A New Structural Damage Isolation Technique Using PP-TSVD

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ABSTRACT

A new damage isolation technique is proposed for system identification in framed structures. The 1-norm of the system parameter vector is introduced as a new regularization function for framed structures. The piecewise polynomial truncated singular value decomposition (PP-TSVD) is applied to filter out noise-polluted solution and to impose the regularization function given in the 1-norm of system parameters. The discrepancy principle and the cross validation are utilized to select a proper truncation number. The validity of the proposed method is demonstrated through isolating damage of a two-span continuous truss.

1. INTRODUCTION

The Tikhonov regularization scheme with a 2-norm regularization function has been proven to be very effective in stabilizing the ill-posedness of inverse problems such as system identification (SI) problems of mechanical systems (Hansen 1998; Park et al. 2001). The regularization scheme usually enhances the continuities of solutions of ill-posed problems by employing proper regularization functions. When it comes to SI for damage isolation, however, enforcement of the continuities of the solutions is not always desirable because damage isolation is essentially an identification procedure of discontinuities in system parameters. Therefore, the usual regularization schemes yield smeared solutions of damage detection problems, which are characterized as false warning and missing damage.

This paper presents a new regularization function to overcome the drawbacks of the Tikhonov regularization scheme with 2-norm regularization functions in the damage isolation. The proposed regularization function is defined as an 1-norm of a system parameter vector, and is imposed to an original SI problem by using the piecewise polynomial-truncated singular value decomposition (PP-TSVD, Hansen 1996). The PP-TSVD is based on the truncated singular value decomposition (TSVD) and the optimization of the 1-norm. In the PP-TSVD, the TSVD filters out noise in measured responses as the Tikhonov regularization, and discontinuity of the solution is recovered during the optimization of the 1-norm. A truncation number in TSVD plays a crucial role in damage isolation since the truncation number controls the stability and accuracy of the damage isolation procedure. The cross validation (Golub and Van Loan 1996) and the discrepancy principle (Morozov 1993) are utilized to estimate the noise level in measurements and to select a proper truncation number, respectively.

The validity of the proposed method is demonstrated through numerical examples on a two-span continuous truss for two damage cases. The proposed method reproduces sharp resolutions in damage isolation while the stability of the SI is properly maintained by the TSVD.

2. REGULARIZATION FUNCTION

In case displacements of a structure are measured at selected observation points for all load cases, stiffness parameters are estimated by minimizing the following least squared errors between measured and calculated displacements (Park et al. 2001).

$$\underset{\mathbf{X}}{\text{Minimize}}\ \Pi_E = \frac{1}{2}\sum_{i=1}^{nlc}\left\|\tilde{\mathbf{u}}_i(\mathbf{X}) - \bar{\mathbf{u}}_i\right\|_2^2 \quad \text{subject to}\ \ \mathbf{R}(\mathbf{X}) \le 0 \tag{1}$$

where Π_E, nlc, $\tilde{\mathbf{u}}_i$, $\bar{\mathbf{u}}_i$ and $\mathbf{R}$ are the error function, the number of load cases, calculated displacements by the mathematical model, measured displacements at observation points for load case i, and a constraint vector for the stiffness parameters, respectively. $\|\cdot\|_2$ denotes the 2-norm of a vector. The system parameter vector $\mathbf{X}$ consists of the stiffness property of each member, X_i. For the convenience of further discussions, the error function defined in (1) is rewritten in a single vector form as

$$\underset{\mathbf{X}}{\text{Minimize}}\ \Pi_E = \frac{1}{2}\left\|\tilde{\mathbf{U}}(\mathbf{X}) - \bar{\mathbf{U}}\right\|_2^2 \text{subject to}\ \ \mathbf{R}(\mathbf{X}) \le 0 \tag{2}$$

where $\tilde{\mathbf{U}}$ and $\bar{\mathbf{U}}$ are vectors obtained by arranging the vectors of the computed displacements and the measured displacements for each load case in a row

To obtain numerically stable and physically meaningful solutions of (2), a proper regularization function should be defined along with the minimization problem. The regularization functions defined by the 2-norm of the system parameter vector have been widely used in conjunction with the Tikhonov regularization scheme. However, the regularization function of the 2-norm yields smeared results in the damage detection, which means that the damage information of a member is transferred to other members. This smearing effect of the 2-norm lowers the resolutions of SI for the structural damage detection. To overcome the drawback of the 2-norm regularization function, this paper presents a regularization function defined by 1-norm of the system parameter vector.

$$\Pi_R = \left\|\mathbf{X} - \mathbf{X}_0\right\|_1 = \sum_{i=1}^{n}\left|X_i - (X_i)_0\right| \tag{3}$$

where $\mathbf{X}_0$ and n are the baseline value vector (Watson 1980) and the order of system parameter vector, respectively, and $\|\cdot\|_1$ denotes the 1-norm of a vector. The regularization function has to be imposed to the minimization problem (2) to stabilize the SI problem.

3. TRUNCATED SINGULAR VALUE DECOMPOSITION

The piecewise polynomial truncated singular value decomposition (PP-TSVD, Hansen 1998) is employed to impose the regularization function (3) in the optimization of the error function. In this method, the incremental solution of the error function is obtained by solving the quadratic sub-problems without the constraints. The noise-polluted solution components are truncated from the incremental solution. Finally, the regularity condition is imposed to restore the truncated solution components and the constraints. The above procedure is defined as follows.

$$\begin{aligned} &\underset{\mathbf{X}}{\text{Minimize}}\ \Pi_R = \|\mathbf{X}-\mathbf{X}_0\|_1 \ \text{ subject to } \mathbf{R}(\mathbf{X}) \le 0 \ \text{ and} \\ &\underset{\mathbf{X}}{\text{Minimize}}\ \Pi_E = \frac{1}{2}\|\tilde{\mathbf{U}}(\mathbf{X})-\overline{\mathbf{U}}\|_2^2 \end{aligned} \tag{4}$$

The incremental solution for the minimization of the error function is obtained by solving the following quadratic sub-problem (Park et al. 2001).

$$\underset{\Delta\mathbf{X}}{\text{Minimize}}\ \ \|\mathbf{S}\Delta\mathbf{X}-\mathbf{U}_{k-1}^r\|_2^2 \tag{5}$$

where $\Delta\mathbf{X}$, and $\mathbf{S}$ are the solution increment and the sensitivity matrix of the displacement fields with respect to the system parameters at the observation points, respectively, and the subscript k denotes the iteration count. The displacement residual $\mathbf{U}_{k-1}^r$ is defined as $\mathbf{U}_{k-1}^r = \overline{\mathbf{U}} - \tilde{\mathbf{U}}_{k-1}$, where $\tilde{\mathbf{U}}_{k-1}$ is the displacement field calculated by the system parameters at the previous iteration.

The truncated singular value decomposition (TSVD) (Hansen 1998) is utilized to filter out the noise-polluted solution components associated with smaller singular values.

$$\Delta\mathbf{X} = \sum_{j=1}^{t}\frac{\mathbf{z}_j^T\mathbf{U}^r}{\omega_j}\mathbf{v}_j + \sum_{j=t+1}^{n}\gamma_j\mathbf{v}_j = \Delta\mathbf{X}_t + \mathbf{q} \tag{6}$$

where t and ω_j are the truncation number and a singular value of $\mathbf{S}$ which has the descending order of $\omega_{\max} = \omega_1 \ge \omega_2 \ge \ldots \ge \omega_n = \omega_{\min} \ge 0$ while $\mathbf{z}_j$ (left singular vector, LSV) and $\mathbf{v}_j$ (right singular vector, RSV) are the j-th column vectors associated with ω_j, and γ_j are undetermined constants.

The incremental form of (4) is expressed with respect to $\mathbf{q}$ as follows.

$$\begin{aligned} &\underset{\mathbf{q}}{\text{Minimize}}\|\mathbf{q}-(\mathbf{X}_0-\mathbf{X}_{k-1}-\Delta\mathbf{X}_t)\|_1 \\ &\text{subject to } \mathbf{V}_t^T\mathbf{q}=0 \ \text{ and } \ \mathbf{X}_l-\mathbf{X}_{k-1}-\Delta\mathbf{X}_t \le \mathbf{q} \le \mathbf{X}_u-\mathbf{X}_{k-1}-\Delta\mathbf{X}_t \end{aligned} \tag{7}$$

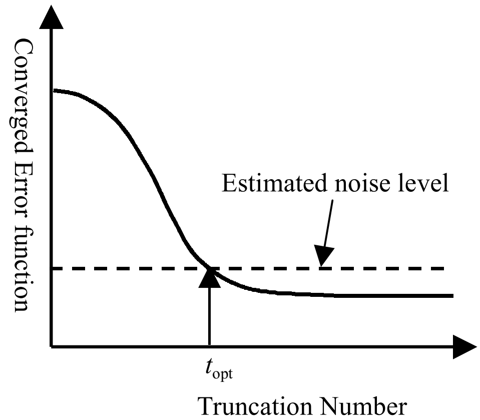


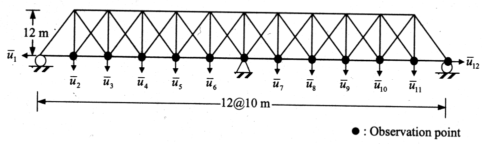
Figure 1. Optimal truncation number by the discrepancy principle

where $\mathbf{V}_t = (\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_t)$. The equality constraint of (7) represents that $\mathbf{q}$ should be a linear combination of the truncated RSVs. (7) is a linear programming with respect to $\mathbf{q}$ and is solved by the simplex method. Once the optimal solution of (7) is obtained from the linear programming, an one-dimensional line search is performed for the error function to accelerate convergence.

The determination of a proper truncation number is a keystone in the TSVD. The truncation number plays a similar role to the regularization factor in the Tikhonov regularization technique. In case a truncation number is too small, most of the useful information on a structure is lost while too large truncation number yields noise-polluted, meaningless solutions (Hansen 1998). Therefore, the truncation number should be determined so that as much useful information of a structure can be retained while most of noise-polluted solution components are truncated. The discrepancy principle, which is proposed for linear SI problems by Morozov, is employed to choose an optimal truncation number (Morozov, 1993). Based on this principle, the optimal truncation number can be determined as the largest one that satisfies the following criterion as shown in Fig 1.

$$\left\| \tilde{\mathbf{U}}(\mathbf{X}_t^*) - \overline{\mathbf{U}} \right\|_2 \geq \left\| \mathbf{e} \right\|_2 \tag{8}$$

where $\mathbf{X}_t^*$ is the converged solution obtained by the PP-TSVD with a fixed truncation number t for each iteration given in (7), and $\left\| \mathbf{e} \right\|_2$ is the 2-norm of noise in measurements. Since the noise magnitudes in measurements are unknown, the cross validation (Golub and Van Loan, 1996) is employed to estimate the 2-norm of noise components. As the discrepancy principle, the cross validation is proposed for linear SI problems. Recently, Haber and Oldenburg (2000) show that the 2-norm of noise components can be estimated in nonlinear SI problems by applying the cross validation to each linearized problem iteratively.



Member	Area(cm²)	Member	Area(cm²)
Top	250	Vertical	200
Bottom	300	Diagonal	220

Figure 2. Geometry, cross sectional areas and measured DOFs of the two-span continuous truss

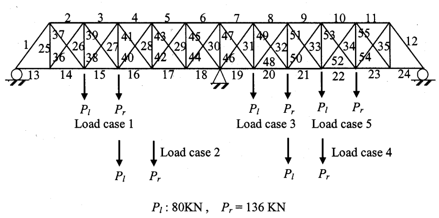


Figure 3. Member ID numbers and load cases of the two-span continuous truss

4. NUMERICAL EXAMPLES

Numerical simulation studies are performed with the proposed method to isolate damage of the two-span continuous truss presented. Fig. 2 shows the geometry, support conditions and the locations of 12 observation points, which are depicted as solid circles in the figure. Horizontal displacements are measured at the roller supports and vertical displacements are measured at the other observation points independently for each load case shown in Fig. 3.

Proportional random noise generated by a uniform probability function between ± noise amplitude is added to the displacement obtained by a mathematical model to simulate real measurements. The noise amplitude of 5% used throughout the presented examples. Based on the estimated noise level by the cross validation, the truncation number is determined by the discrepancy principle. The identification results by the proposed method are compared with those by the 2-norm regularization scheme, in which all the algorithms are exactly the same as the proposed method except that the 2-norm is used as the regularization function. Two damage cases are presented. Damage case I contains 70% damage in bottom member 16 and 30% damage in bottom member 21. A diagonal member (member 48) and a bottom member (member 22) are damaged by 30% in damage case II.

Fig. 4 shows the results of the damage isolation for damage case I. The 1-norm regularization scheme yields sharp drops of the stiffness parameters only at the damaged members, while the stiffness parameters of undamaged members in the vicinity of the damaged members are reduced in the 2-norm regularization scheme. In particular, most of the damage

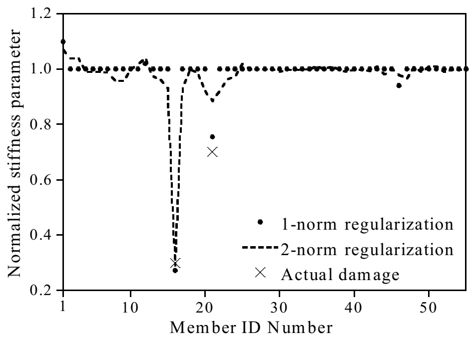


Figure 4. Identified stiffness parameters of damage case I

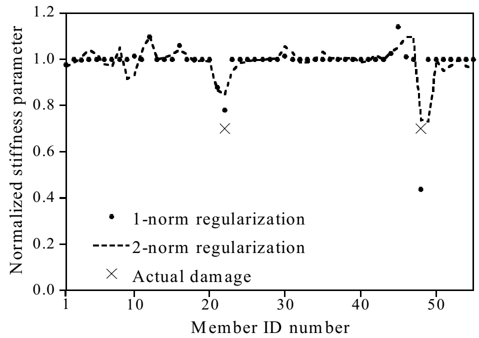


Figure 5. Identified stiffness parameters for damage case II

information of member 21 is smeared out to members 20 and 22 in the 2-norm regularization scheme. The damaged members are isolated exactly, and the damage severity is accurately estimated by the 1-norm regularization scheme. Some of the undamaged members are isolated as damaged members, and the damage severity of member 21 is rather underestimated by the 2-norm regularization scheme.

The results of damage isolation are illustrated in Fig. 5 for damage case II. The 1-norm regularization scheme yields sharp damage isolation for member 48, while the damage information on member 48 is smeared out to members 49 and 9 by the 1-norm regularization scheme. Member 21, which is a bottom member connected to member 22, is isolated as a damaged one by both scheme. Nevertheless, the 1-norm regularization scheme yields sharper damage isolation in an overall sense than the 2-norm regularization scheme.

5. CONCLUSIONS

A new damage isolation technique with an 1-norm of a system parameter vector is proposed for system identification in framed structures. The PP-TSVD is employed to perform SI with 1-norm regularization function. The discrepancy principle is utilized to select a truncation number for the TSVD, and the noise level in measurements is estimated by the cross validation. Numerical simulation studies are performed with the proposed algorithm to isolate damage in a two-span continuous truss. The proposed method yields shaper damage isolation than the 2-norm regularization scheme in the presented examples. Although this paper presents the damage isolation only for a truss, the proposed method can be applied to any types of framed structures. Also, it is believed that a stable and accurate damage detection algorithm could be formulated by the proposed 1-norm regularization function.

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