REGULARIZATION

The parameter estimation with the output error estimator is typically ill-posed inverse problem. Ill-posed problems suffer from three instabilities: nonexistence of solution, non-uniqueness of solution and/or discontinuity of solution when measured data is polluted by noise [2]. So far, many authors have attempted to overcome an instability problem by imposing upper and lower constraints on the system parameters. However, Neuman [6] and Hjelmstad [4] have shown that constraints are not sufficient to guarantee a meaningful solution.

In this study it is utilized regularization technique in order to overcome ill-posedness in optimization processing. The following regularization function is used for the current identification of a structure.

$$\Pi_R = \frac{\beta}{2}\left\|\mathbf{x} - \mathbf{x}_0\right\|_T^2 \tag{2}$$

where β and $\mathbf{x}_0$ denote the regularization factor and the system parameters representing baseline stiffness properties of a structure, respectively. $\|\cdot\|_T$ is the Tikhonov norm of a matrix.

The regularization effect in parameter estimation process is determined by the regularization factor. Some rigorous methods to find an optimal regularization factor have been proposed for linear inverse problems. The geometric mean scheme (GMS) proposed by Park is adopted to determine the optimal regularization factor [8]. In the GMS, the optimal regularization factor is defined as the geometric mean between the maximum singular value and the minimum singular value of the Gauss-Newton hessian matrix of the error function given in Eq. (1).

$$\beta = \sqrt{S_{\max} \cdot S_{\min}} \tag{3}$$

where β, $S_{\max}$, $S_{\min}$ denote regularization factor, maximum singular value and minimum singular value which is not zero, respectively.

By adding the regularization function to the error function, the regularized output error estimator is defined as follows :

$$\underset{\mathbf{x}}{\text{Minimize}}\ \Pi = \frac{1}{2}\sum_{i=1}^{nmd}\left\|\boldsymbol{\phi}_i(\mathbf{x}) - \hat{\boldsymbol{\phi}}_i\right\|^2 + \frac{\beta}{2}\left\|\mathbf{x}-\mathbf{x}_0\right\|_T^2 \quad \text{subject to } \mathbf{R}(\mathbf{x}) \le 0 \tag{4}$$

SENSITYVITY

To solve the constrained nonlinear optimization problem expressed by Eq. (4), the recursive quadratic programming (RQP) and the Fletcher active set strategy are employed [1]. In RQP we need the sensitivity of the Eq. (1) with respect to system parameters. The sensitivity is shown in Eq. (5)

$$\Pi_{,x} = \sum_{i=1}^{nmd}\left\|\boldsymbol{\phi}_i - \hat{\boldsymbol{\phi}}_i\right\| \cdot \boldsymbol{\phi}_{i,x} \tag{5}$$

where the subscript $(\ )_{,x}$ denotes the partial derivative with respect to a system parameter.

In case where the mode shape vector is normalized by mass matrix of the structural system, several methods to calculate the sensitivity are already proposed, such as the modal method, the modified modal method, and the Nelson's method [6] and so on. When the mode shape vector is normalized by mass matrix, the sensitivity matrix of the vector is as follows:

$$\boldsymbol{\phi}_{j,x} = -\sum_{i\neq j}^{nmd}\frac{\boldsymbol{\phi}_i^T\mathbf{K}_{,x}\boldsymbol{\phi}_j}{(\lambda_i - \lambda_j)\boldsymbol{\phi}_i^T\mathbf{M}\boldsymbol{\phi}_i}\boldsymbol{\phi}_i \quad (\ i \neq j\ ) \tag{6}$$

where λ , $\mathbf{M}$, and $\mathbf{K}_{,x}$ denote eigen value, the mass matrix, and the sensitivity matrix of the stiffness matrix of a structural system with respect to a system parameter, respectively. However, in the system identification where the partially measured mode shape is used to determine design variables we cannot use the sensitivity of the normalized mode shape by the mass matrix.

If we assume that $\bar{\boldsymbol{\phi}}$ is normalized by arbitrary matrix $\mathbf{C}$, then we can express its sensitivity $\bar{\boldsymbol{\phi}}_{,x}$ as follows:

$$\bar{\boldsymbol{\phi}}_{j,x} = \frac{1}{\boldsymbol{\phi}_i^T\mathbf{C}\boldsymbol{\phi}_i}\left(\boldsymbol{\phi}_{j,x}\sqrt{\boldsymbol{\phi}_i^T\mathbf{C}\boldsymbol{\phi}_i} - \boldsymbol{\phi}_j\frac{\boldsymbol{\phi}_i^T\mathbf{C}\boldsymbol{\phi}_{i,x}}{\sqrt{\boldsymbol{\phi}_i^T\mathbf{C}\boldsymbol{\phi}_i}}\right) \tag{7}$$

If we substitute m_c for $\sqrt{\boldsymbol{\phi}_i^T\mathbf{C}\boldsymbol{\phi}_i}$, the Eq. (5) can be represented as the following equation.

$$\bar{\boldsymbol{\phi}}_{i,x} = \frac{1}{m_c}\boldsymbol{\phi}_{i,x} - \frac{1}{m_c^3}\left(\boldsymbol{\phi}_i^T\mathbf{C}\boldsymbol{\phi}_{i,x}\right)\boldsymbol{\phi}_i \tag{8}$$

MEASURED DATA PERTURBATION SCHEME

If a sufficient number of measured data sets are available for the same measurement condition, the effect of measurement noise on identification results may be reduced by averaging the measured data. In real situations, however, only limited sets of noisy measurement data are available. Therefore, it is usually difficult to determine whether the changes of the system parameters are caused by measurement noise or by actual damage.

The data perturbation method, which has been proposed by Hjelmstad and Shin [5] for a numerical simulation study, is employed. In the data perturbation method, a series of the system identification is performed with generated data sets around a given set of measured displacements by perturbing the given data with a small magnitude. As a result, the identified system parameters are interpreted statistically with their distributions.

To obtain samples of a system parameter for its statistical distribution, the measured data perturbation iteration is performed with the following perturbed measurement data.

$$(\bar{u}_j)_i^k = (\bar{u}_j)_i\,(1+\eta_j^k) \tag{9}$$

where $(\bar{u}_j)_i^k$ and η_j^k are a perturbed displacement for load case *i* and a random number, respectively, for the *j*-th component of the measured displacement at the *k*-th iteration.

DAMAGE SEVERITY

After mean and standard deviation are evaluated using the measured data perturbation scheme, it is the next progression to determine which member is damaged. Because baseline value of the system parameter is a prior knowledge, it is possible to define damage by comparing the estimated mean value of a member with its baseline value. When any estimated system parameter value is less than its baseline value, it is regarded that the system parameter encounters damage. According to the statistical interpretation proposed by Yeo [9], we can calculate the damage severity as follows :

$$S_D = \frac{x_0 - \bar{x}}{x_0} \times 100(\%) \tag{10}$$

where x_0 and $\bar{x}$ are the baseline value and the estimated mean value of a design system parameter.

EXAMPLE

The numerical example structure is 2-story frame structure as shown in Fig. 1. Since the axial rigidity can be neglected in a frame structure, the only flexural rigidity EI of each member is selected as the system parameter. The damage is assumed that the rotational rigidity of support 11 is lost perfectly by any severe load. It is assumed that the damage is reflected as the reduction of the flexural stiffness of the member connected to the support.

Fig. 1 and Fig. 2 show the geometry condition and boundary conditions and the finite element model of the structure, respectively. It is assumed that the structure is made of steel. Therefore Young's modulus of each element is assumed uniformly 206Gpa. A cross-section of each element is 0.02 m × 0.02 m rectangle. Moment of inertia from the neutral axis is 1.33×10^{-8} m^4. 5% proportional random noise for each mode shape is added to the noise-free mode shape vector. The baseline values of all the system parameters are 2746.67 N·m^2 , and which is used as the initial values for design parameters. A posteriori information used as the measurement is modal displacement including horizontal and vertical displacement, and rotational displacement at nodes 3,5,13 and 15. The total number of degrees of freedom of the structure is 48, and the number of measured degrees of freedom is 12. First three modes are used in the ROEE in Eq. (4).

Fig. 3 represents the estimation result as the normalized values of system parameters. The figure shows that system parameters of several members is reduced than their baseline values. However only the three members (5, 9, 10) are assessed as damaged members using the statistical approach of Yeo [9]. Their damage severities are shown in Fig. 4. The two members are just ones connected with the hinged support and the other member is the side bay member. Maybe the reason that the member 5 is assessed to a damaged member is the sparseness of measurement data in parameter estimation.

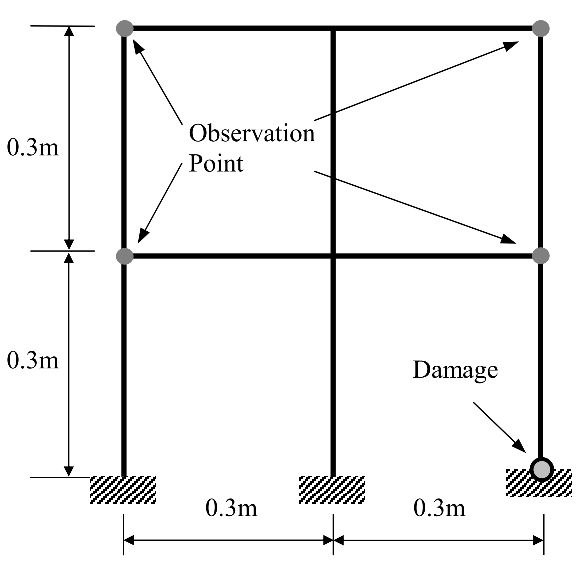

Fig. 1 Geometry and boundary condition

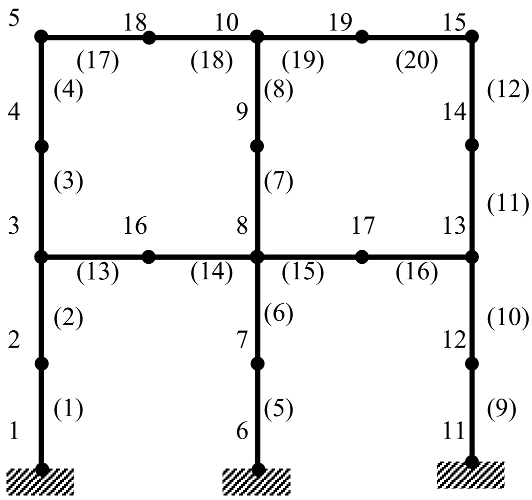

Fig. 2 FEM modeling

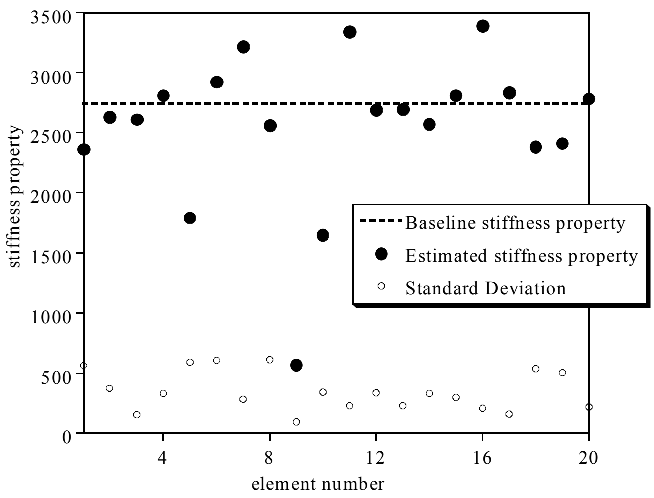

Fig. 3 Estimated result

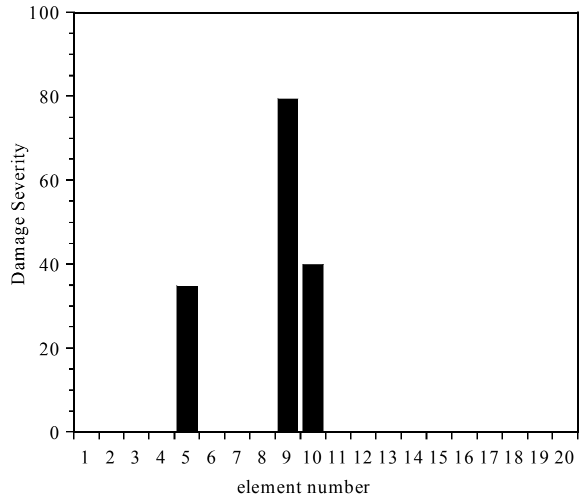

Fig. 4 Damage severity

CONCLUSION

The damage detection and assessment algorithm using mode shape vector is proposed. The Tikhonov regularization technique is employed to alleviate the ill-posedness of the inverse problem in SI. The GMS is utilized to determine the optimal regularization factor. The method to calculate the sensitivity matrix of the mode shape vector, which is normalized by an arbitrary matrix, is presented. Data perturbation scheme is used to assess the structural damage statistically. Statistical approach is used to determine damaged members and assess the damage severity.

In spite of successful detection of the deterioration of the frame is assessed as the reduction of flexural rigidity of the member connected to the support but the proposed algorithm still has many problems. Because the system identification scheme using modal data contains drawbacks caused by insensitiveness of lower mode shape to changes of structural properties, proposed algorithm needs to higher mode shape to distinguish the mode shape by changes of structural properties but it can't be obtained in real situation. So it is needed to update proposed algorithm to overcome these problems.

REFERENCE

1. Banan, M.R. and Hjelmstad, K.D. (1993), "Identification of structural systems from measured response," Structural Research Series NO. 579, Dept. of Civil Eng. Univ. of Illinois at Urbana-Champaign, May

2. Bui, H.D. (1994), "Inverse problems in the mechanics of materials: An introduction",

CRC Press, Boca Raton

3. Groetsch, C.W. (1984). "The theory of Tikhonov regularization for Fredholm equations of the first kind", Pitman Advanced Publishing, Boston

4. Hjelmstad, K.D. (1996). "On the uniqueness of modal parameter estimation." J. of Sound and Vibration 192(2), 581-598

5. Hjelmstad, K.D., and Shin, S.B. (1997). "Damage Detection and Assessment of Structures from static response." J. of Engineering Mechanics, ASCE, 123(6), 568-576

6. Nelson, R.B. (1976). "Simplified Calculation of Eigenvector Derivatives," AIAA Journal, Vol. 14, Sept. 1976, pp. 1201-1205

7. Neuman, S. P., and Yakowitz S. (1979). "A statistical approach to the inverse problem of aquifer Hydrology." Water Resources Research, 15(4), 845-860

8. Park, H.W., Shin, S.B. and Lee, H.S., "Determination of Optimal Regularization Factor for System Identification of Linear Elastic Continua with the Tikhonov Function," International Journal for Numerical Methods in Engineering, Vol. 51, No.10, 2001, pp.1211-1230.

9. Yeo, I.H., Shin, S.B., Lee, H.S., Chang, S.P., "Statistical Damage Assessment of Framed Structures from Static Responses," Journal of Engineering Mechanics, ASCE, Vol. 126, No. 4, pp. 414-421, 2000