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AERONAUTICS APPLICATIONS

L_1-Regularization Functions for the SI of Structures

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ABSTRACT

A new class of regularization functions is proposed for the system identification structures. The regularity conditions based on the L_1-norm for the system property framed and continuous structures are investigated. The discretized regularity conditi is expressed by the 1-norm of the system parameters. The L_1-truncated singular val decomposition (L_1-TSVD) is employed to filter out noise-polluted solution compone and to impose the regularity condition. The validity of the proposed method is demo strated through isolating damage in a 2-span continuous truss and an inclusion in square plate.

INTRODUCTION

Various system identification (SI) algorithms have been proposed during last f decades to estimate stiffness properties of structures. The SI algorithms are defined the minimization of the least squared errors between measured and calculated respor es of structures. Unfortunately, the system identification is a type of ill-posed problen which suffers from discontinuity and non-uniqueness of solutions [1-4].

To overcome the ill-posedness of the SI, the regularization techniques are wide adopted [2]. In the regularization technique, a regularization function is considered define function spaces of SI. Most of previous studies usually use the 2-norm of syste parameters representing the discrete form of the L_2-norm of the solution space as t regularization function. The regularization functions defined by the L_2-norms of sy tem parameter effectively stabilize the ill-posedness of SI. However, the regularizati schemes using the 2-norm of system parameters yield smeared solutions of SI [3-4].

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which information on the stiffness properties of a member is shared with other members. The smearing effect is caused by smoothing characteristics of the L_2-norm. It is difficult to estimate structural properties accurately with smeared solutions of SI.

This paper presents a new class of regularization functions defined by L_1-norms of solution spaces of SI and their discrete form given by the 1-norms of the system parameter vectors. The L_1-regularization functions prevent the stiffness information of a member from smearing out to other members by restricting arbitrary changes of system parameters. Mathematical backgrounds of the L_1-regularization scheme are discussed. Since the 1-norm of a vector is not differentiable, the Newton-type solution scheme, which requires the gradient information of object function, is not suitable for the SI with the L_1-regularization function.

The L_1-truncated singular value decomposition (L_1-TSVD) is adopted to optimize the error function with the L_1-regularity condition. Two optimization problems are defined in the L_1-TSVD. The first optimization problem calculates the solution components of a quadratic sub-problem of the error function that are not polluted by measurement noise by truncating smaller singular values. The L_1-regularity condition is imposed in the second optimization, which is performed by the simplex method for each quadratic sub-problem of the error function. A truncation number in L_1-TSVD plays a crucial role in SI since the truncation number controls the stability and accuracy of the solution. The discrepancy principle [8] is employed to define an optimal truncation number.

The validity of the proposed method is demonstrated through numerical examples on a framed and a continuous structure. The proposed method reproduces sharp resolutions in the solution of SI while the stability is properly maintained by the L_1-TSVD.

L_1-REGULARITY CONDITION

The stiffness properties of a structure are estimated in SI by minimizing the least squared errors between measured and calculated responses under known loading conditions.

$$\underset{\mathbf{X}}{\text{Minimize}}\, \Pi_E = \frac{1}{2}\sum_{i=1}^{nlc}\left\|\tilde{\mathbf{u}}_i(\mathbf{X}) - \bar{\mathbf{u}}_i\right\|_2^2 \quad \text{subject to } \mathbf{R}(\mathbf{X}) \le 0 \tag{1}$$

Here, $\tilde{\mathbf{u}}_i$, $\bar{\mathbf{u}}_i$, $\mathbf{X}$, $\mathbf{R}$ and nlc are calculated displacements by the mathematical model, measured displacements at observation points for load case i, a system parameter vector, a constraint vector for the system parameters and the number of load cases, respectively, while $\|\cdot\|_2$ denotes the 2-norm of a vector. The system parameter vector $\mathbf{X}$ represents the discretized stiffness properties of a structure. The displacement field of a structure is calculated by a stiffness equation, which may be derived using the FEM or similar discretization methods.

$$\mathbf{K}(\mathbf{X})\mathbf{u}_i = \mathbf{P}_i \quad \text{for} \quad i = 1,\cdots,nlc \tag{2}$$

where $\mathbf{P}_i$ is the nodal load vector of load case i.

The SI problems defined by the minimization problem (1) exhibit strong instabilities characterized by the discontinuity and non-uniqueness of solutions. The instabilities become severe when the measured responses contain noise and/or the number of measured responses are smaller than that of the degrees of freedom in the mathematical model. To avoid the instabilities of SI problems, a proper solution space for a SI problem should be supplied along with the minimization problem (1). The Tikhonov regularization techniques are widely employed to specify a proper function space of the solution of (1). In Tikhonov regularization technique, a regularization function that is a 2-norm of the system parameter vector is added to the original error function in (1), and the optimization is performed for the modified objective function.

$$\underset{\mathbf{X}}{\text{Minimize}}\, \Pi = \frac{1}{2}\sum_{i=1}^{nlc}\left\|\tilde{\mathbf{u}}_i(\mathbf{X}) - \bar{\mathbf{u}}_i\right\|_2^2 + \frac{1}{2}\left\|\mathbf{X}-\mathbf{X}_0\right\|_2^2 \text{ subject to } \mathbf{R}(\mathbf{X}) \le 0 \tag{3}$$

where $\mathbf{X}_0$ denotes a priori information of the system parameters. The second term of (3) represents the regularization function.

The modified minimization problem (3) is equivalent to impose the following regularity of condition [5,6] to the original SI problem given in (1).

$$\left\|x - x_0\right\|_{L_2(V)}^2 = \int_V (x-x_0)^2 dV \approx \left\|\mathbf{X}-\mathbf{X}_0\right\|_2^2 < \infty \tag{4}$$

where x and x_0 denotes the system parameter and its a prior information before discretization, and the approximation symbol should be understood as the discretiztion procedure. The regularization function in (4) specifies that the solution of the SI should be a square integrable function. The regularity condition of a square integrable function can be given alternatively by the L_1-norm of the gradient of the system parameters.

$$\left\|\nabla(x-x_0)\right\|_{L_1(V)} = \int_V \left|\nabla(x-x_0)\right| dV < \infty \tag{5}$$

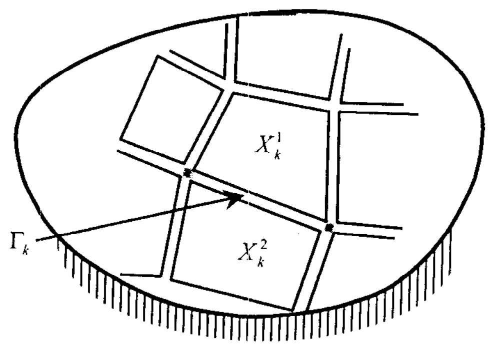


Figure 1. Inter-element boundaries in a structural domain

where ∇ is gradient operator. In case the domain of a structure is discretized by finite elements and the system parameters are constant within elements, the L_1-regularity condition (5) is discretized as follows.

$$\begin{aligned}\int_V |\nabla(x-x_0)|dV &\approx \sum_e \int_{V_e} |\nabla(x-x_0)|dV + \sum_{k=1}^{n_B} |(X_k^1 - X_k^2) - ((X_k^2)_0 - (X_k^2)_0)| l_k \\ &= \sum_{k=1}^{n_B} |(X_k^1 - (X_k^2)_0) - (X_k^2 - (X_k^2)_0)| l_k\end{aligned} \quad (6)$$

where n_B, and l_k are the number of inter-element boundaries and the length of the k-th inter-element boundary, respectively, while X_k^1 and X_k^2 are system parameters of two elements sharing the k-th inter-element boundary as shown in Fig. 1. Since the system property is assumed to be constant in a element, the domain integral in (6) vanishes. The second term of (6) represents jumps of the system parameters across inter-element boundaries. The discretized L_1-regularity condition is expressed by the 1-norm of the system parameter vectors using a discrete derivative operator $\mathbf{L}$.

$$\sum_{k=1}^{n_B} |(X_k^1 - (X_k^2)_0) - (X_k^2 - (X_k^2)_0)| l_k = \|\mathbf{L}(\mathbf{X}-\mathbf{X}_0)\|_1 \quad (7)$$

where $\|\cdot\|_1$ denotes the L_1-norm of a vector. The discrete derivative operator is determined based on the element connectivity and the geometry of elements.

The stiffness properties of members in framed structures are assumed to be concentrated at the centroid of cross section of each member. Therefore, the stiffness properties in a structural domain are the Dirac delta functions in the planes of member cross sections in a framed structure. Since the Dirac delta function is not a square integrable function, the regularity condition defined by the L_2-norm of system parameter is too stringent for a framed structure. A proper regularity condition of the system parameters for a framed structure should be defined by the L_1-norm as follows.

$$\|x-x_0\|_{L_1(V)} = \int_V |x-x_0|dV = \sum_e \int_{V_e} |x-x_0|dV = \sum_{i=1}^{n_M} |X_i - (X_i)_0| l_i < \infty \quad (8)$$

where n_M, X_i, $(X_i)_0$, and l_i are the number of members in a framed structure, the system parameter, baseline value of the system parameter and the length of member i, respectively. The discretized regularity condition is rewritten in the weighted 1-norm of the system parameter vector.

$$\sum_{i=1}^{n_M} |X_i - (X_i)_0| l_i = \|\mathbf{L}(\mathbf{X}-\mathbf{X}_0)\|_1 \quad (9)$$

where $\mathbf{L}$ is a diagonal weighting matrix that has diagonal entries equal to the lengths of elements. The discretized L_1-regularity condition (7) or (9) should be imposed to minimization problem (1) to obtain numerically stable and physically meaningful solutions of SI.

L_1-TRUNCATED SINGULAR VALUE DECOMPOSITION (L_1-TSVD)

This paper presents a new algorithm, which is referred to as the L_1-TSVD, to impose the L_1-regularity condition iteratively in the optimization of the error function using the TSVD. In the proposed method, the incremental solution of the error function is obtained by solving the quadratic sub-problems without the constraints. The noise-polluted solution components are truncated from the incremental solution. Finally, the regularity condition is imposed to restore the truncated solution components and the constraints. The above procedure is defined as follows.

$$\begin{aligned} &\underset{\mathbf{X}}{\text{Minimize}}\ \Pi_R = \|\mathbf{L}(\mathbf{X}-\mathbf{X}_0)\|_1 \ \text{subject to}\ \mathbf{R}(\mathbf{X}) \le 0 \ \text{and} \\ &\underset{\mathbf{X}}{\text{Minimize}}\ \Pi_E = \frac{1}{2}\|\tilde{\mathbf{U}}(\mathbf{X}) - \bar{\mathbf{U}}\|_2^2 \end{aligned} \tag{10}$$

where $\tilde{\mathbf{U}}$ and $\bar{\mathbf{U}}$ are vectors obtained by arranging the vectors of the computed displacements and the measured displacements for each load case in a row, respectively.

The incremental solution for the minimization of the error function is obtained by solving the following quadratic sub-problem [2].

$$\underset{\Delta\mathbf{X}}{\text{Minimize}}\ \|\mathbf{S}\Delta\mathbf{X} - \mathbf{U}'_{k-1}\|_2^2 \tag{11}$$

where $\Delta\mathbf{X}$, and $\mathbf{S}$ are the solution increment and the sensitivity matrix of the computed responses with respect to the system parameters at the observation points, respectively, and the subscript k denotes the iteration count. $\mathbf{U}'_{k-1}$ is the displacement residual vector between measured and calculated response vector at the previous iteration. The truncated singular value decomposition (TSVD) [3] is utilized to filter out the erroneous solution components associated with smaller singular values.

$$\Delta\mathbf{X} = \sum_{i=1}^{t} \frac{\mathbf{z}_i^T \mathbf{U}'}{\omega_i} \mathbf{v}_i + \sum_{i=t+1}^{n} \gamma_i \mathbf{v}_i = \Delta\mathbf{X}_t + \mathbf{q} \tag{12}$$

where t and ω_i are the truncation number and a singular value of $\mathbf{S}$ which has the descending order while $\mathbf{z}_i$ (left singular vector, LSV) and $\mathbf{v}_i$ (right singular vector, RSV) are the j-th column vectors associated with ω_i [7], and γ_i is a undetermined constant. The discrepancy principle [8] is employed to determine the optimal truncation number. The incremental form of (10) is expressed in terms of $\mathbf{q}$ as follows.

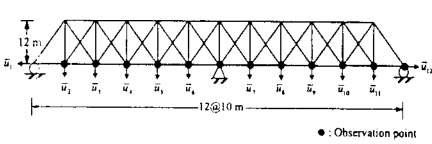
$$\begin{aligned} &\underset{\mathbf{q}}{\text{Minimize}} \|\mathbf{L}[\mathbf{q} - (\mathbf{X}_0 - \mathbf{X}_{k-1} - \Delta\mathbf{X}_t)]\|_1 \\ &\text{subject to}\ \mathbf{V}_t^T\mathbf{q} = 0 \ \text{and}\ \mathbf{X}_l - \mathbf{X}_{k-1} - \Delta\mathbf{X}_t \le \mathbf{q} \le \mathbf{X}_u - \mathbf{X}_{k-1} - \Delta\mathbf{X}_t \end{aligned} \tag{13}$$

where $\mathbf{V}_t = (\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_t)$. The equality constraint of (13) represents that $\mathbf{q}$ should be a linear combination of the truncated RSVs. (13) is a linear programming with respect to $\mathbf{q}$, and is solved by the simplex method [9]. Once the optimal solution of (13) is obtained from the linear programming, an one-dimensional line search is performed for the error function to accelerate convergence.

NUMERICAL EXAMPLES

Numerical simulation studies are performed with the proposed method to identify stiffness properties of a two-span continuous truss and an inclusion in a square plate. Fig. 2 shows the geometry, support conditions and the locations of observation points of a two-span continuous truss. Horizontal displacements are measured at the roller supports and vertical displacements are measured at the other observation points independently for each load case shown in Fig. 3. Fig. 4 shows the geometry, boundary conditions and material properties of a square plate. Fig 5 shows the element layout of the structural domain and the locations of observation points of the square plate, which are depicted as solid circles. Both horizontal and vertical displacements are measured at all observation points independently for each load case shown in Fig. 4.

Proportional random noise generated by a uniform probability function between ±5% noise amplitude is added to the displacement obtained by a mathematical model to simulate real measurements. The identification results by the proposed method are



Member	Area(cm²)	Member	Area(cm²)
Top	250	Vertical	200
Bottom	300	Diagonal	220

Figure 2. Geometry, cross sectional areas and measured DOFs of the two-span continuous truss

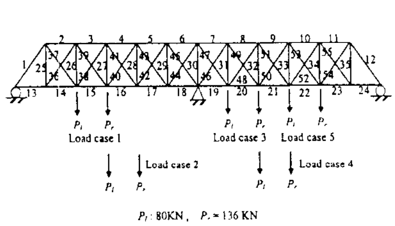


Figure 3. Member ID numbers and load cases of the two-span continuous truss

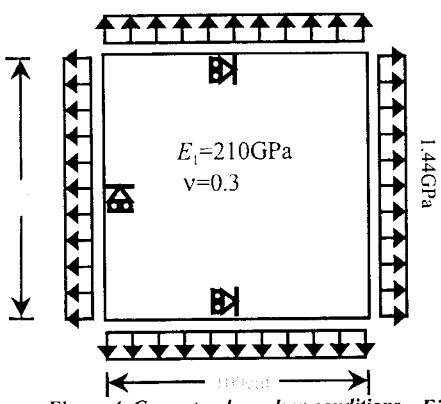


Figure 4. Geometry, boundary conditions and material properties of a square plate

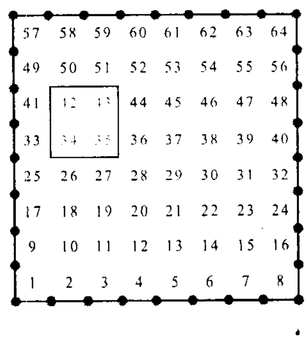


Figure 5. The predefined subgroups and the locations of observation points of a square plate

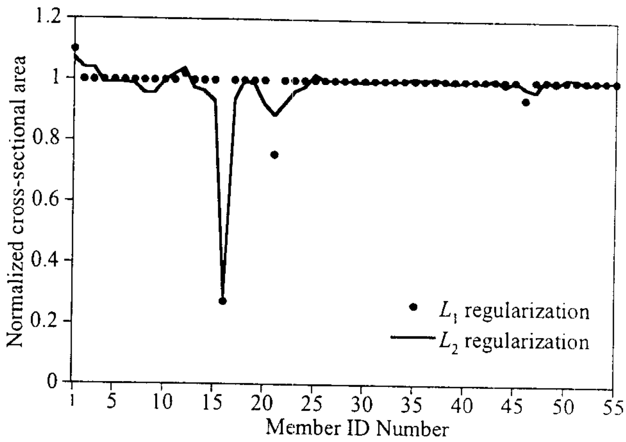


Figure 6. Identified cross-sectional area of a two span continuous truss

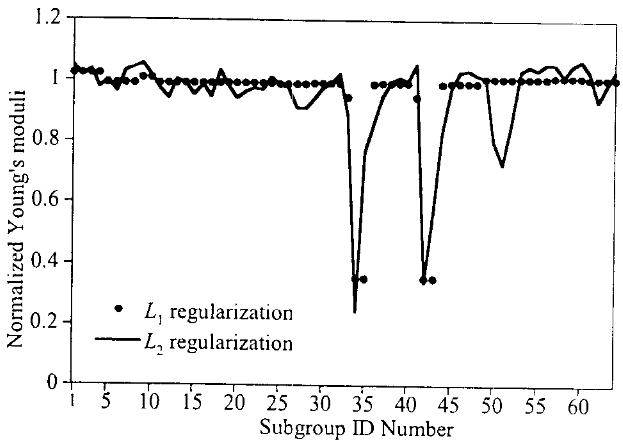


Figure 7. Identified Young's moduli of subgroups in a square plate

compared with those by the L_2-regualarization scheme, in which all the algorithms are exactly the same as the proposed method except that the 2-norm is used as the discrete regularization function.

Damage is simulated with 70% and 30% reduction in the sectional areas of two bottom members (member 16 and member 21) in the two-span continuous truss. The system parameters for this case are assumed to be cross-sectional areas of all members. As shown in Fig. 6, the L_1-regularization scheme yields sharp drops of the system parameters only at the damaged members, while the system parameters of undamaged members in the vicinity of the damaged members are reduced in the L_2-regularization scheme. In particular, most of the damage information of member 21 is smeared out to members 20 and 22 in the L_2-regularization scheme. The damaged members are isolated exactly, and the reduced cross-sectional areas are accurately estimated by the L_1-regularization scheme.

For the square plate, the inclusion is located at elements 34, 35, 42, and 43, and the Young's modulus of the inclusion is 34% of that of the matrix material. The Young's modulus of each element is selected as the system parameters As shown in Fig. 7, the L_1-regularization scheme yields sharp drops in the Young's moduli only at location of the inclusion. However, the L_2-regularization scheme yields considerably smeared results. The Young's moduli of elements 35 and 43 are considerably overestimated, and the Young's modulus of element 51 erroneously dropped. The location of the inclusion is isolated exactly, and the Young's modulus of the inclusion is accurately estimated by the L_1-regularization scheme.

CONCLUSION

A new regularization scheme based on the L_1-norm of the system property is proposed for the SI of structures. The L_1-TSVD is proposed for SI with L_1-regularization. Numerical simulation studies are performed with the proposed algorithm to identify damage in a two-span truss and an inclusion in a square plate. The proposed method accurately isolates damage and an inclusion in the examples while the regularization scheme with the 2-norm of the system parameters yields smeared results. It is believed that the L_1-regularity condition is capable of restoring exact structural information and alleviating the ill-posedness of SI problems.

REFERENCES

1. Bui, H. D. 1994. *Inverse problems in the mechanics of materials: An introduction*, CRC Press, Boca Raton
2. Park, H.W., Shin, S. B., and Lee, H. S. 2001. "Determination of an optimal regularization factor in system identification with Tikhonov Regularization for linear elastic continua," *International J. for Numerical Methods in Engineering*, 51(10), pp.1211-1230.
3. Hansen, P. C. 1998. *Rank-deficient and discrete ill-posed problems : numerical aspects of linear inversion*, SIAM, Philadelphia.
4. Groetsch, C. W. 1984. *The theory of Tikhonov regularization for Fredholm equations of the first kind*, Pitman Advanced Publishing, Boston
5. Johnson, C. 1987. *Numerical solution of partial differential equations by the finite element methods*, Cambridge University Press, New York.
6. Oden, J. T. 1979. *Applied functional analysis : a first course for students of mechanics and engineering science*, Prentice-Hall, Englewood Cliffs, NJ.
7. Golub, G. H., and Van Loan, C. F. 1996. *Matrix computations, 3rd ed.*, The Johns Hopkins University Press, London.
8. Morozov, V. A. 1993. *Regularization methods for ill-posed problems*, M. Stessin, translator, CRC Press, Boca Raton.
9. Barrondale, I., and Roberts, F. D. K. 1973. "An improved algorithm for discrete l_1 linear approximation," *SIAM J. Numerical Analysis*, 10(5), pp.839-848.