Structural health monitoring using dynamic responses with regularized autoregressive model

J.S. Kang
School of Civil, Urban and Geosystem Engineering, Seoul National University, Seoul, Korea

H.S. Lee
School of Civil, Urban and Geosystem Engineering, Seoul National University, Seoul, Korea

ABSTRACT: This paper presents a new structural health monitoring algorithm using accelerations with regularized autoregressive model. Autoregressive model is employed to estimate whether the system is changed or not by changes of residual error between measured and calculated acceleration. The key difficulty of structural health monitoring system is how to handle noises, whereas measured acceleration contain a mix of information related to both the damage in the structure and the perturbations due to the environment. A time window technique is utilized to prevent perturbations due to the environment in available measurement data. A regularization technique is adopted to alleviate the effect of noises and to stabilize the result of monitoring system. An generalized extreme value distribution(GEV) is also adopted to raise reliability of decision making. The validity of proposed algorithm is demonstrated by a numerical simulation on a two-span continuous truss bridge.

1 INTRODUCTION

Over the last few decades, there has been a significant increase in the health monitoring and safety management field of the complex structure. The primary goal of the structural health monitoring is to find changes of system parameters and to decide its soundness at earliest possible stage. There are two categories in structural health monitoring and damage assessment whether structural model, such as stiffness, damping and mass information exist or not. One is model based scheme and the other is non-model based scheme. In model based scheme, system parameters are estimated by inverse analysis based on the sensitivity method from a mathematical model. In non-model based scheme, structural soundness is evaluated by pattern recognition and statistical approach from only measured signals without a structural model.

Model based system identification problem is a type of inverse problems, which are usually ill-posed problem. An ill-posed problem is characterized by the non-uniqueness, non-continuous and instability of solutions. Various regularization techniques have been developed to overcome this ill-posedness of inverse problem. In spite of ill-posedness can be alleviated by regularization techniques successfully, model-based system identification schemes are not applied in real situation because of modeling error that difference between mathematical model and real structural model. Recently, a lot of structural health monitoring researches with statistical pattern recognition using purely measured signals has been attempted in the center of Los Alamos national laboratory in USA. Autoregressive model is widely used in time series pattern analysis (Box, 1994).

The sequence of non-model based structural health monitoring system is divided into six steps that Data acquisition, prediction modeling, feature extraction, control distribution construction, monitoring and decision making. Measured signals are obtained from sensors and a prediction model is made of the autoregressive model. Coefficients of the autoregressive model and residual errors are estimated by a prediction model. Statistical treatments of obtained residual errors must be done for more reliable structural health monitoring. Finally, the decision making of soundness of considered structure in real time by monitoring residual errors continuously will be performed.

Various algorithms for structural health monitoring using static or dynamic responses are proposed. But the main problem of structural health monitoring system is how to handle noises, whereas measured signals contain a mix of information related to both the damage in the structure and the perturbations due to the environment. A new structural health monitoring algorithm with time window technique (Kang, et al, 2005) is employed. In time window technique, the residual errors are predicted sequentially within a finite time period which called time window. The time window advances forward at each time step to predict residual errors repeatedly. Perturbations of environment are commonly changed gradually during long time period and time window size is relatively very smaller than environmental perturbation period so it is assumed that perturbation of environment can be neglected within the time window.

Decision whether the considered structure is sound or not using residual errors in every time step is also very important. Extreme value distribution (Castillo, 1988) is utilized for making decision boundary of soundness of the target structure. Extreme value distribution is utilized to detect outliers because damage information almost lie on the tail of distribution and extreme value distribution is well established in tail distribution. A generalized extreme value distribution(GEV) (Park, et al, 2005) which unify three known extreme value distributions, Gumbel, Weibull and Flechet is utilized for simplicity.

The validity and accuracy of the proposed algorithm is demonstrated through a numerical simulation studies on a two-span truss bridge. The numerically generated acceleration data with noise under Kobe earthquake ground acceleration are utilized as measured signals for the numerical simulation example.

2 AUTOREGRESSIVE MODEL

2.1 *Definition*

Autoregressive(AR) model is utilized to evaluate structural health monitoring system using acceleration signals during a long period. Autoregressive model is widely used stochastic model that can be extremely useful in the representation of certain practically occurring series. And autoregressive model has a great merit that white noises pass through the autoregressive process. So autoregressive model respond to only changes of system.

In this model, the current value of the process is expressed as a finite, linear combination of previous values of the process and a random error e_t. Let us denote the values of a process at equally spaced times t, t-1, t-2, … by x_t, x_{t-1}, x_{t-2}, …. Then

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \cdots + \phi_p x_{t-p} + e_t \tag{1}$$

is called autoregressive model of order p. Where, ϕ is coefficients of autoregressive model, e_t is random error in the measured signal at time t and p is order of autoregressive model.

2.2 *Least square method*

Autoregressive model is expressed with coefficients as weighted regressive form. There are several methods to calculate coefficients of the autoregressive model. Least square method is utilized because it is very simple and clear. From Equation 1, residual error between estimated value from autoregressive model and measured value at time t is as follows.

$$e_t = x_t - \left(\phi_1 x_{t-1} + \phi_2 x_{t-2} + \cdots + \phi_p x_{t-p}\right) \tag{2}$$

The first term in the right side of Equation 2 is a measured signal at time t and the second term is the estimated value from autoregressive model at time t. After expansion of Equation 2 into considered time periods and minimize residual errors, the linear object function by least square method is obtained as shown in Equation 3.

$$\Pi = \underset{\phi}{Min} \sum_{t=p+1}^{N} \|e_t\| = \underset{\phi}{Min} \sum_{t=p+1}^{N} \left\{x_t - \boldsymbol{\psi}_t^T(x)\, \boldsymbol{\varphi}(\phi)\right\}^2 \tag{3}$$

Where, $\boldsymbol{\psi}_t(x) = [-x_{t-1} \cdots - x_{t-p}]^T$, $\boldsymbol{\varphi}(\phi) = [\phi_1 \cdots \phi_p]^T$ and N is total number of measured signals in considered time period. N must be greater than twice of the order p of the autoregressive model. The optimal solution of Equation 3 is obtained like Equation 4 by least square method.

$$\boldsymbol{\varphi}(\phi) = \left[\sum_{t=p+1}^{N} \boldsymbol{\psi}_t(x) \boldsymbol{\psi}_t^T(x) \right]^{-1} \sum_{t=p+1}^{N} \boldsymbol{\psi}_t(x) x_t \tag{4}$$

After decision of coefficients of autoregressive model, foregoing signals can be predicted by using definition of autoregressive model in Equation 1. If there is no damage in the structure then residual errors are very small. Residual errors will be highly increased when some problem occurs in the structure. By using this phenomenon, autoregressive model can be utilized in structural health monitoring system.

3 TIME WINDOW TECHNIQUE

3.1 *Objective*

The key difficulty in structural health monitoring is perturbation of measured signals by unknown effects such as environmental and instrumental effects. Measurement errors can be reduced according to improvement of sensor technology but perturbation of environment cannot be reduced. Measured signals are gradually changed according to various factors of environment such as day and night, season, temperature and humidity and so on. Even if there is no changes in the considered structure, measured signals can be swayed by this environmental situation. Almost previous methods suffer from this difficulty of environmental factors. Though a algorithm is performed well in experimental data in laboratory, it cannot be applied in real structure because of perturbations of environment.

Various pattern recognition algorithms attempt to solve this problem, but they still have some problems to apply in real structures. The reasons are accuracy and economical efficiency. Pattern recognition technique requires so many base solutions. The more base solutions we have, the more accuracy pattern recognition algorithm has. Learning process which called finding suited solutions from the base solutions takes so long time. Because environmental conditions cannot be exactly same in previous time, solutions obtained by learning process still have modeling errors. And the final goal of structural health monitoring is finding defects as soon as possible, so the process must be fast in order to apply in real systems.

Time windowing technique is adopted to solve this problems. Environmental factors are commonly gradually changed during very long time period. In autoregressive model with time window technique, the residual errors are predicted sequentially within a finite time period, which is called a time window. The time window overlaps and advances forward at each time step to predict residual errors step by step. Time window size is relatively very smaller than time period of environmental perturbations so it is assumed that changes of environment within the time window cannot happen.

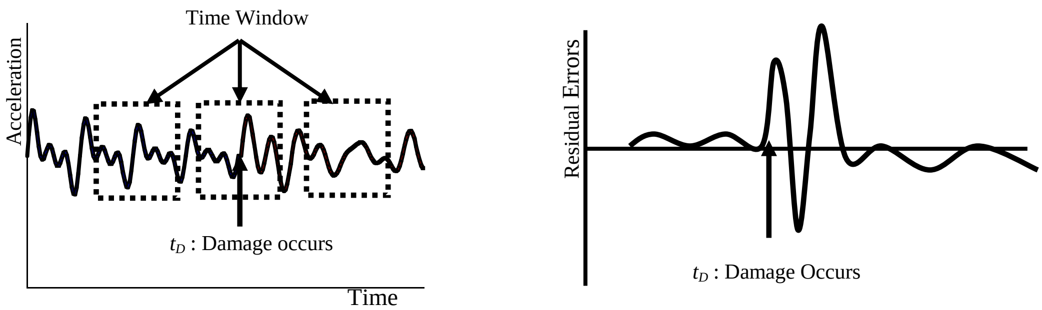

Figure 1. Outline of Time window technique and variation of residual erros

3.2 *Procedure of time window technique*

The optimal solution of coefficients of autoregressive model with time window technique can be obtained by following the same sequence in previous mentioned thus the final form is shown in Equation 5.

$$\boldsymbol{\varphi}_i(\phi) = \left[\sum_{t=t_i+p+1}^{t_f} \boldsymbol{\psi}_t(x) \boldsymbol{\psi}_t^{\ T}(x) \right]^{-1} \sum_{t=t_i+p+1}^{t_f} \boldsymbol{\psi}_t(x) x_t \quad (5)$$

Where, t_i, is the start time of each time window and t_f is the end time. $\boldsymbol{\varphi}_i$ is time varying coefficients which is modified according to advances of time window.

There is outline of the time window technique in Figure 1. Residual errors are estimated sequentially by using autoregressive model in moving time window. Let us assume that an abrupt change is occurred at time t_d by sudden effects. Before time t_d, there is no change in the system so residual errors between measured signals and estimated data by prediction model remain small. As time goes on, measured signals changed by abrupt change will be included in time window. Measured signals after time t_d discord from predicted value from previous estimated autoregressive model so residual errors are highly increased. After more time elapsed, time window fully passed t_d, all the measured signals in time window are filled with damaged information so residual errors return to small because of recalculated coefficients of autoregressive model with signals of damaged information. Figure 1 show schematic diagram of time window technique that time window advances forward at each time step to predict residual errors repeatedly and variation of residual errors according to time axis.

4 DECISION MAKING

4.1 *Extreme value distribution*

To decide whether the considered structure is sound or not using estimated results from prediction model is also very important. No matter how prediction model may work perfectly, it is useless without support of rigorous decision making algorithm. It is unreasonable to decide health of the structure by merely the magnitude of residual errors. For more reliable decision making of structural health monitoring, statistical approach is inevitable. Distribution of the residual errors must be found statistically from sparse residual errors and pick up outliers from the distribution in a given significant level.

Outliers almost lie in the tail of the distribution of residual errors. Extreme value distribution is utilized for an more accurate selection of outliers because extreme value distribution is well established for tail distribution.

4.2 *Generalized extreme value distribution*

Generally, it is known that any distribution follows one of three extreme value distributions, Gumbel, Weibull and Flechet distribution. It is very annoyed to find what the best distribution in three distributions to estimate the distribution of residual errors is. The three types of distribution can be expressed in one single form, called von-mises form as shown in Equation 6. Equation 6a and 6b expresses distribution for maxima and minima, respectively. According to the value of c, von-mises form can change to the three extreme value distribution forms. c > 0, c < 0, c = 0 we get Frechet, Weibull and Gumbel distribution, respectively.

$$\overline{G}(x;\lambda,\delta,c) = \exp\left\{ -\left[1 + c\left(\frac{x-\lambda}{\delta} \right) \right]^{-1/c} \right\} \quad \left(1 + c\left(\frac{x-\lambda}{\delta} \right) \geq 0, \quad \delta > 0 \right) \quad (6a)$$

$$\underline{G}(x;\lambda,\delta,c) = 1 - \exp\left\{ -\left[1 + c\left(\frac{\lambda-x}{\delta} \right) \right]^{-1/c} \right\} \quad \left(1 + c\left(\frac{\lambda-x}{\delta} \right) \geq 0, \quad \delta > 0 \right) \quad (6b)$$

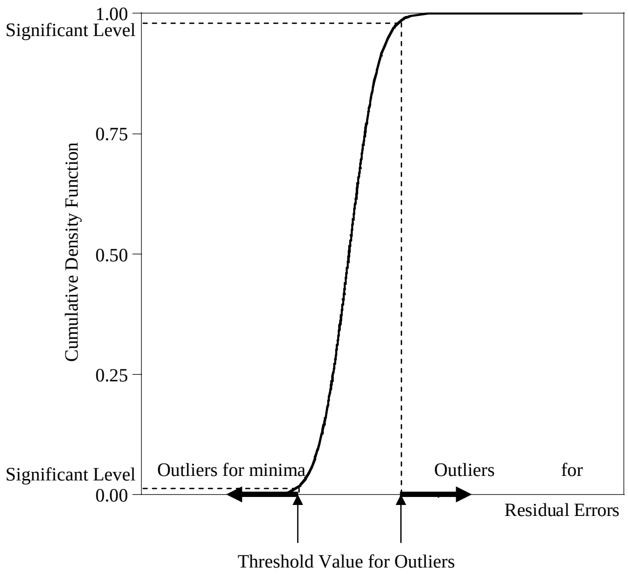

Figure 2. Outline of outlier detection

4.3 *Optimization*

Optimization process is utilized to find three coefficients, λ, δ and c of generalized extreme value distribution. By minimizing difference between generalized extreme value distribution and empirical cumulative density function like Equation 7, optimal three coefficients can be obtained.

$$Min\frac{1}{2}\left[\mathbf{G}(x;\lambda,\delta,c)-\mathbf{p}\right]^{T}\mathbf{W}\left[\mathbf{G}(x;\lambda,\delta,c)-\mathbf{p}\right] \tag{7}$$

Where, **G** is generalized extreme value distribution, **p** is empirical cumulative density function of the calculated residual errors and **W** is the weighting matrix.

4.4 *Outlier detection*

From the optimized distribution of the residual errors, threshold value of maxima and minima can be found by one to one match from given significant level. Residual errors which are greater than threshold value of maxima are defined as outliers of maxima and smaller than threshold value of minima are defined as outliers of minima. It can be judged that some abrupt change occurs in the considered structure by detected outliers. A brief outline of this process is shown with graph in Figure 2. A real-time structural health monitoring system can be consisted by repeating this sequence continuously.

5 EXAMPLE

5.1 *Two-span continuous truss*

The validity of the proposed structural health monitoring algorithm is verified through a simulation study with a two-span continuous truss shown in Figure 3. Typical material properties of steel (Young's modulus = 210 GPA, Specific mass = 7.85 Kg/m^3) are used for all truss members. The cross sectional areas of top, bottom, vertical and diagonal members are 112.5 cm^2, 93.6 cm^2, 62.5 cm^2 and 75.0 cm^2, respectively. The natural frequencies of the truss range from 6.6 Hz to 114.7 Hz. Sampling rate is 200 Hz to involve all of the high frequency modes information. The damping characteristics are simulated by modal damping ratio 3%~30% in each mode, continuously.

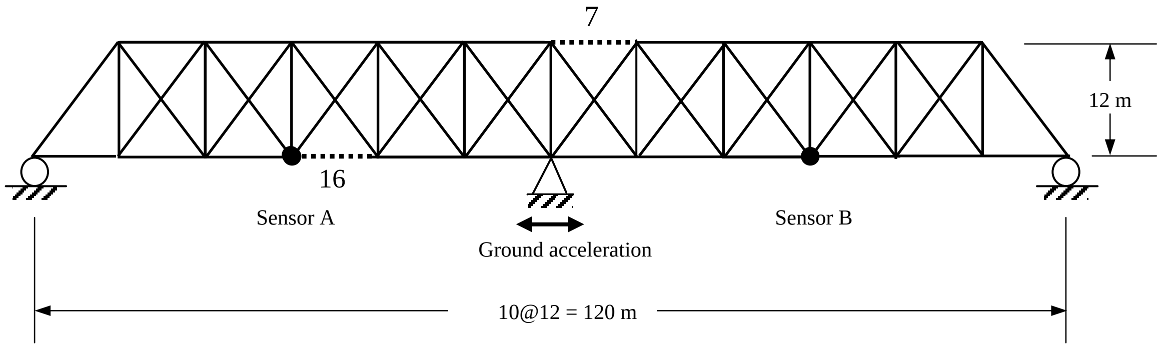

Figure 3. two-span continuous truss

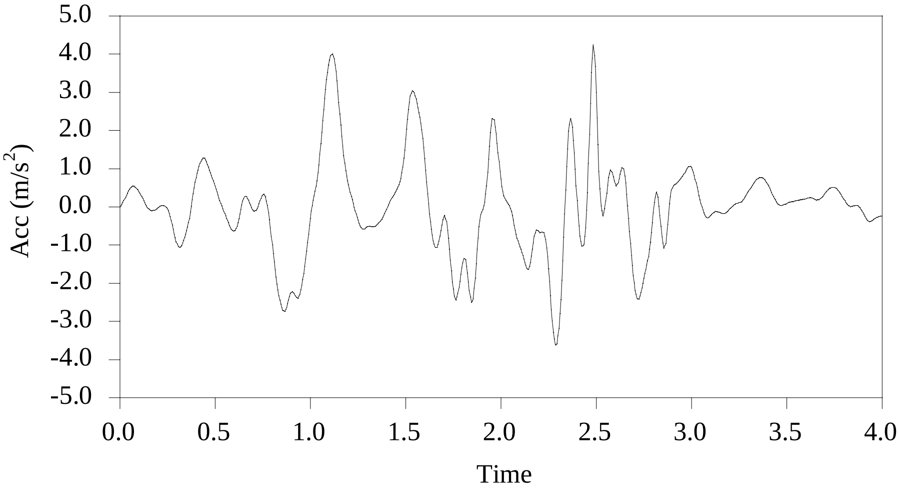

Figure 4. Input ground acceleration by Kobe earthquake

It is assumed that accelerations are measured with Kobe earthquake ground acceleration at centered hinge support in the horizontal direction for simulating under earthquake situation. Input ground acceleration is shown in Figure 4. This ground accelerations which is a maximum magnitude part are extracted from 40 second full data. Sensing points are center of the both span in bottom nodes of the truss. It is assumed that abrupt change occurs in the considered structure at 3 second. Damage is implemented as reduction of cross sectional area. The cross sectional areas of Upper member 7 and lower member 16 are reduced by 40% and 50%, respectively. Damaged members are represented as dotted line in Figure 3. Vertical direction accelerations are measured numerically in the time period from 0 sec to 4 sec. The measurement errors are simulated by adding 5% random proportional noise to accelerations calculated by the finite element model.

Residual errors estimated from autoregressive model using measured accelerations with time window technique are shown in Figure 5. Dotted lines in Figure 5 are threshold values of outliers in given 99.9% significant level. Upper dotted line is threshold values of maxima and lower dotted line is threshold values of minima. Outlier detection process is performed in both direction maxima and minima. Residual errors around 3 second are greatly distinguished from other time steps in the result from sensor A. It is definitely represented that some abrupt change occurs in target structure around 3 second. There are also some small peaks around 1.25 and 2.5 second. They might be false alarms affected by large magnitude of input ground motion described in Figure 4. By the way, it cannot be detected abrupt changes by the result of outlier detection using signals from sensor B. The damage information of member 7 and 16 cannot carry to location of sensor B. Therefore health monitoring system must be consisted by multi sensors to detect localized abrupt changes.

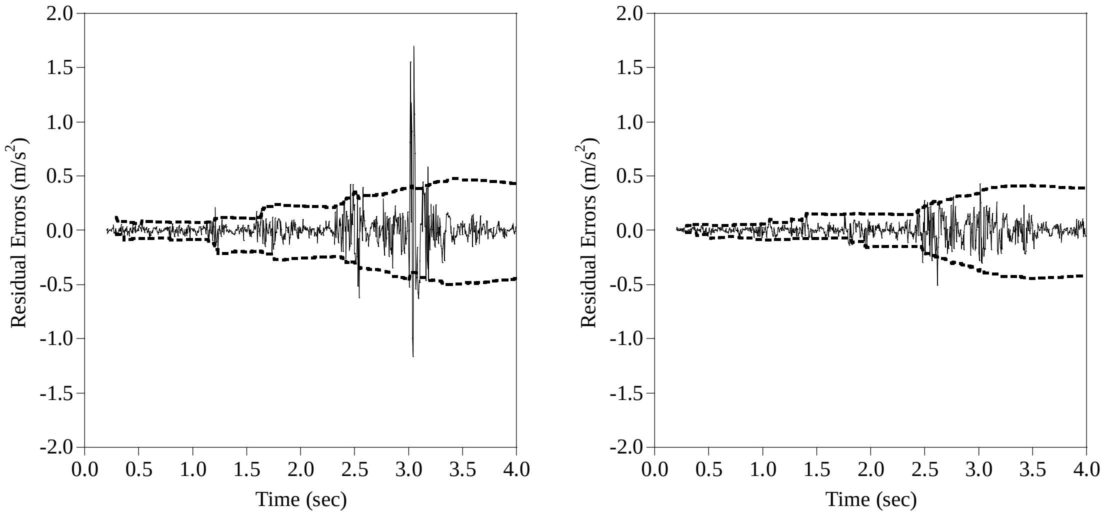

Figure 5. Result of outlier detection from sensor A and sensor B

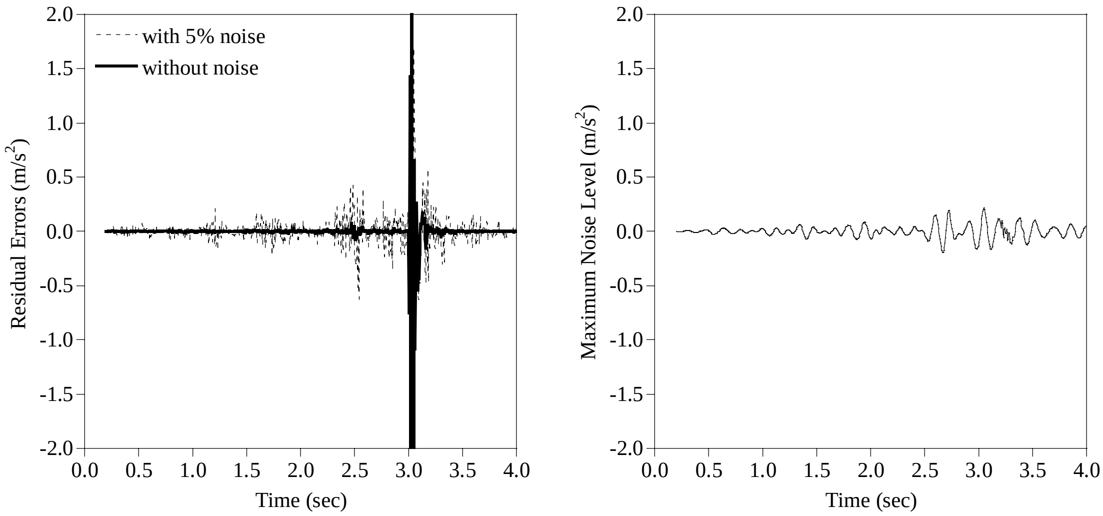

Figure 6. Result from sensor A according to noise and maximum noise level

The very important characteristic of autoregressive model is that white noises pass through the autoregressive model. This means that autoregressive model cannot react to white noises because regressive model has no relationship with white noises. There is a comparison with noise levels in Figure 6. In case without noise, estimation result is almost perfect along all the time period even if the magnitude of ground motion is huge. In case with 5% proportional random noise, there are some perturbations around 1.25, 1.75 and 2.5 second. The maximum noise level of 5% proportional noise is shown in right side of Figure 6. The perturbations due to 5% random noise are lager than possible maximum perturbations from given 5% noise. This means there are some amplification factors by noise in the algorithm. A regularization technique is needed to alleviate this amplification. Various regularization techniques are proposed to alleviate instabilities or amplification. Almost of them try to regularize the estimation system. But this case is quite different. Decision making is decided not by autoregressive model but by residual errors. Even if regularization of autoregressive model is well worked, residual errors cannot be stabilized. So new regularization technique which can alleviate the amplification by noises is needed.

6 CONCLUSION & FURTHUR STUDY

New structural health monitoring algorithm which is free from perturbations of environment is proposed. Residual errors are estimated using an autoregressive model with a time window technique. Perturbations of environment can be neglected within time window relatively smaller than time period of data acquisition. Generalized extreme value distribution is utilized for more reliable decision making of soundness of the structure. The validity of proposed algorithm is demonstrated by numerical simulation example in two-span continuous truss. Residual errors are greatly increased at the time that abrupt change occurs and it is available to find abrupt change by outlier detection of residual errors. Residual errors can be amplified by noises, so it might be proposed new regularization technique to alleviate perturbation of residual errors due to noises.

7 REFEFENCES

Enrique Castillo, 1988, *Extreme Value Theory in Engineering*, ACADEMIC PRESS, 1250 Sixth Avenue, San Diego, CA 92101

George E. P. Box, Gwilym M. Jenkins and Gregory C. Reinsel, 1994, *Time Series Analysis forecasting and control*, Prentice-Hall, Englewood Cliffs, New Jersey 07632

Hyun Woo Park and Hoon Sohn, 2005, “Parameter Estimation of the Generalized Extreme Value Distribution for Structural Health Monitoring”, *Submitted to Journal of Probabilistic Engineering Mechanics*.

Joo Sung Kang, Seung-Keun Park, Soobong Shin, Hae Sung Lee, 2005, “Structural system identification in time domain using measured acceleration”, Journal of Sound and Vibration 288, pp 215-234.