Structural Health Monitoring using Dynamic Responses with Autoregressive Model

Joo Sung KANG and Hae Sung LEE

ABSTRACT

This paper presents a new structural health monitoring algorithm using dynamic responses, especially acceleration. Damage is defined as abrupt changes in some parameters of the considered structure. Abrupt change means that change of system parameter occurs either instantaneously or at least very fast with respect to the sampling rate of the measurements due to severe events such as earthquake, typhoon, crash and so on. Autoregressive model is employed to estimate whether the system is damaged or not by changes of residual error between measured and calculated acceleration. The key difficulty of structural health monitoring system is how to handle noises, whereas measured acceleration contain a mix of information related to both the damage in the structure and the perturbations due to the environment. A time windowing technique is utilized to prevent perturbations due to the environment in available measurement data. In time windowing procedure, the residual error is predicted sequentially within a finite time period, which is called a time window. The time window advances forward at each time step to predict residual step by step to detect damage of system parameters in time domain. An generalized extreme value distribution(GEV) is also adopted to raise reliability of proposed algorithm. The validity of proposed algorithm is demonstrated by a numerical simulation on a two-span truss bridge.

Key words *: structural health monitoring, abrupt change, autoregressive model, damage detection, time windowing, generalized extreme value distribution*

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INTRODUCTION

Over the last few decades, there has been a significant increase in the safety management field of the complex structure. The primary goal of the structural health monitoring is to find changes of system parameters and to decide its soundness at earliest possible stage. There are two categories in structural health monitoring and damage assessment whether structural model, such as stiffness, damping and mass information exist or not. One is model based scheme and the other is non-model based scheme. In model based scheme, system parameters are estimated by inverse analysis based on the sensitivity information from a mathematical model. In non-model based scheme, structural soundness is evaluated by pattern recognition and statistical approach from only measured signals without a structural model.

Model based system identification problem is a type of inverse problems, which are usually ill-posed problem. An ill-posed problem is characterized by the non-uniqueness, non-continuous and instability of solutions. Various regularization techniques have been developed to overcome this ill-posedness of inverse problem. In spite of ill-posedness can be alleviated by regularization techniques successfully, model-based system identification schemes are not applied in real situation because of modeling error that difference between mathematical model and real structural model. Recently, a lot of structural health monitoring researches with statistical pattern recognition using purely measured signals except a structural mathematical model are attempted in the center of Los Alamos national laboratory in USA. Autoregressive model is widely used in time series pattern analysis.

Various algorithms for structural health monitoring using static or dynamic signals are proposed. But the main problem of structural health monitoring system is how to handle noises, whereas measured signals contain a mix of information related to both the damage in the structure and the perturbations due to the environment. To prevent from effects of environment, new structural health monitoring algorithm with time windowing technique is employed. In time windowing technique, the residual errors are predicted sequentially within a finite time period. The time window advances forward at each time step to predict residuals repeatedly. Perturbations of environment are commonly changed gradually during long time period. Time window size is relatively very smaller than environmental perturbation period so it is assumed that perturbation of environment can be neglected within the time window.

Extreme value distribution is utilized for making decision boundary of soundness of the target structure. A generalized extreme value distribution(GEV) which unify three known extreme value distribution, Gumbel, Weibull and Flechet is used.

The validity and accuracy of the proposed algorithm is demonstrated through a numerical simulation studies on a two span truss bridge. The numerically generated acceleration data under kobe earthquake with noise are utilized as measured signals for the numerical simulation example.

AUTOREGRESSIVE MODEL

Autoregressive(AR) model is utilized to evaluate structural health monitoring system using acquired acceleration signals during a long period. Autoregressive model is widely used stochastic model that can be extremely useful in the representation of certain practically occurring series. In this model, the current value of the process is expressed as a finite, linear aggregate of previous values of the process and a random error e_t. Let us denote the values of a process at equally spaced times t, t-1, t-2, … by x_t, x_{t-1}, x_{t-2}, …. Then

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \cdots + \phi_p x_{t-p} + e_t \tag{1}$$

is called autoregressive model of order p. Where, ϕ is coefficients of autoregressive model, e_t is random error in the measured signal at time t and p is order of autoregressive model.

Least Square Method

Autoregressive model is expressed with coefficients as weighted regressive form. There are several methods to calculate coefficients of the autoregressive model. Least square method is utilized because it is very simple and clear. From Eq.1, residual error between estimated value from autoregressive model and measured value at time t is as follows.

$$e_t = x_t - \left(\phi_1 x_{t-1} + \phi_2 x_{t-2} + \cdots + \phi_p x_{t-p}\right) \tag{2}$$

First term in the right side of Eq.2 is a measured signal at time t and second term is the estimated value from autoregressive model at time t. After expansion of Eq.2 into considered time periods and minimize residual errors, the linear object function by least square method is obtained as shown in Eq.3.

$$\begin{gathered}\Pi = \underset{\phi}{Min} \sum_{t=p+1}^{N} \left\| e_t \right\| = \underset{\phi}{Min} \sum_{t=p+1}^{N} \left\{ x_t - \boldsymbol{\psi}_t^T(x)\, \boldsymbol{\varphi}(\phi) \right\}^2 \\ \boldsymbol{\psi}_t(x) = [-x_{t-1} \cdots - x_{t-p}]^T,\ \boldsymbol{\varphi}(\phi) = [\phi_1 \cdots \phi_p]^T\end{gathered} \tag{3}$$

Where, N is total number of measured signals in considered time period. N must be greater than twice of the order p of the autoregressive model. The optimal solution of Eq.3 is obtained like Eq.4 by least square method.

$$\boldsymbol{\varphi}(\phi) = \left[\sum_{t=p+1}^{N} \boldsymbol{\psi}_t(x) \boldsymbol{\psi}_t^T(x) \right]^{-1} \sum_{t=p+1}^{N} \boldsymbol{\psi}_t(x) x_t \tag{4}$$

After decision of coefficients of autoregressive model, foregoing signals can be predicted by using definition of autoregressive model in Eq.1. If there is no damage in the structure then residual errors are very small. Residual errors will be highly increased when some problem occurs in the structure. By using this phenomenon, autoregressive model can be utilized in structural health monitoring system.

The flow chart of structural health monitoring system with autoregressive model is shown in Fig.1. Measured signals are obtained from sensors and a prediction model is made of the autoregressive process. Coefficients of the autoregressive model and residual errors are estimated by a prediction model. Statistical treatments of obtained residual errors must be done for more reliable structural health monitoring. Finally, the decision making of soundness of considered structure in real time by monitoring residual errors continuously will be performed.

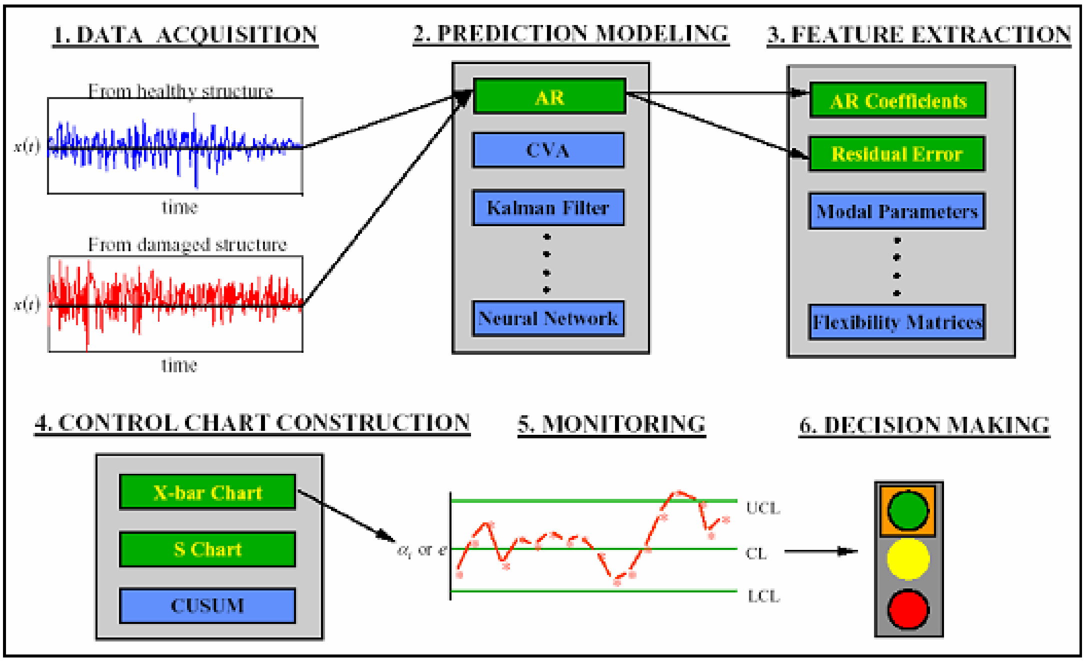

Figure 1. Structural health monitoring procedure with autoregressive model

TIME WINDOWING TECHNIQUE

The key difficulty in structural health monitoring is perturbation of measured signals by environmental factors. Measured signals are slightly changed according to various factors of environment such as day and night, season, temperature and humidity and so on even if there is no problem in the considered structure. Almost previous methods suffer from this difficulty of environmental factors. Though a algorithm is performed well in experimental data, it cannot be applied in perturbed signals in real structure. Time windowing technique is adopted to solve this problem. Environmental factors are commonly gradually changed during very long time period. In time windowing technique, residual errors are estimated using measured signals within finite time period which is called a time window. Time window size is relatively very smaller than environmental perturbation period so it is assumed that effects of environment can be neglected.

There is outline of the time windowing technique in Fig.2. Residual errors are estimated sequentially using measured signals in moving time window. Let us assume that an abrupt change occurs at time t_d by sudden effects. There is no change of system before abrupt change time t_d so residual errors between measured signals and predicted data by prediction model. Time goes on, measured signals after abrupt change time t_d will be included in time window. Then measured signals after time t_d discord from predicted value of previous autoregressive model and residual errors are increased. After more time elapsed and time window fully passed damaged time t_d, all the measured signals in time window are filled with damaged information so residual errors are decrease by recalculated coefficients of autoregressive model with signals of damaged information. Fig.2 shows that time window advances forward at each time step to predict residuals repeatedly. Fig.3 shows that a variation of residual errors according to time axis with a abrupt change during measured time period.

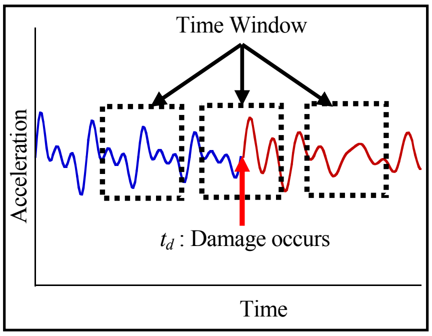

Figure 2. Time windowing technique

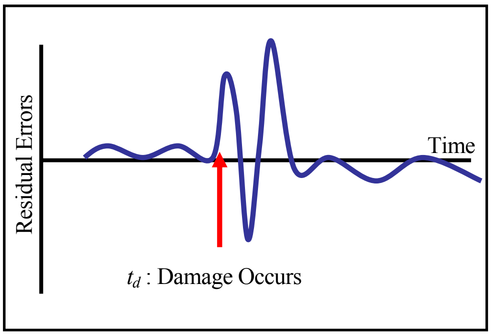

Figure 3. Residual error

GENERALIZED EXTREME VALUE DISTRIBUTION

Decision making using residual errors in every time step sequentially must be performed whether the considered structure is sound or not. It is unreasonable to decide health of the structure by merely the magnitude of residual errors. For more reliable decision making of structural health monitoring, we must find the distribution of residual errors and pick up outliers from the distribution of residual errors in a given significant level. Outliers almost lie in the tail of the distribution of residual errors. Extreme value distribution is utilized for an accurate selection of outliers because extreme value distribution is well established for tail distribution. Generally, it is known that any distribution follows one of three extreme value distributions, Gumbel, Weibull and Flechet distribution. The three type of distribution can be expressed in one single form, called von-mises form as shown in Eq.5.

$$\begin{aligned}
&\text{- Maxima : } \overline{G}(x;\lambda,\delta,c) = \exp\left\{-\left[1+c\left(\frac{x-\lambda}{\delta}\right)\right]^{-1/c}\right\} \quad \left(1+c\left(\frac{x-\lambda}{\delta}\right)\geq 0, \quad \delta>0\right) \\
&\text{- Minima : } \underline{G}(x;\lambda,\delta,c) = 1-\exp\left\{-\left[1+c\left(\frac{\lambda-x}{\delta}\right)\right]^{-1/c}\right\} \quad \left(1+c\left(\frac{\lambda-x}{\delta}\right)\geq 0, \quad \delta>0\right)
\end{aligned} \tag{5}$$

Optimization process is utilized to find three coefficients λ, δ, c. By minimizing difference between extreme value distribution and empirical cumulative density function like Eq.6, optimal distribution of residual errors can be obtained.

$$Min\frac{1}{2}\left[\mathbf{G}(x;\lambda,\delta,c)-\mathbf{p}\right]^{T}\mathbf{W}\left[\mathbf{G}(x;\lambda,\delta,c)-\mathbf{p}\right] \tag{6}$$

Where, **G** is generalized extreme value distribution, **p** is empirical cumulative density function of estimated residual errors and **W** is the weighting matrix.

From the optimal distribution of residual errors, threshold value of residual errors can be found in given significance level. Residual errors beyond this threshold value are defined as outliers. It can be said that abrupt change occurs in the considered structure by detected outliers. A real-time structural health monitoring system can be consist by repeating this sequence continuously..

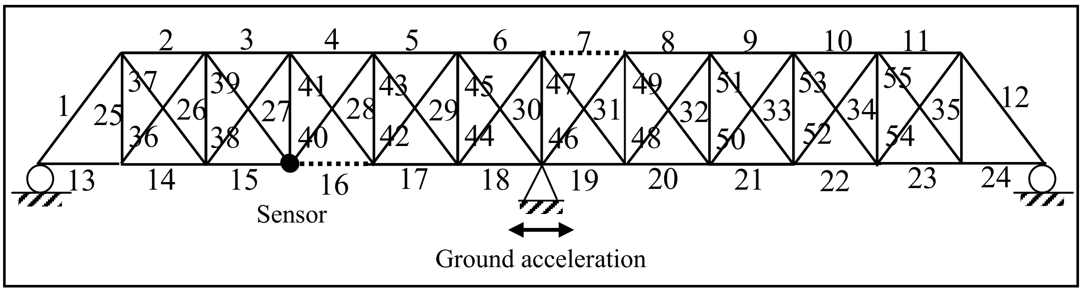

Figure 4. 2-span continuous truss

EXAMPLE

The validity of the proposed structural health monitoring algorithm is examined through a simulation study with a two-span continuous truss shown in Fig. 4. Typical material properties of steel (Young's modulus = 210 GPa, Specific mass = 7.85Kg/m^3) are used for all truss members. The cross sectional areas of top, bottom, vertical and diagonal members are 112.5 cm^2, 93.6 cm^2, 62.5 cm^2 and 75.0 cm^2, respectively. The natural frequencies of the truss range from 6.6 Hz to 114.7 Hz. It is assumed that accelerations are measured using kobe earthquake ground acceleration at centered hinge support in the horizontal direction for simulating under earthquake situation. Input ground acceleration is shown in Fig.5. The damping characteristics are modal damping 3%~30%, continuously. The sensor point is center of the left span in bottom nodes of the truss. Abrupt change occurs in the considered structure at 3 sec. Vertical direction accelerations are obtained numerically in the time period from 0 sec to 4 sec with the interval of 1/200 sec. The measurement errors are simulated by adding 5% random proportional noise to accelerations calculated by the finite element model.

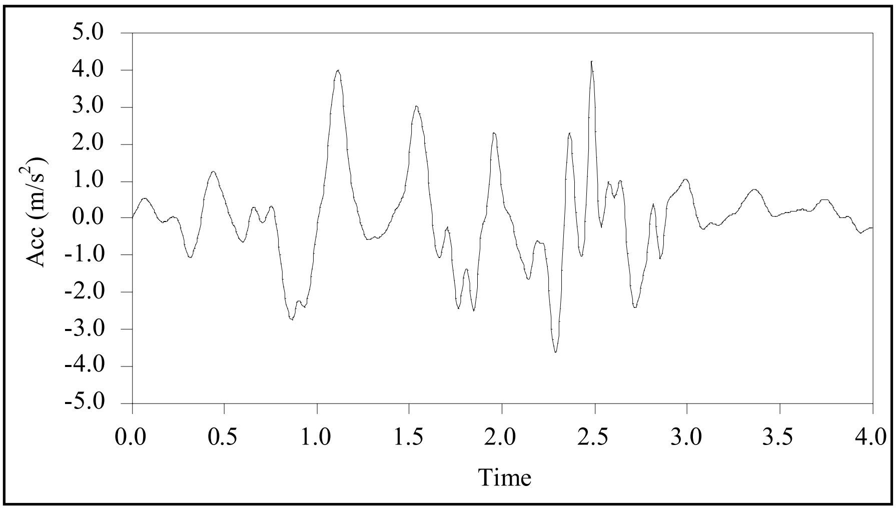

Figure 5. Input ground acceleration by kobe earthquake

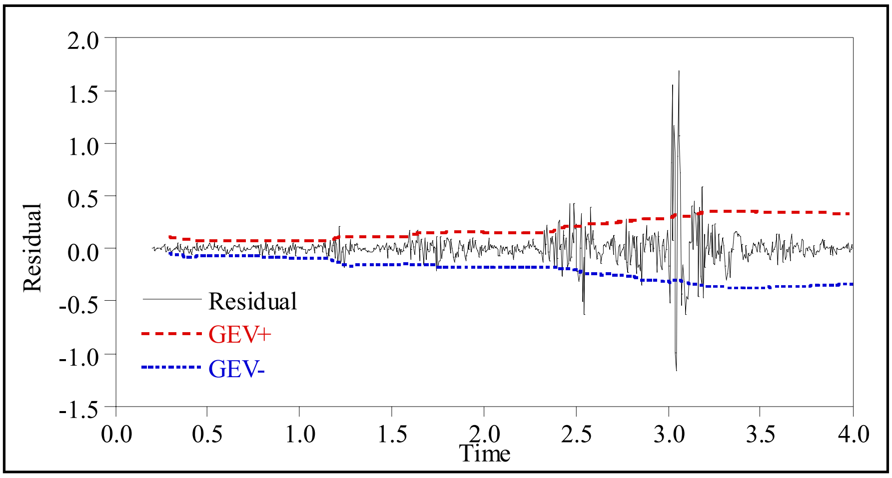

Figure 6. Outlier detection

Residual errors estimated from autoregressive model using measured accelerations are shown in Fig.6. Residual errors greatly increase at 3 sec because of simulating damage. Dashed line in Fig.6 is threshold values of outliers in given 99% significant lever. Residual errors at 3 sec. are definitely decided as outliers in 99% significant level. There are other outliers around 2.5 sec., maybe they are affected by large magnitude of input ground motion.

CONCLUSIONS

New structural health monitoring algorithm which is free from perturbations of environment is proposed. Residual errors are estimated using an autoregressive model and a time windowing technique. Perturbations of environment can be neglected within time window relatively smaller than time period of data acquisition. Generalized extreme value distribution is utilized for more reliable decision making of soundness of the structure. The validity of proposed algorithm is demonstrated by 2-span continuous truss numerical simulation example. Residual errors are greatly increased at the time of abrupt change occuring and it is available to find abrupt change by outlier detection of residual errors. Residual errors can be modified by external loading condition as well as abrupt change of the structure. So it is needed that effects of external loading condition must be separated from the measured signals.

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