PROBABILISTIC ANALYSIS OF FATIGUE CRACK GROWTH USING MOMENT METHOD

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Summary This paper presents an incremental approach to predict the path and probabilistic distribution of fatigue crack growth using the dual boundary element method. A new iterative scheme using a secant method is proposed to model curved crack growth. To predict the distribution and failure probability of fatigue crack, the second-order moment method using lognormal distribution is incorporated with the proposed incremental formulation.

INTRODUCTION

Fatigue crack growth is one of the most important factors in the design of the steel structures. Numerous experiment and researches have been performed for the prediction of fatigue crack growth [1]. This paper presents an incremental approach to predict the path and probabilistic distribution of the fatigue crack growth using a secant approach and moment method.

A fatigue crack usually does not grow in a straight line due to mixed-mode loadings or asymmetric geometry. Most numerical studies on crack growth adopt the incremental approach [2], in which the direction of crack growth is assumed as the tangent direction at the current crack tip without iterative modification. In this approach, the increment should be very small to describe the crack path because there is no correction. Some iterative scheme has been proposed to consider curved crack growth [3]. However, the increment of crack growth was considered as constant during the iterative modification. For each given increment of fatigue loading, the size of crack increment is evaluated in the integral of Paris-Erdogan equation along the crack path and should be modified according to the change of crack path. In this paper, a new iterative scheme is proposed considering the change of both the direction and growth rate.

Fatigue tests show great dispersion in the distribution of fatigue life due to many factors including imperfection of material [4]. Probabilistic approaches, such as the first-order reliability method (FORM) and the Monte-Carlo simulation (MCS), are devoted to estimate the failure probability. On the other hand, moment methods are adopted to estimate the moments of the response. Hong et al. [5] proposed a second-order third-moment method for the estimation of the failure probability. However, geometric changes according to crack growth were not considered because they used a very simplified form of Paris-Erdogan equation. In this paper, moment method is incorporated with the incremental formulation, in which geometric changes are taken into account.

FATIGUE CRACK GROWTH

The dual boundary element method (DBEM) [3] is employed for the analysis of two-dimensional plates with cracks. Paris-Erdogan law and maximum circumferential stress criterion are adopted for the crack growth rate and tangent direction, respectively. The mixed-mode stress intensity factors (SIF) are evaluated by J-integral. Crack growth is simulated by making an incremental crack growth for a given increment of fatigue loading. In each loading step, both crack increments and growth path are updated iteratively. In the first iteration, the direction and size of crack increment are determined by using the SIF obtained at the current crack tip. In the later iterative modification, the SIF at the new crack tip is also utilized as well as the SIF at the previous one. For a given loading step, the size of increment is determined by evaluating iteratively the integral of Paris-Erdogan equation with trapezoidal rule. Each crack path is assumed as a parabola, but discretized as a straight line for the simplicity of integration in the DBEM. The parabola is modified iteratively utilizing the tangent direction at the new crack tip until the new direction converges to the tangent of the parabola assumed previously.

PROBABILISTIC ANALYSIS

Randomness in the initial crack, fatigue loading and material properties have an effect on the distribution of crack growth. The moment method gives an approximate solution to the problem of determining the behaviour of the dependent random variables. In the second-order moment method, response function is approximated by second-order Taylor expansion with respect to random variables. Approximations of the moments of the response function can be determined from those of the random variables. For simplicity, only the initial crack length is selected as a random variable. For each loading step, in the proposed incremental formulation, the moments of crack length are approximated by those of the previous loading step. The derivatives used in the approximations of moments are derived from the Paris-Erdogan equation. In this study, two- and three-parameter lognormal distribution is used to estimate the crack growth. In FORM, several crack-growth analyses for a given service life are required for the optimization to find the design point in the initial distribution. On the other hand, in moment method, failure probability is directly predicted from the distribution derived at each loading step, and so only one crack-growth analysis is required. It is assumed that failure occurs when the fatigue crack reaches a critical length defined by the toughness of a material.

EXAMPLE

The proposed method is applied to 20cm× 40cm rectangular steel plate (E = 21,000 kN/cm^2, ν = 0.3) with single edge-crack in the middle of the left side. This plate is subject to constant-amplitude loading which ranges from zero to 16.5 kN/cm^2. The coefficients of the Paris-Erdogan equation are $C = 1.886\times10^{-10}$ and m = 3.0. Mean and standard deviation (SD) of the initial crack length are 0.2cm and 0.02cm, respectively. Critical crack length is about 2.13cm. This plate is discretized by 293 nodes and 140 quadratic boundary elements.
Fig. 1 shows the initial and derived crack distribution obtained by MCS with 10,000 samples after 400,000 loading. In case initial distribution is normal, derived distribution approaches lognormal. Table 1 shows that the proposed moment method well approximates the mean and SD while there is 13% difference in skewness. It is believed that mean and SD is more important than skewness in most engineering problems. Also, the difference in skewness has a very little effect on the estimation of failure probability as shown in Fig 2. Failure probabilities obtained by the proposed method are compared with the results of FORM in Fig. 2. In FORM, 4 ~ 6 times of crack-growth analyses are executed for a given service life. If failure probabilities are calculated for 10 service fatigue lives, around 50 times of crack-growth analyses are requited. Two-parameter lognormal shows good fit for the initial normal distribution. In case of initial lognormal distribution, three-parameter lognormal is appropriate.

CONCLUSIONS AND FURTHER STUDY

Proposed moment method produces a good approximation of the crack distribution compared with the results of MCS. For the calculation of the failure probability, moment method needs much less computational cost than FORM. It is shown that lognormal is appropriate for the fatigue crack distribution. Additional researches on multivariable reliability problems should be performed.

References

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Table 1. Crack distribution obtained from the proposed moment method and MCS

	Initial distribution (N=0)			Derived distribution (N=400,000)		
	Mean (cm)	SD (cm)	Skewness	Mean (cm)	SD (cm)	Skewness
Moment method	0.2000	0.0200	0.0000	1.3173	0.3764	0.8805
MCS (Size=10,000)	0.2001	0.0199	-0.0607	1.3170	0.3660	1.0140

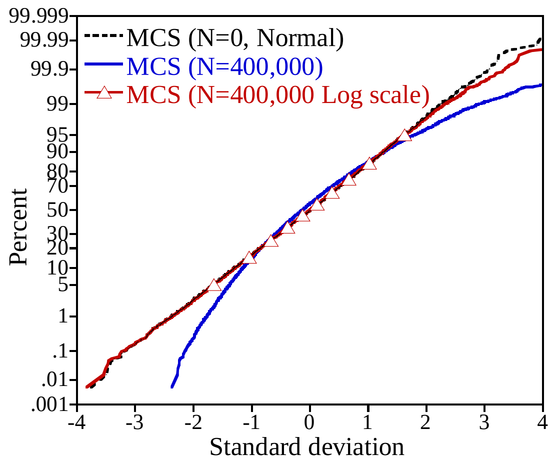

Fig. 1. Probabilistic paper of the crack distribution

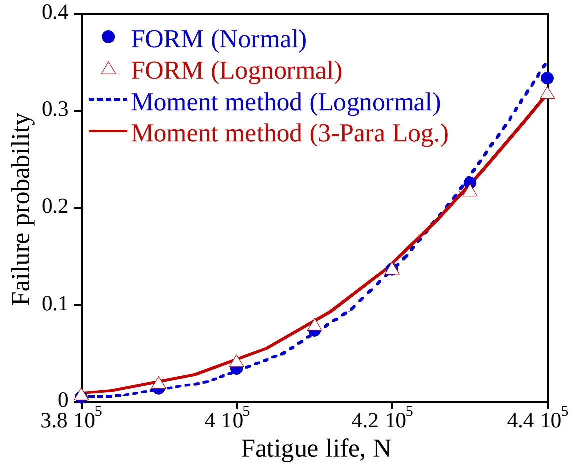

Fig. 2. Failure probability