System Identification in Time Domain for Structural Damage Assessment Using L_1-Regularization and Time Windowing Technique

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This paper presents a system identification (SI) scheme in time domain using measured acceleration data. The error function is defined as the time integral of the least square errors between the measured acceleration and the calculated acceleration by a mathematical model. Damping parameters as well as stiffness properties of a structure are considered as system parameters. The structural damping is modeled by the Rayleigh damping. A new regularization function defined by the L_1-norm of the first derivative of system parameters with respect to time is proposed to alleviate the ill-posed characteristics of inverse problems and to accommodate discontinuities of system parameters in time. The time window concept is proposed to trace variation of system parameters in time.

Key Words: System Identification, Regularization, Time Window

1. Introduction

Immediate safety assessment structures after an earthquake is extremely important in evaluating serviceability and functionality of social infrastructures. Nowadays, not only ground acceleration but also acceleration of important social infrastructures is monitored during earthquakes. It would be very helpful for quick restoration of social activities if structural damage caused by an earthquake is accessed with the measured acceleration during an earthquake in real time or near real time.

Various damage assessment schemes based on system identification (SI) have been extensively investigated for social infrastructures during the last few decades[1),2),3),4)]. The modal analysis approaches[2),3)] have been widely adopted to detect structural damage using measured acceleration of structures. The modal analysis approaches, however, suffer from drawbacks caused by insensitiveness of modal data to changes of structural properties.

To overcome the drawbacks of the modal analysis approaches, this paper presents a system identification scheme in time domain using measured acceleration data. The error function is defined as the time integral of the least square errors between the measured acceleration and the calculated acceleration by a mathematical model. The structural damping is modeled by the Rayleigh damping. A regularization technique[1),4),5)] is employed to overcome the ill-posedness of inverse problems. A regularization function defined by the L_1-norm of first time derivatives of stiffness parameters is proposed to accommodate abrupt changes of system parameters in time. The L_1-truncated singular value decomposition (TSVD)[5)] is adopted to optimize the error function with the L_1-regularization function. To trace the variation of stiffness parameters in time, a time windowing technique is introduced. In the time windowing technique, SI is performed sequentially within a finite time interval, which is called a time window. The time window advances forward at each time step to identify changes of system parameters in time. The validity and accuracy of the proposed method are demonstrated through two numerical simulation study.

2. Parameter Estimation Scheme In Time Domain

The discretized equation of motion of a structure subjected to ground acceleration a_g caused by an earthquake is expressed as follows.

$$\mathbf{Ma} + \mathbf{C}(\mathbf{x}_c)\mathbf{v} + \mathbf{K}(\mathbf{x}_s)\mathbf{u} = -\mathbf{Ma}_g \tag{1}$$

where **M**, **C** and **K** represent the mass, damping and stiffness matrix of the structure, respectively, and **a**, **v** and **u** are the relative acceleration, velocity and displacement of the structure to ground motion, respectively. The damping parameters and the stiffness parameters of the structure are denoted by $\mathbf{x}_c$ and $\mathbf{x}_s$ in (1), respectively. Newmark β-method is used to integrate the equation of motion. Since the operational vibrations of a structure are negligible compared to those induced by an earthquake, the initial condition of (1) is set to zero.

In case ground acceleration as well as accelerations of a given structure at some discrete observation points are measured, the unknown system parameters of a structure including stiffness and damping properties are identified through minimizing least squared errors between computed and measured acceleration. In case the system parameters are invariant in time, the parameter estimation procedure is represented by the following optimization problem[4].

$$\underset{\mathbf{x}}{\mathrm{Min}}\ \Pi_E(t) = \frac{1}{2}\int_0^t \left\| \tilde{\mathbf{a}}(\mathbf{x}) - \bar{\mathbf{a}} \right\|_2^2 dt \ \text{ subject to } \ \mathbf{R}(\mathbf{x}) \le 0 \tag{2}$$

where $\tilde{\mathbf{a}}$, $\bar{\mathbf{a}}$, **x** and **R** are the calculated acceleration and the measured acceleration at observation points relative to ground acceleration, system parameter vector and constraint vector, respectively, with $\|\cdot\|_2$ representing the 2-norm of a vector. Linear constraints are used to set physically significant upper and lower bounds of the system parameters. The minimization problem defined in (2) is a constrained nonlinear optimization problem because the acceleration vector $\tilde{\mathbf{a}}$ is a nonlinear implicit function of the system parameters.

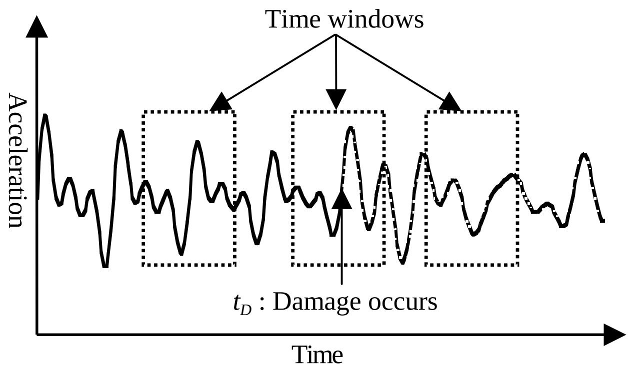

Fig. 1 Time window concept

In case the system parameters vary with time, the time window technique is proposed. Fig.1 illustrates the time window concept. In this technique the minimization problem for the estimation of the system parameters is defined in a finite time interval, which is referred to as a time window.

$$\begin{aligned} &\underset{\mathbf{x}}{\mathrm{Min}}\ \Pi_E(t) = \frac{1}{2}\int_t^{t+d_w} \left\| \tilde{\mathbf{a}}(\mathbf{x}(t)) - \bar{\mathbf{a}} \right\|_2^2 dt \\ &\text{subject to } \ \mathbf{R}(\mathbf{x}(t)) \le 0 \end{aligned} \tag{3}$$

Here, t and d_w is the initial time and the window size of a given time window. It is assumed that system parameters are constant in a time window, and that system parameters estimated by (3) represent the system parameters at time t. As the time window advances forward sequentially in time, the variations of system parameters in time are identified.

3. L_1-Regularization Scheme

The parameter estimation defined by the minimization problems is a type of ill-posed inverse problems. Ill-posed problems suffer from three instabilities: non-existence of solution, non-uniqueness of solution and discontinuity of solution[5] when measured data are polluted by noise. Because of the instabilities, the optimization problem given in (2) and (3) may yield meaningless solutions or diverge in optimization process. Attempts have been made to overcome instabilities of inverse problems merely by imposing upper and lower limits on the system parameters. However, it has been demonstrated by several researchers that the constraints on the system parameters are not sufficient to guarantee physically meaningful and numerically stable solutions of inverse problems[5].

The regularization technique proposed by Tikhonov is widely employed to overcome the ill-posedness of inverse problems. In the Tikhonov regularization technique, a positive definite regularization function is added to the original optimization problem.

$$\begin{aligned} &\underset{\mathbf{x}}{\mathrm{Min}}\ \ \Pi(t) = \Pi_E(t) + \lambda \Pi_R(t) \\ &\text{subject to } \ \mathbf{R}(\mathbf{x}) \le 0 \end{aligned} \tag{4}$$

where Π_R and λ are a regularization function and a regularization factor, respectively. Various regularization functions are used for different types of inverse problems. Kang et al[4] proposed the following regularization function defined by the L_2-norm for the SI in

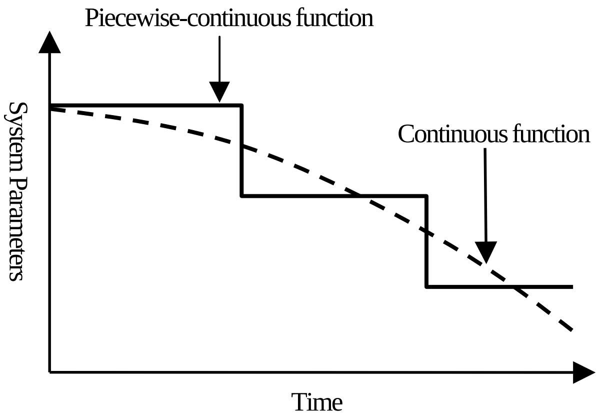

Fig. 2 Continuous and piecewise-continuous functions

time domain.

$$\Pi_R(t) = \frac{1}{2}\int_0^t \left\| \frac{d\mathbf{x}}{dt} \right\|_2^2 dt \tag{5}$$

The regularization function defined in (5) is able to represent continuously varying system parameters in time. Since, however, the system parameters may vary abruptly (Fig.2) with time during earthquakes due to damage, a regularization function that can accommodate piecewise continuous functions in time is required to access damage that occurs during an earthquake. To represent discontinuity of system parameters in time, this paper proposes an L_1-regularization function of the first derivative of system parameters with respect to time.

$$\Pi_R(t) = \frac{1}{2}\int_t^{t+d_w} \left\| \frac{d\mathbf{x}}{dt} \right\|_1 dt \tag{6}$$

where $\|\cdot\|_1$ representing the 1-norm of a vector.

Since the error function is nonlinear with respect to stiffness parameters, a Newton-type optimization algorithm, which requires gradient information of an objective function, is usually employed in SI. As the L_1-regularization function is non-differentiable, the objective function in the Tikhonov regularization scheme defined in (4) contains a non-differentiable function, and thus a Newton-type optimization algorithm cannot be applied. To avoid this difficulty, this paper employs the L_1-TSVD to impose the L_1-regularization function in the optimization of the error function[5),6)]. In the proposed method, the incremental solution of the error function is obtained by solving the quadratic subproblems without the constraints. The noise-polluted solution components are truncated from the incremental solution. Finally, the regularization function is imposed to restore the truncated solution components and the constraints. The above procedure is defined as follows.

$$\begin{aligned} &\min_{\mathbf{x}} \Pi_R(t) = \frac{1}{2}\int_t^{t+d_w} \left\| \frac{d\mathbf{x}}{dt} \right\|_1 dt \\ &\text{subject to } \mathbf{R}(\mathbf{x}) \le 0 \text{ and} \\ &\qquad \min_{\mathbf{x}} \Pi_E(t) = \frac{1}{2}\int_t^{t+d_w} \left\| \tilde{\mathbf{a}}(\mathbf{x}) - \bar{\mathbf{a}} \right\|_2^2 dt \end{aligned} \tag{7}$$

The error function and the regularization function are easily discretized in time domain using simple numerical methods. The truncated solution of the minimization problem of the error function is obtained by the truncated singular value decomposition, while the simplex method is employed to solve the minimization problem of the L_1-regularization function with constraints. Detailed solution procedures are presented in Ref. 5 and 6.

4. Damping Model

It is a difficult task to model damping properties of real structures. In fact, existing damping models cannot describe actual damping characteristics exactly, and are approximations of real damping phenomena to some extents[7)]. Since the damping has an important effect on dynamic responses of a structure, the damping properties should be considered properly in the parameter estimation scheme. In most of previous studies on the parameter estimation, the damping properties of a structure are assumed as known properties, and only stiffness properties are identified. However, the damping properties are not known a priori and should be included in system parameters in the SI.

Among various classical damping models, the modal damping and the Rayleigh damping are the most frequently adopted model. In the modal damping, a damping matrix is constructed by using generalized modal masses and mode shapes. In Rayleigh damping, a damping matrix is defined as a linear combination of the mass matrix and stiffness matrix as follows.

$$\mathbf{C} = a_0\mathbf{M} + a_1\mathbf{K} \tag{8}$$

The damping coefficients of the Rayleigh damping can be determined when any two modal damping ratios and the corresponding modal frequencies are specified.

In case the modal damping is employed in the parameter estimation, the number of the system parameters associated with the damping is equal to that of the total number of DOFs, which increases the total

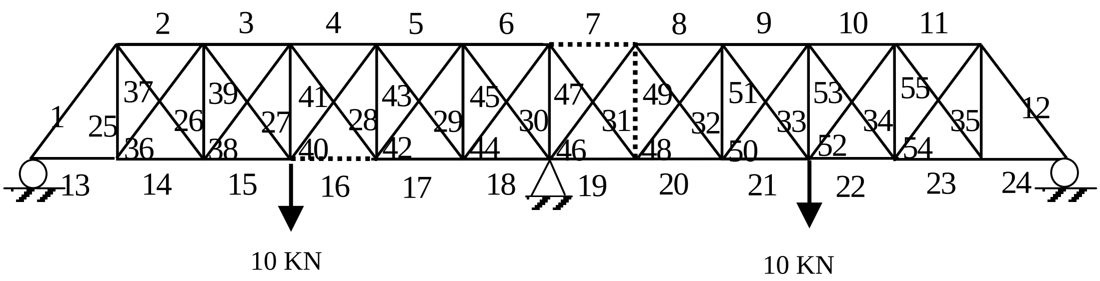

Fig.3 2-span continuous truss

number of unknowns in the optimization problem given in (8). Since neither modal damping nor Rayleigh damping can describe actual damping exactly, and the modal damping requires more unknowns than the Rayleigh damping in the parameter estimation, this study employs the Rayleigh damping for the SI. The Rayleigh damping yields a linear fit to the exact damping of a structure. To approximate actual damping of a structure more accurately, Caughey damping, which is the general form of the rayleigh damping[7)], may be adopted.

$$\mathbf{C} = \mathbf{M}\sum_{i=0}^{J-1} a_i (\mathbf{M}^{-1}\mathbf{K})^i , \quad J \le ndof \tag{9}$$

where *ndof* is the total number of degrees of freedom of the given structure. For J=2, the Caughey damping becomes identical to the Rayleigh damping.

5. Examples

Two numerical simulation studies are presented to illustrate validity of the L_1-regularization function and the time window technique.

5.1 two-span continuous truss

The validity of the proposed L_1-regularization function is examined through a simulation study with a two-span continuous truss shown in Fig. 3. Typical material properties of steel (Young's modulus = 210 GPa, Specific mass = 7.85Kg/m^3) are used for all members. The cross sectional areas of top, bottom, vertical and diagonal members are 112.5 cm^2, 93.6 cm^2, 62.5 cm^2 and 75.0 cm^2, respectively. The natural frequencies of the truss range from 6.6 Hz to 114.7 Hz. Damage of the truss is simulated with 40%, 50% and 34 % reductions in the sectional areas of member 7, 16 and 31, respectively. The damaged members are depicted by dotted lines in Fig. 7. It is assumed that the stiffness and the damping properties of the truss do not vary during the measurement. Accelerations of the truss are measured from a free vibration induced by a sudden release of applied loads of 10 KN shown in Fig. 7. The measurement errors are simulated by adding 8% random noise generated from a uniform probability function to accelerations calculated by the finite element model. The observation points are located at 12 bottom nodes of the truss. Both *x*- and *y*- direction accelerations are measured in the time period from 0 sec to 0.2 sec with the interval of 1/200 sec. The modal damping ratios used in the calculation of measured accelerations are shown in Fig. 6.

In case either the regularization scheme or damping estimation is not included in the SI, the optimization procedure does not converge or converges to meaningless solutions. Therefore, only the results with the regularization scheme and damping estimation are presented here. Fig.4 and Fig.5 show the variation of the identified stiffness properties of damaged member 7 and undamaged member 46 by the L_1- and L_2-regularization function, respectively. Although rather large measurement noise of 8% is presented, both regularization functions are able to identify the severity of damage of the damaged member accurately, and yield no significant differences in identified results. Estimated stiffness properties of the other damaged members are similar to those of damaged member 6, which are not presented in this paper. For a undamaged member, however, the L_1-regularization function yields much faster convergence rate in time than the L_2-regularization function. Since the parameter estimation is performed in finite time interval in the time window technique, the identification results of all system parameters should converge within a given time window. Therefore, fast convergent characteristic of the L_1-regularization function is extremely important in the time window technique. Fig. 6 shows the exact modal damping ratios used for the calculation of measured accelerations together with identified modal damping ratios by the Rayleigh damping. The initial modal damping ratios calculated by the assumed Rayleigh damping coefficients are also drawn in the same

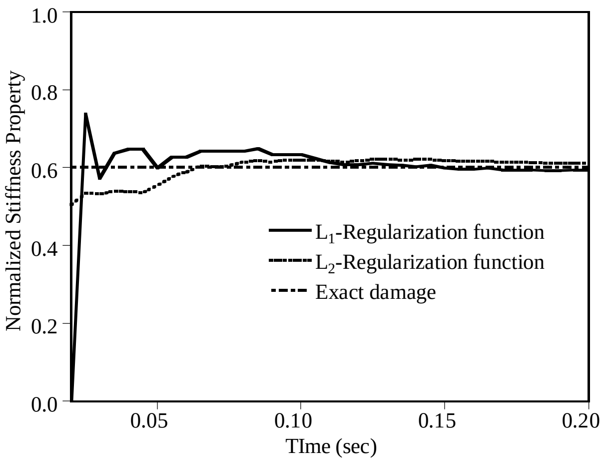

Fig.4 Variation of stiffness property of member 7

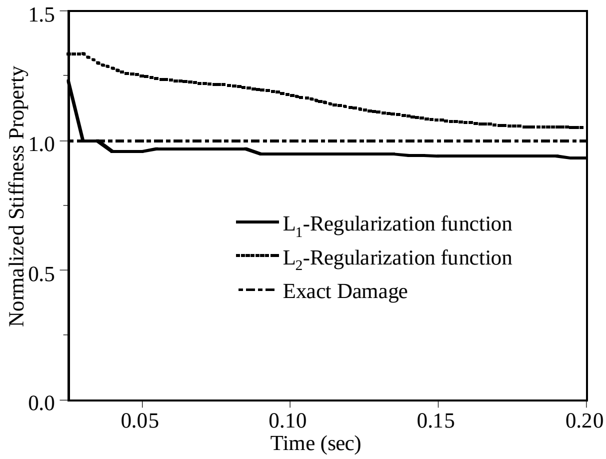

Fig.5 Variation of stiffness property of member 46

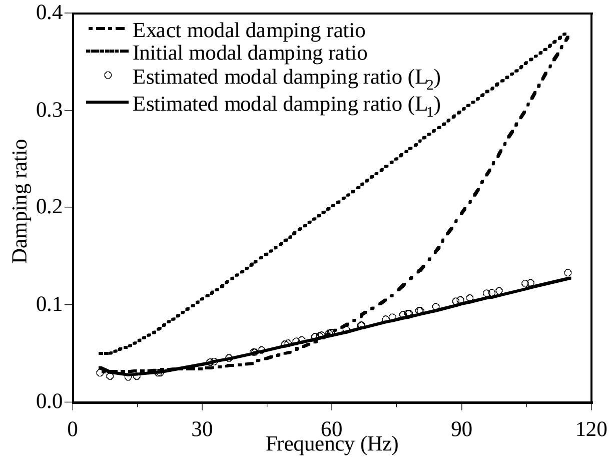

Fig.6 Estimated damping ratio

figure. Both regularization functions yield almost identical results for the damping ratios, and well approximate the real modal damping. The identified stiffness properties of all members at the final time step are shown in Fig. 7. Since stiffness properties of the damaged members reduce prominently compared with the oscillation magnitudes of the other members, the damaged mem-

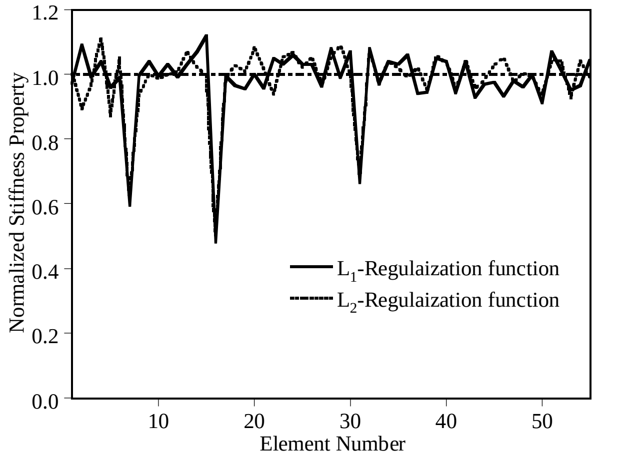

Fig.7 Estimated stiffness properties at the final time step

bers are clearly assured by both regularization functions. Since the L_1- and the L_2-regularization function specify different continuity conditions in time domain only, identified results at the final time by the two regularization schemes are very similar to each other.

5.2 SDOF system

Fundamental investigations on the time window technique are performed in this example through a SDOF system. Since it is difficult to apply a regularization scheme in a SDOF system, a regularization function is not applied to the error function. Fig. 8 shows the SDOF system used in this example. The initial stiffness of spring, the mass and the damping ratio of the SDOF system are 297.88 N/m, 10.24 Kg and 3 %, respectively. A free vibration of the system is introduced by sudden release of an applied load of 100 N. Damage of the spring occurs at time 5 sec and the stiffness of the spring reduces by 50%. The acceleration of the system is measured for 20 sec at the sampling rate of 40/sec. Noise in measurement is not considered, and the stiffness of the spring is the only system parameter in this example.

Fig.9 shows the estimated stiffness of the spring with time window size of 0.25 sec and 1.0 sec. The time window size of 0.25 sec yields severely oscillatory results after damage, which are suppressed by the longer time window size of 1.0 sec. At the current stage of

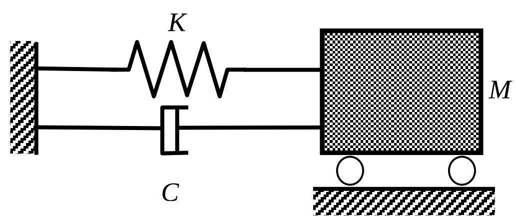

Fig. 8 SDOF system

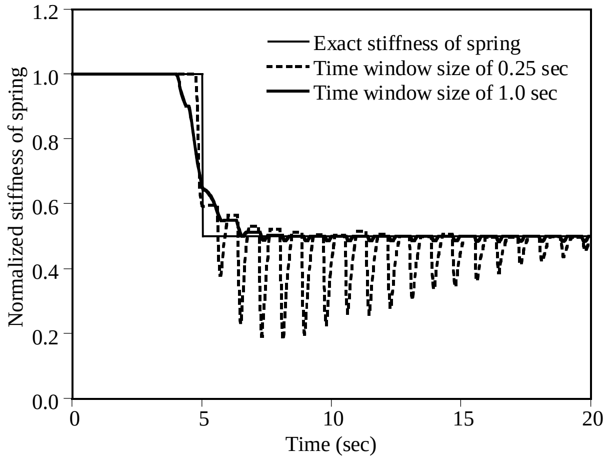

Fig.9 Estimated spring stiffness by time window

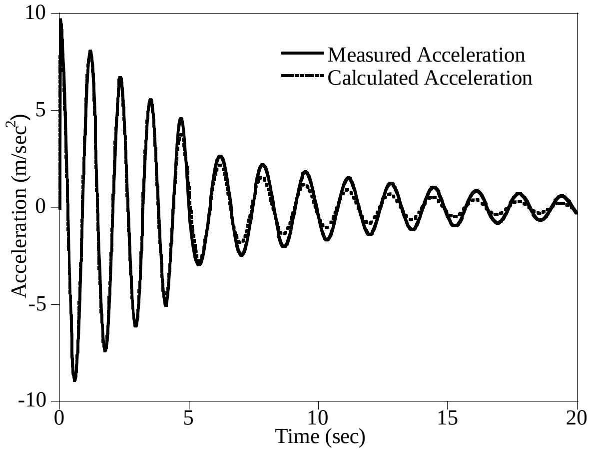

Fig.10 Measured and calculated acceleration

research, it is believed that the oscillations are caused by inaccurate estimation of the acceleration at the time of damage that acts as noise in the initial condition. In case that a longer time window is used, the effect of noise in the initial condition is smeared out by larger amount of measured data included in the window. Therefore, the regularization scheme proposed in this paper can suppress the oscillations of identified results after damage if properly applied. Fig. 10 shows accelerations calculated by the estimated stiffness. The accelerations after damage are underestimated.

6. Conclusion

The L_1-regularization function and the time window technique are proposed for SI in time domain using measured acceleration data is proposed. The system parameters include the damping parameters as well as the stiffness parameters of a structure. The Rayleigh damping is used to estimate the damping characteristics of a structure. The least square errors of the difference between calculated acceleration and measured acceleration is adopted as an error function. The regularization technique is employed to alleviate the ill-posedness of the inverse problem in SI. The L_1-TSVD is utilized to optimize a non-differential object function.

The L_1-regularization function exhibits very compromising characteristic of fast convergence, which is very crucial in the time window technique. Although the time window technique has not been fully developed yet, the example presented in this paper shows capabilities of the time window technique for the identification of damage caused by earthquakes.

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