An Incremental Formulation for the Prediction of 2D Fatigue Crack Growth

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ABSTRACT

This paper presents a new incremental formulation to predict the growth path of a two-dimensional fatigue crack under mixed-mode loading. Displacement and traction boundary integral equations are employed for the governing equations of a linear elastic cracked body. The Paris equation and the maximum principle stress criterion are adopted to determine the growth rate and direction of fatigue crack, respectively. The three governing equations, i.e., the dual boundary integral equations, the Paris equation, and the equation of the maximum principle stress criterion, are nonlinear with respect to the growing crack path. Fatigue crack growth is discretized for given loading cycles and the growth path for each crack increment is modeled as a parabola parameterized by the crack-tip position. An iterative solution scheme based on the Newton-Raphson method is proposed.

INTRODUCTION

Fatigue crack growth is one of the most important factors in the design of the steel structures. A fatigue crack usually does not grow straight due to mixed-mode loadings or asymmetric geometry. Numerous experiments and researches have been performed for the prediction of fatigue crack growth [1]. Most numerical studies are based on incremental simulation such as the tangent approach, in which each segment of crack path is approximated as a straight line [2] or circular arc [3]. Since the growth direction is assumed as the tangent direction at the current crack tip without iterative modification, increments should be very small to describe the curved crack path. Some iterative scheme has been proposed for curved crack growth [4,5,6]. However, in those schemes, the increments were considered as constant during the iterative modification. For a given number of loading cycles, the size of increment is evaluated from the Paris equation along the crack path and that should be modified according to the changes of growth path. The authors developed an incremental formulation based on successive iteration, in which both the direction and size of crack increment are iteratively updated [7]. However, the three nonlinear governing equations of a two-dimensional fatigue crack are solved separately in the previous study. In this study, the authors present a new incremental formulation based on the Newton-Raphson method to solve the three nonlinear governing equations at the same time.

GOVERNING EQUATIONS FOR FATIGUE CRACK GROWTH

A linear elastic body with a crack is subject to fatigue loads on its surface. The displacement and traction boundary integral equations (BIEs) [2,4] are employed to obtain responses of the craked body. The displacement BIE is applied on the exterior boundary of the body, while the traction BIE is used on one side of the traction-free crack surfaces. The Paris equation is adopted to define the growth rate of a fatigue crack. The integral form of the Paris equation is utilized to calculate the arc length of the growing path of a fatigue crack. The direction of crack growth is determined by the maximum principal stress criterion (MPSC), which states that crack grows in the direction perpendicular to the maximum principal direction. The Paris equations and the equation of MPSC are expressed with respect to mixed-mode stress intensity factors, which are functions of crack opening displacements (CODs) at the crack tip. The single path for fatigue crack growth is parameterized with respect to the arc length of fatigue crack. The unknowns, i.e., the growth path of fatigue crack, displacements and tractions of boundary, are determined by solving three governing equations, i.e., the Paris equation, the equation of MPSC and the dual BIEs.

DISCRETIZATION AND INCREMENTAL FORMULATION

The dual BIEs for a cracked body are discretized by the boundary element methods using straight-line elements. Crack growth is simulated by making a crack extension for a given number of fatigue loading. The growing crack path for each loading increment is modelled by a parabolic curve defined by the crack-tip position. Since the discretized equations of the BIEs and the crack growth criterions are nonlinear with respect to the nodal unknowns and the crack-tip position, the incremental forms of the discretized equations are derived to solve the nonlinear system of equations by the Newton-Raphson method. The shape sensitivities of the dual BIEs with respect to crack-tip position are derived. In each loading step, each parabola for a crack increment is iteratively updated by the Newton-Raphson method. The mixed-mode SIFs are evaluated by displacement extrapolation method [2]. The direction of the parabolic crack increment is determined with the MPSC to satisfy the continuity condition of the crack path. The size of increment is determined from the integration of the Paris equation with the trapezoidal rule. Each crack path for each loading step is assumed as a parabola, but discretized as a straight line for the simplicity of integration of the DBIEs.

This study provides a rigorous approach for the prediction of the growth path of fatigue crack. The validity of the proposed method will be demonstrated by numerical examples.

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