1-NORM REGULARIZATION FUNCTION FOR AN INVERSE PROBLEM OF ELASTIC CONTINUA WITH DISCONTINUOUS SYSTEM PARAMETERS

Hyun Woo PARK [1], Man Woo PARK [2], Byeong Kyu AHN [3], Hae Sung LEE [4]

1) *Korea Bridge Design & Engineering Research Center, Seoul National University, Seoul Korea 151-742*
2) *Structure Department, DONG IL Engineering Consultants, Co., Ltd, Seoul Korea 138-200, Former graduate student, Dept. of Civil Eng., Seoul National University*
3) *Structural Design Department, Korea Engineering Consultants Corp., Seoul Korea 143-715, Former graduate student, Dept. of Civil Eng., Seoul National University*
4) *School of Civil, Urban, and Geosystem Engineering, Seoul National University, Seoul Korea 151-742*

Corresponding Author: Hae Sung LEE, chslee@snu.ac.kr

ABSTRACT

It is well known that a numerically stable and physically meaningful solution of an inverse problem can be obtained by specifying a proper solution space, which is defined by the norm of the system parameter. The *a priori* information, which includes the smoothness and an initial estimate of a solution, is used to define the regularization function. The initial estimate of a solution is hereafter referred to as a baseline solution.

A material property of a continuous structure may vary in piecewise-continuous fashion, and thus has to be square integrable in the structural domain. The regularization function has been commonly used in a 2-norm based regularization scheme for the identification of piecewise continuous system parameters [1]. The Tikhonov regularization technique and the conventional truncated singular value decomposition are typical 2-norm based regularization schemes. Although the 2-norm based regularization schemes effectively stabilize ill-posedness of an inverse problem such as system identification, they often produce blurry solutions [2], in which information on the system parameter of a group is smeared into that of other groups. Therefore, the regularization schemes based on the 2-norm of the system parameter vector are not suitable for reconstructing the discontinuous distribution of system parameters accurately.

The solution space for a square integrable system parameter around the baseline solution is alternatively defined by the L_1-norm of the gradient of the system parameter as follows:

$$\Psi(D) = \left\|\nabla(D - D_0)\right\|_{L_1(V)} = \int_V \left\|\nabla(D - D_0)\right\|_1 dV \le a < \infty \tag{1}$$

where Ψ, D, D_0, ∇, $\|\cdot\|_{L_1(V)}$ and $\|\cdot\|_1$ denote the regularization function, a material property, and the baseline solution for the material property in a structural domain V, the gradient operator, the L_1–norm of a function defined in a structural domain V and the 1-norm of a vector, respectively.

To integrate the L_1-norm of the gradient of the material property in a domain discretized by sub-domains, consider a domain with two sub-domains as shown in Figure 1. It is assumed that the material property is constant in each sub-domain. The two sub-domains and

their interface boundary are denoted as V_1, V_2 and S, respectively, in Figure 2. A narrow transition region of width 2ε along boundary S is considered to integrate (1). The material property in the transition region is assumed to vary linearly in the normal direction of S, and to be constant in the tangential direction of S, which leads to the following expression:

$$D(n,s) = \frac{D^{\mathrm{II}} - D^{\mathrm{I}}}{2\varepsilon} n + \frac{D^{\mathrm{II}} + D^{\mathrm{I}}}{2} \tag{2}$$

where D^{I} and D^{II} represent the discretized material property of sub-domain I and II, respectively. For simplicity of presentation, D_0 is omitted during the discretization procedure. The L_1-norm given in (1) is discretized as follows.

$$\int_V \|\nabla D\|_1 dV = \int_V \left(\left|\frac{\partial D}{\partial x}\right| + \left|\frac{\partial D}{\partial y}\right|\right) dV = \lim_{\varepsilon \to 0} \int_{V_{\mathrm{I}}^T + V_{\mathrm{II}}^T} \left(\left|\frac{\partial D}{\partial x}\right| + \left|\frac{\partial D}{\partial y}\right|\right) dV = \left|D^{\mathrm{II}} - D^{\mathrm{I}}\right| (l_x + l_y) \tag{3}$$

where l_x and l_y denote the projected length of the interface boundary onto x- and y-axis, respectively, while V_{I}^T and V_{II}^T are the transition region in sub-domain I and II, respectively.

In case the domain of the body is discretized into several sub-domains as shown in Figure 1, the regularization function in (1) is discretized by using (3) as follows.

$$\int_V \|\nabla (D - D_0)\|_1 dV \approx \sum_{k=1}^{n_B} \left| (D_k^{\mathrm{I}} - (D_k^{\mathrm{I}})_0) - (D_k^{\mathrm{II}} - (D_k^{\mathrm{II}})_0) \right| ((l_k)_x + (l_k)_y) \tag{4}$$

where n_B and l_k denote the number of inter-group boundaries and the length of the k-th inter-group boundary, respectively, while $(l_k)_x$ and $(l_k)_y$ represent the projected lengths of the k-th inter-group boundary onto x-axis and y-axis, respectively. D_k^{I}, D_k^{II}, $(D_k^{\mathrm{I}})_0$ and $(D_k^{\mathrm{II}})_0$ represent the material properties to be identified and the associated baseline solutions of two groups sharing the k-th inter-group boundary S_k as shown in Figure 1.

The validity of the proposed 1-norm regularization function (4) will be demonstrated through an identification problem of an inclusion in a plate under a plane stress condition.

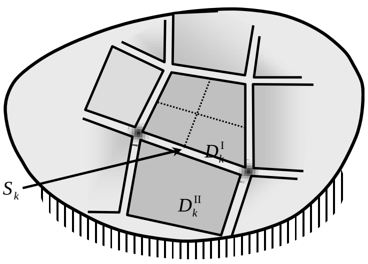

Figure 1. Discretization of a domain with predefined groups and inter-group boundaries

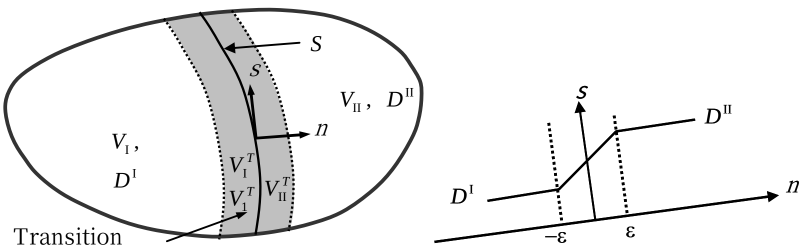

Figure 2. Transition region along the interface boundary between two sub-domains

REFERENCES

1. Park, H.W., Shin, S.B., Lee, H.S. "Determination of an optimal regularization factor in system identification with Tikhonov function for linear elastic continua," *International Journal for Numerical Methods in Engineering* 2001; **51**(10):1211-1230.
2. Park, H.W. and Lee, H.S. "1-Norm Based Regularization Scheme for System Identification of Structures with Discontinuous System Parameters," submitted for publication in *International Journal for Numerical Methods in Engineering* 2005.