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Inverse Analysis for Identifying Soil Properties in Excavation Analysis by Elasto-plastic Soil Spring Model

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ABSTRACT

This paper presents an inverse analysis for identification of soil properties in excavation analysis by elasto-plastic soil spring model. The proposed method is based on the least square method to find soil properties that minimize errors between computed displacement by the finite element model and measured displacement. The integration of square of the errors over analysis domain is used as an objective function. Since the number of observation points is usually less than that of degrees of freedom of structures, eigenvectors of earth-retaining structures are used to interpolate measured displacements over analysis domain. To validate the proposed method, a numerical example with numerically simulated displacement data is presented.

INTRODUCTION

The elasto-plastic soil spring model for excavation analysis is used widely because of simplicity of the model. However, there usually exist notable differences between behaviors of earth-retaining structures computed by the finite element method based on elasto-plastic soil spring model and real behaviors of earth-retaining structures since the elasto-plastic soil spring cannot represent real mechanical behaviors of soil and complicated real situations cannot be included in the model. To predict behaviors of earth-retaining structures accurately during excavation process, it is necessary to employ an inverse analysis that adjusts material properties of soil to compensate the limitation of soil-spring model and the uncertainty of material properties of soil based on measured displacement of a structure in an average sense. This paper presents an inverse analysis technique to identify optimal material properties of soil that minimize the differences between analysis results and real behaviors of earth-retaining structures.

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Since an inverse analysis is based on a direct analysis, a rigorous excavation analysis technique based on elasto-plastic soil spring model is crucial for dependable inverse analysis. In this paper, the excavation analysis technique proposed by Kim *et al* (Kim 1996), which is proven to have the unique solution regardless of excavation steps, is employed. This paper intends to develop an inverse analysis algorithm, and the excavation analysis technique proposed by Kim *et al* (Kim 1996) is not presented here.

Displacement and material properties of soil are selected as the response variable and design variables, respectively. Currently, most of the inverse analysis algorithms are based on the minimization of least-squared errors between measured displacement and its counterpart computed by the finite element model at discrete observation points (Banan 1994). Because this approach estimates design variables based on local behaviors of structures, it may be difficult to take account of global behaviors of a structure. To find out the optimal material properties of soils based on the global behaviors of an earth-retaining structure, this paper presents a new class of the objective function that is an integration of the squared error between calculated displacement and measured displacement over a whole structure.

Since the number of observation points is usually less than the number of degrees of freedom of structures, measured displacements at observation points are expanded to the nodal points of the finite element model by use of eigenvectors of the stiffness matrix of an earth-retaining structure. The measured displacement function is interpolated by the shape functions used in the finite element model and the expanded displacements at the nodal points.

OBJECTIVE FUNCTION AND ITS INCREMENTAL FORM

To determine design variables that minimize errors between measured displacement and computed displacement by the elasto-plastic soil spring model, the following objective function is employed.

$$\Pi = \sum_{k=1}^{n} \frac{1}{2} \int_{\Omega} (u^k - \bar{u}^k)(u^k - \bar{u}^k) d\Omega \tag{1}$$

where n, Ω, u^k and $\bar{u}^k$ are the current excavation step, the analysis domain, the computed displacement by elasto-plastic soil spring model and the measured displacement function, respectively. The design variables that minimize the objective function are obtained as follows.

$$\frac{\partial \Pi}{\partial X_i} = \sum_{k=1}^{n} \int_{\Omega} u_{,i}^k (u^k - \bar{u}^k) d\Omega = 0 \qquad i = 1,\cdots,m \tag{2}$$

Here, X_i, $u_{,i}^k$ and m denote design variables, the sensitivity of displacement and the number of design variables, respectively. The comma in subscript represents differentiation with respect to design variable. Descretization of displacement and the sensitivity of displacement with suitable interpolation functions leads to the following matrix equation.

$$\sum_{k=1}^{n} [\mathrm{S}^k]^T [\mathrm{M}](\mathrm{u}^k) = \sum_{k=1}^{n} [\mathrm{S}^k]^T (\theta^k) \tag{3}$$

where $[S^k]$ is the matrix of sensitivity of displacement with respect to all design variables, and

$$[M] = \int_{\Omega} (N)^T (N) d\Omega \ , \quad (\theta^k) = \int_{\Omega} (N)^T \bar{u}^k d\Omega$$

(3) is a nonlinear algebraic simultaneous equation since displacement and its sensitivity are nonlinear functions of design variables. The design variables that satisfy (3) are obtained by applying the Newton-Raphson method to (3). The iteration updates of field variables are defined as

$$(\ )_r = (\ )_{r-1} + \Delta(\ ) \tag{4}$$

where r is iteration count for the inverse analysis. The incremental form of (3) is obtained by substitution of (4) into (3) and including only first-order incremental terms.

$$\sum_{k=1}^{n} ([\Delta S^k][M](u^k)_{r-1} + [S^k]_{r-1}^T [M](\Delta u^k) - [\Delta S^k]^T (\theta^k)) =$$
$$\sum_{k=1}^{n} ([S^k]_{r-1}^T (\theta^k) - [S^k]_{r-1}^T [M](u^k)_{r-1}) \tag{5}$$

Since the increment of displacement in (5) denotes change in displacement due to changes in design variables, the increment of displacement is expressed in terms of the sensitivity of displacement and the increment of design variables.

$$(\Delta u^k) = \sum_{j=1}^{m} \frac{\partial (u^k)_{r-1}}{\partial X_j} \Delta X_j = [S^k]_{r-1} (\Delta X) \tag{6}$$

The increment of the sensitivity of displacement can be written in terms of the increment of design variables using the second sensitivity of displacement. However, because it is difficult to evaluate the second sensitivity of displacement in history-dependent materials, the increment of the sensitivity of displacement is neglected in this paper. Neglecting the increment of the sensitivity of displacement in (5) and substituting (6) into (5), the final incremental expression of (3) is obtained.

$$\sum_{k=1}^{n} [S^k]_{r-1}^T [M][S^k]_{r-1} (\Delta X) = \sum_{k=1}^{n} ([S^k]_{r-1}^T (\theta^k)_{r-1} - [S^k]_{r-1}^T [M](u^k)_{r-1}) \tag{7}$$

The sensitivity of displacement is easily obtained by differentiating the converged incremental expression of the equilibrium equation (Vidal 1992, Vidal 1993).

INTERPOLATION OF MEASURED DISPLACEMENT FUNCTION

To evaluate (θ^k) in the right-hand side of (7), the displacement function has to be measured. Since displacements are measured at finite observation points, the measured displacement function is approximated with eigenvectors of the stiffness matrix of a structure and the shape functions used in a finite element model. For the convenience of derivation of equations, it is assumed that locations of observation points always coincide those of nodes of a finite element model.

The increment of measured displacement at each node of finite element model can be expressed by the linear combination of the eigenvectors of the stiffness matrix of an earth-retaining structure.

$$(\Delta\bar{u}^k) = \sum_{i=1}^{ndof} c_i^k(\phi^k)_i = [\phi^k](c^k) \tag{8}$$

where $ndof$ and $(\phi^k)_i$ are the number of degrees of freedom in the finite element model and the eigenvectors of the stiffness matrix, respectively. In case that displacements are measured at all nodes, the coefficients c_i^k are obtained by inverting (8). However, because the number of observation points is less than the number of degrees of freedom of the finite element model, the increment of displacement at each node is approximated with n lower order eigenvectors.

$$(\Delta\bar{u}^k) \approx \sum_{i=1}^{n} c_i^k(\phi^k)_i = [\bar{\phi}^k](\bar{c}^k) \tag{9}$$

Here, n is the number of observation points, and $[\bar{\phi}^k]$ is the matrix that consists of n lower order eigenvectors. The n coefficients in (9) are obtained by equating the measured displacement to interpolated displacement by (9) at each observation point.

$$(\bar{c}^k) = [\tilde{\phi}^k]^{-1}(\Delta\tilde{u}^k) \tag{10}$$

where $[\tilde{\phi}^k]$ is matrix that consists of the components of the n lower order eigenvectors at observation points, and $(\Delta\tilde{u}^k)$ is the measured displacement vector at observation points. Using (9), (10) and the same shape function used in finite element model, the measured displacement function is approximated as

$$\Delta\bar{u}^k = (\mathrm{N})(\Delta\bar{u}^k) = (\mathrm{N})[\bar{\phi}^k][\tilde{\phi}^k]^{-1}(\Delta\tilde{u}^k) \tag{11}$$

By use of (11), (θ^k) in the right-hand side of (7) is expressed as follows.

$$\begin{aligned}(\theta^k) &= \int_\Omega (\mathrm{N})^T \bar{u}^k d\Omega \\ &= \int_\Omega (\mathrm{N})^T \bar{u}^{k-1} d\Omega + \int_\Omega (\mathrm{N})^T \Delta\bar{u}^{k-1} d\Omega \\ &= (\theta^{k-1}) + [\mathrm{M}][\bar{\phi}^k][\tilde{\phi}^k]^{-1}(\Delta\tilde{u}^k)\end{aligned} \tag{12}$$

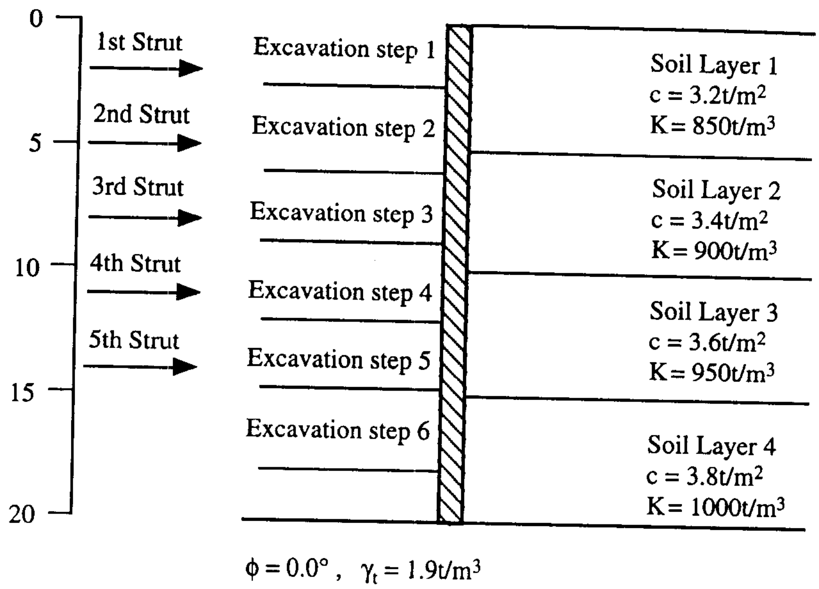

Figure 1. Material properties of soil layers, excavation step and geometry of the earth-retaining structure

NUMERICAL EXAMPLES

The proposed method is applied to an excavation analysis. The excavation site consists of four different soil layers. Material properties of each soil layer and the earth-retaining wall, location of struts, and excavation step are given in Fig. 1. The earth-retaining wall and struts are modeled as a beam and elastic springs, respectively. The wall is discretized by 20 finite elements that employ the Hermitian shape function. Displacement of the wall is measured at 9 observation points which locate at every 2m. The numerically computed displacement at each excavation step with the given material properties of the soil layers and the finite element model is used as the measured displacement data.

The material properties of each soil layer evaluated after the fifth excavation step by the proposed method are compared with the given material properties in table 1. The errors of K are larger that those of c. This is because K has little influence on the behaviors of the structure compared to c. Table 2 shows the errors of the axial force of each strut after the sixth excavation step computed by the material properties of each soil layer estimated after the fifth excavation step. The material properties estimated by the proposed method yield very accurate results for the axial force of each strut within 5% error. The average error of displacement evaluated by the following formula after the sixth excavation step is 2.5 %.

$$E = \sqrt{\frac{\int_{\Omega} (u_{exa} - u_{inv})^2 d\Omega}{\int_{\Omega} u_{exa}{}^2 d\Omega}} \times 100 \tag{13}$$

Table 1. The estimated material properties of each soil layer after the fifth excavation step

Layer	Material properties	Given value	Estimated value	Error (%)
1	*K*	850	2917	243
	c	3.20	0.95	70
2	*K*	900	1458	62
	c	3.40	4.01	18
3	*K*	950	476	50
	c	3.60	3.42	5
4	*K*	1000	1154	15
	c	3.80	3.82	1

Table 2. The errors of the axial force of each strut after the sixth excavation step

Strut	Exact value	Estimated value	Error (%)
1	-200	-209	5
2	163	166	2
3	169	170	1
4	90	90	0
5	41	41	3

where u_{exa} and u_{inv} are displacement computed by the given material properties and by the estimated material properties by present method.

CONCLUSION

A new parameter estimation algorithm is proposed to identify the optimal material properties of soil for the excavation analysis. The proposed method is based on the minimization of the integration of the squared error between measured displacement and computed displacement by the finite element model. The measured displacement function is interpolated by the eigenvectors of the structure and the measured displacement at the observation points. The proposed method is applied to a numerically simulated example. There are some errors in estimated material properties by the proposed method. However, the response variables such as displacement and the axial forces of the struts show very good agreement with the exact solution. The proposed method can be easily applied to other parameter estimation problems such as damage detection of structures.

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