Cover page

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Authors: Seung-Keun Park
School of Civil, Urban & Geosystem Engineering
Seoul National University
San 56-1, Shillim-dong, Kwanak-gu
Seoul, 151-742, Korea

Hyun-Moo Koh
School of Civil, Urban & Geosystem Engineering
Seoul National University
San 56-1, Shillim-dong, Kwanak-gu
Seoul, 151-742, Korea

Hae Sung Lee
School of Civil, Urban & Geosystem Engineering
Seoul National University
San 56-1, Shillim-dong, Kwanak-gu
Seoul, 151-742, Korea

Structural damage detection using time windowing technique from measured acceleration during earthquake

Seung-Keun Park[1], Hyun-Moo Koh[2] and Hae Sung Lee[3]

ABSTRACT

This paper presents a system identification (SI) scheme in time domain using measured acceleration data. The error function is defined as the time integral of the least square errors between the measured acceleration and the calculated acceleration by a mathematical model. Damping parameters as well as stiffness properties of a structure are considered as system parameters. The structural damping is modeled by the Rayleigh damping. A regularization function defined by the L_2-norm of the system parameters with respect to time is proposed to alleviate the ill-posed characteristics of inverse problems and to accommodate discontinuities of system parameters in time. The time window concept is proposed to trace variation of system parameters in time. Numerical simulation study is performed through a planar truss.

Key Words: System Identification, Regularization, Time Window

INTRODUCTION

Immediate safety assessment structures after an earthquake is extremely important in evaluating serviceability and functionality of social infrastructures. Nowadays, not only ground acceleration but also acceleration of important social infrastructures is monitored during earthquakes. It would be very helpful for quick restoration of social activities if structural damage caused by an earthquake is accessed with the measured acceleration during an earthquake in real time or near real time.

Various damage assessment schemes based on system identification (SI) have been extensively investigated for social infrastructures during the last few decades. The modal analysis approaches have been widely adopted to detect structural damage using measured acceleration of structures. The modal analysis approaches, however, suffer from drawbacks caused by insensitiveness of modal data to changes of structural properties.

To overcome the drawbacks of the modal analysis approaches, this paper presents a system identification scheme in time domain using measured acceleration data. The error function is defined as the time integral of the least square errors between the

[1] Seung-Keun Park, School of Civil, Urban & Geosystem Engineering, Seoul National University, Seoul, 151-742, Korea.
[2] Hyun-Moo Koh, School of Civil, Urban & Geosystem Engineering, Seoul National University, Seoul, 151-742, Korea.
[3] Hae Sung Lee, School of Civil, Urban & Geosystem Engineering, Seoul National University, Seoul, 151-742, Korea. Corresponding author

measured acceleration and the calculated acceleration by a mathematical model. The structural damping is modeled by the Rayleigh damping. A regularization technique is employed to overcome the ill-posedness of inverse problems. A regularization function defined by the L_2-norm of system parameters with time is proposed to accommodate abrupt changes of system parameters in time. The L_2-truncated singular value decomposition (TSVD) is adopted to optimize the error function with the L_2-regularization function. To trace the variation of stiffness parameters in time, a time windowing technique is introduced. In the time windowing technique, SI is performed sequentially within a finite time interval, which is called a time window. The time window advances forward at each time step to identify changes of system parameters in time. The validity and accuracy of the proposed method are demonstrated through numerical simulation study.

PARAMETER ESTIMATION SCHEME IN TIME DOMAIN

The discretized equation of motion of a structure is expressed as follows.

$$\mathbf{Ma} + \mathbf{C}(\mathbf{x}_c)\mathbf{v} + \mathbf{K}(\mathbf{x}_s)\mathbf{u} = \mathbf{p} \tag{1}$$

where **M**, **C** and **K** represent the mass, damping and stiffness matrix of the structure, respectively, and **a**, **v** and **u** are the relative acceleration, velocity and displacement of the structure to ground motion, respectively. The damping parameters and the stiffness parameters of the structure are denoted by $\mathbf{x}_c$ and $\mathbf{x}_s$ in (1), respectively. Newmark β-method is used to integrate the equation of motion.

In case ground acceleration as well as accelerations of a given structure at some discrete observation points are measured, the unknown system parameters of a structure including stiffness and damping properties are identified through minimizing least squared errors between computed and measured acceleration. In case the system parameters are invariant in time, the parameter estimation procedure is represented by the following optimization problem.

$$\underset{\mathbf{x}}{\mathrm{Min}}\ \Pi_E(t) = \frac{1}{2}\int_0^t \left\|\tilde{\mathbf{a}}(\mathbf{x}) - \bar{\mathbf{a}}\right\|_2^2 dt \ \text{ subject to } \ \mathbf{R}(\mathbf{x}) \le 0 \tag{2}$$

where $\tilde{\mathbf{a}}$, $\bar{\mathbf{a}}$, $\mathbf{x}$ and $\mathbf{R}$ are the calculated acceleration and the measured acceleration at observation points relative to ground acceleration, system parameter vector and constraint vector, respectively, with $\|\cdot\|_2$ representing the 2-norm of a vector. Linear constraints are used to set physically significant upper and lower bounds of the system parameters. The minimization problem defined in (2) is a constrained nonlinear optimization problem because the acceleration vector $\tilde{\mathbf{a}}$ is a nonlinear implicit function of the system parameters. In case the system parameters vary with time, the time window technique is proposed. Fig.1 illustrates the time window concept. In this technique the minimization problem for the estimation of the system parameters is defined in a finite time interval, which is referred to as a time window.

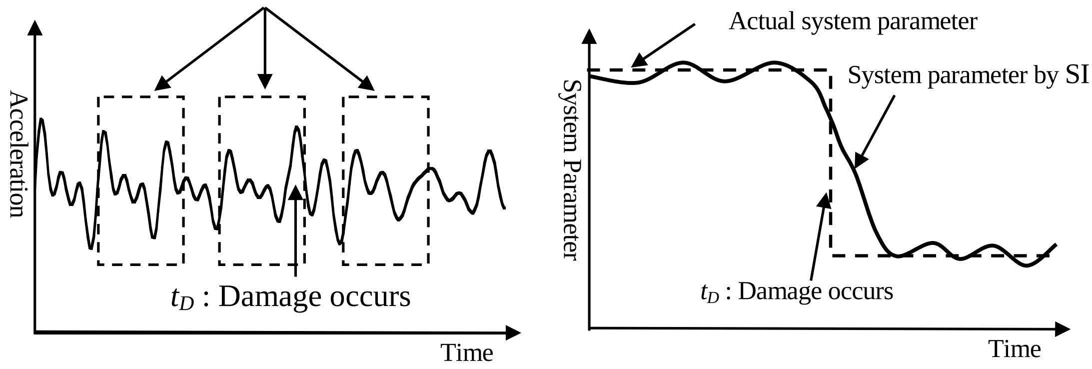

Figure 1. Time window concept

$$\underset{\mathbf{x}}{\text{Min}}\ \Pi_E(t) = \frac{1}{2}\int_t^{t+d_w} \left\| \tilde{\mathbf{a}}(\mathbf{x}(t)) - \bar{\mathbf{a}} \right\|_2^2 dt \ \text{ subject to } \ \mathbf{R}(\mathbf{x}(t)) \le 0 \tag{3}$$

Here, t and d_w is the initial time and the window size of a given time window. It is assumed that system parameters are constant in a time window, and that system parameters estimated by (3) represent the system parameters at time t. As the time window advances forward sequentially in time, the variations of system parameters in time are identified.

L_2-REGULARIZATION SCHEME

The parameter estimation defined by the minimization problems is a type of ill-posed inverse problems. Ill-posed problems suffer from three instabilities: nonexistence of solution, non-uniqueness of solution and discontinuity of solution when measured data are polluted by noise. Because of the instabilities, the optimization problem given in (2) and (3) may yield meaningless solutions or diverge in optimization process. Attempts have been made to overcome instabilities of inverse problems merely by imposing upper and lower limits on the system parameters. However, it has been demonstrated by several researchers that the constraints on the system parameters are not sufficient to guarantee physically meaningful and numerically stable solutions of inverse problems.

A regularization technique based on truncated singular value decomposition is employed to overcome the ill-posedness of inverse problems. In this regularization technique, the error function in (4) and a positive definite regularization function is minimized simultaneouly.

$$\underset{\mathbf{x}}{\text{Min}}\ \ \Pi_R(t) \ \text{ subject to } \ \mathbf{R}(\mathbf{x}) \le 0 \ \text{and} \ \underset{\mathbf{x}}{\text{Min}}\ \ \Pi_E(t) \tag{4}$$

where Π_R is a regularization function. Various regularization functions are used for different types of inverse problems. Kang et al proposed the following regularization function defined by the L_2-norm for the SI in time domain.

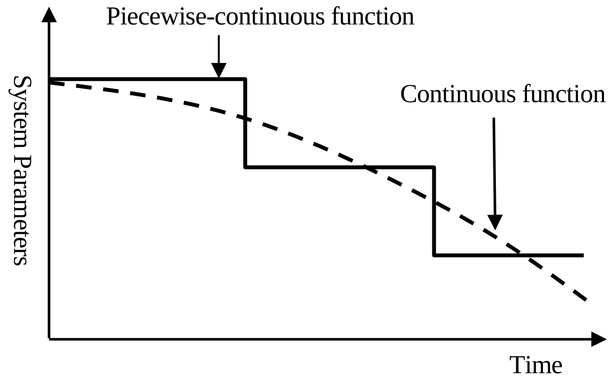

Figure 2. Continuous and piecewise-continuous function

$$\Pi_R(t) = \frac{1}{2}\int_0^t \left\| \frac{d\mathbf{x}}{dt} \right\|_2^2 dt \tag{5}$$

The regularization function defined in (5) is able to represent continuously varying system parameters in time. Since, however, the system parameters may vary abruptly (Fig.2) with time during earthquakes due to damage, a regularization function that can accommodate piecewise continuous functions in time is required to access damage that occurs during an earthquake. To represent discontinuity of system parameters in time, this paper proposes an L_2-regularization function of system parameters with respect to time.

$$\Pi_R(t) = \frac{1}{2}\int_0^t \|\mathbf{x} - \mathbf{x}_0\|_2^2 dt \tag{6}$$

where $\|\cdot\|_2$ representing the 2-norm of a vector.

The error function and the regularization function are easily discretized in time domain using simple numerical methods. The truncated solution of the minimization problem of the error function is obtained by the truncated singular value decomposition. Detailed solution procedures are presented in References. The optimal truncation number can be determined as follows:

$$ntr = \{l \mid \omega_{l+1} < \sqrt{\omega_1 \cdot \omega_p} < \omega_l\} \tag{7}$$

where ω_l is the l-th singular value of sensitivity matrix of error function.

DAMPING MODEL

It is a difficult task to model damping properties of real structures. In fact, existing damping models cannot describe actual damping characteristics exactly, and are approximations of real damping phenomena to some extents. Since the damping has an important effect on dynamic responses of a structure, the damping properties should be considered properly in the parameter estimation scheme. In most of previous studies on the parameter estimation, the damping properties of a structure are assumed as known properties, and only stiffness properties are identified. However, the damping properties are not known a priori and should be included in system parameters in the SI.

Among various classical damping models, the modal damping and the Rayleigh damping are the most frequently adopted model. In the modal damping, a damping matrix is constructed by using generalized modal masses and mode shapes. In Rayleigh damping, a damping matrix is defined as a linear combination of the mass matrix and stiffness matrix as follows.

$$\mathbf{C} = a_0\mathbf{M} + a_1\mathbf{K} \tag{8}$$

The damping coefficients of the Rayleigh damping can be determined when any two modal damping ratios and the corresponding modal frequencies are specified.
In case the modal damping is employed in the parameter estimation, the number of the system parameters associated with the damping is equal to that of the total number of DOFs, which increases the total number of unknowns in the optimization problem given in (8). Since neither modal damping nor Rayleigh damping can describe actual damping exactly, and the modal damping requires more unknowns than the Rayleigh damping in the parameter estimation, this study employs the Rayleigh damping for the SI. The Rayleigh damping yields a linear fit to the exact damping of a structure.

In case either the regularization scheme or damping estimation is not included in the SI, the optimization procedure does not converge or converges to meaningless solutions. Therefore, only the results with the regularization scheme and damping estimation are presented here.

EXAMPLE

Two Numerical simulation studies are presented to illustrate validity of the time windowing technique. Numerical simulation study is performed through a two-span continuous truss subject to free vibration. The integration constants of the Newmark β-method, β=1/2, γ=1/4, are used for all cases.

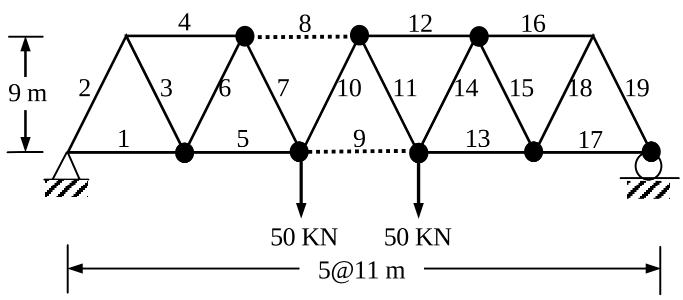

Figure 3. The planar truss structure

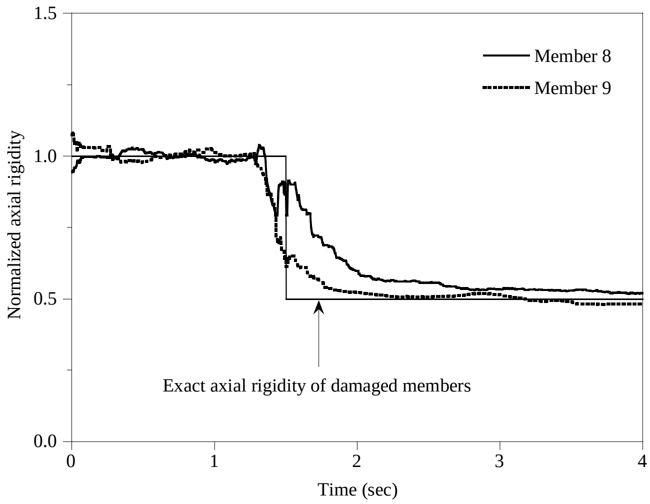

Figure. 4. Variation of axial rigidities of damaged members

The validity of the proposed time windowing technique is examined through a simulation study with a planar truss shown in Fig. 3. Typical material properties of steel (Young's modulus = 200 GPa, Specific mass = $7.86 \times 10^3 Kg/m^3$) are used for all members. The cross sectional areas of all members are 0.157 m^2. The natural frequencies of the truss range from 6.8 Hz to 114.4 Hz. Damage of the truss is simulated with 50% reductions in the sectional areas of member 8 and 9, respectively. The damaged members are depicted by dotted lines in Fig. 3. It is assumed that the damage suddenly occurs at t=1.5 sec. Accelerations of the truss are measured from a free vibration induced by a sudden release of applied loads of 50 KN shown in Fig. 3. The measurement errors are simulated by adding 5% random noise generated from a normal distribution function to accelerations calculated by the finite element model. The observation points are depicted as a solid dot. The vertical direction of accelerations are measured in the time period from 0 sec to 4 sec with the interval of 1/400 sec. The horizontal direction is only measured at the right support. Fig. 6 shows the exact modal damping ratios used for the calculation of measured accelerations together with identified modal damping ratios by the Rayleigh damping. The initial modal damping ratios calculated by the assumed Rayleigh damping coefficients are also drawn in the same figure. The size of time window is 0.25 sec.

The variations of axial rigidities of the two damaged with time are drawn in Figure 4. From the figure, it is clearly seen that the damage occurs at t=1.5 sec, and that the estimated stiffness parameters of damaged members 8 and 9 converge to the actual values as time steps proceed. Figure 5 shows the axial rigidity of each member identified at the final time t = 4.0 sec. The vertical axes of both Figure 4 and Figure 5 represent the normalized axial rigidity with respect to the initial value of each member. The identified axial rigidities oscillate moderately within the range of ±10 % for all undamaged members. Since, however, axial rigidities of the damaged members are reduced prominently compared with those of the other members, the damaged members are clearly distinguished from undamaged members.

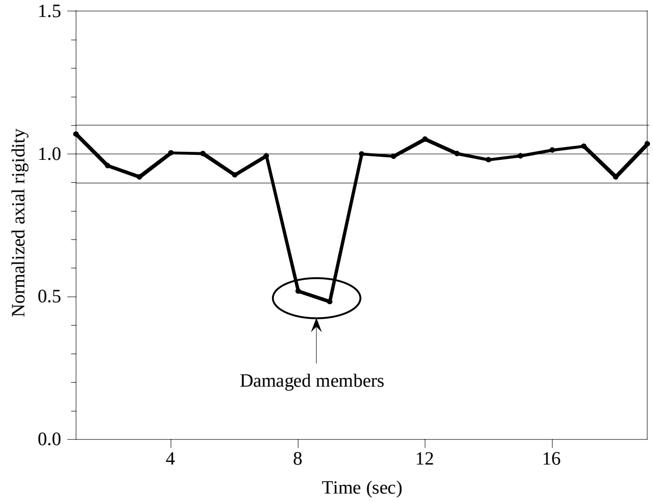

Figure. 5. Identified axial rigidities at the final time step, t = 4 s.

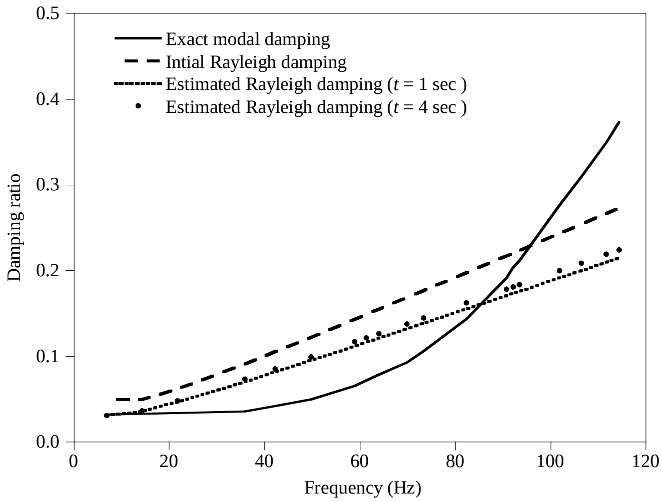

Figure. 6. Identified modal damping ratio

Figure 6 shows the variations of the identified damping ratios frequency at t = 1 sec and t = 4.0 sec. The identified Rayleigh damping approximates the exact modal damping closely.

CONCLUSION

The L_2-regularization function and the time window technique are proposed for SI in time domain using measured acceleration data is proposed. The system parameters include the damping parameters as well as the stiffness parameters of a structure. The Rayleigh damping is used to estimate the damping characteristics of a structure. The least square errors of the difference between calculated acceleration and measured acceleration is adopted as an error function. The regularization technique is employed to alleviate the ill-posedness of the inverse problem in SI. The L_2-TSVD is utilized to solve the given optimization problem.

The proposed method exhibits very compromising characteristics in detecting damage, and is able to estimate the stiffness properties accurately even though the damping characteristics are approximated by the Rayleigh damping. The example presented in this paper shows capabilities of the time window technique for the identification of abrupt damage.

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