Structural system identification in time domain using a time windowing technique from measured acceleration

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ABSTRACT: This paper presents a system identification (SI) scheme in time domain using measured acceleration data. The error function is defined as the time integral of the least square errors between the measured acceleration and the calculated acceleration by a mathematical model. Damping parameters as well as stiffness properties of a structure are considered as system parameters. The structural damping is modeled by the Rayleigh damping. A regularization function defined by the L_1-norm of the first derivative of system parameters with respect to time is proposed to alleviate the ill-posed characteristics of inverse problems and to accommodate discontinuities of system parameters in time. The bilinear fitting method (BFM) is adopted to select an optimal truncation number of the L_1-TSVD. The time window concept is proposed to trace variation of system parameters in time. Numerical simulation study is performed through a two-span continuous truss.

1 INTRODUCTION

Immediate safety assessment structures after an earthquake is extremely important in evaluating serviceability and functionality of social infrastructures. Nowadays, not only ground acceleration but also acceleration of important social infrastructures is monitored during earthquakes. It would be very helpful for quick restoration of social activities if structural damage caused by an earthquake is accessed with the measured acceleration during an earthquake in real time or near real time.

Various damage assessment schemes based on system identification (SI) have been extensively investigated for social infrastructures during the last few decades. The modal analysis approaches have been widely adopted to detect structural damage using measured acceleration of structures. The modal analysis approaches, however, suffer from drawbacks caused by insensitiveness of modal data to changes of structural properties.

To overcome the drawbacks of the modal analysis approaches, this paper presents a system identification scheme in time domain using measured acceleration data. The error function is defined as the time integral of the least square errors between the measured acceleration and the calculated acceleration by a mathematical model. The structural damping is modeled by the Rayleigh damping. A regularization technique is employed to overcome the ill-posedness of inverse problems. A regularization function defined by the L_1-norm of first time derivatives of stiffness parameters is proposed to accommodate abrupt changes of system parameters in time. The L_1-truncated singular value decomposition (TSVD) is adopted to optimize the error function with the L_1-regularization function. The bilinear fitting method (BFM) is adopted to select an optimal truncation number of the L_1-TSVD. To trace the variation of stiffness parameters in time, a time windowing technique is introduced. In the time windowing technique, SI is performed sequentially within a finite time interval, which is called a time window. The time window advances forward at each time step to identify changes of system parameters in time. Numerical simulation study is performed through a two-span continuous truss.

2 PARAMETER ESTIMATION SCHEME IN TIME DOMAIN

2.1 *Error function in time domain*

The discretized equation of motion of a structure subjected to ground acceleration a_g caused by an earthquake is expressed as follows.

$$\mathbf{Ma} + \mathbf{C}(\mathbf{x}_c)\mathbf{v} + \mathbf{K}(\mathbf{x}_s)\mathbf{u} = -\mathbf{Ma}_g \tag{1}$$

where **M**, **C** and **K** represent the mass, damping and stiffness matrix of the structure, respectively, and **a**, **v** and **u** are the relative acceleration, velocity and displacement of the structure to ground motion, respectively. The damping parameters and the stiffness parameters of the structure are denoted by $\mathbf{x}_c$ and $\mathbf{x}_s$ in (1), respectively. Newmark β-method is used to integrate the equation of motion. Since the operational vibrations of a structure are negligible compared to those induced by an earthquake, the initial condition of (1) is set to zero.

In case ground acceleration as well as accelerations of a given structure at some discrete observation points are measured, the unknown system parameters of a structure including stiffness and damping properties are identified through minimizing least squared errors between computed and measured acceleration. In case the system parameters are invariant in time, the parameter estimation procedure is represented by the following optimization problem.

$$\underset{\mathbf{x}}{\text{Min}}\ \Pi_E(t) = \frac{1}{2}\int_0^t \left\|\tilde{\mathbf{a}}(\mathbf{x}) - \bar{\mathbf{a}}\right\|_2^2 dt \ \text{ subject to } \ \mathbf{R}(\mathbf{x}) \le 0 \tag{2}$$

where $\tilde{\mathbf{a}}$, $\bar{\mathbf{a}}$, **x** and **R** are the calculated acceleration and the measured acceleration at observation points relative to ground acceleration, system parameter vector and constraint vector, respectively, with $\|\cdot\|_2$ representing the 2-norm of a vector. Linear constraints are used to set physically significant upper and lower bounds of the system parameters. The minimization problem defined in (2) is a constrained nonlinear optimization problem because the acceleration vector $\tilde{\mathbf{a}}$ is a nonlinear implicit function of the system parameters.

In case the system parameters vary with time, the time window technique is proposed. Fig.1 illustrates the time window concept. In this technique the minimization problem for the estimation of the system parameters is defined in a finite time interval, which is referred to as a time window.

$$\underset{\mathbf{x}}{\text{Min}}\ \Pi_E(t) = \frac{1}{2}\int_t^{t+d_w} \left\|\tilde{\mathbf{a}}(\mathbf{x}(t)) - \bar{\mathbf{a}}\right\|_2^2 dt \ \text{ subject to } \ \mathbf{R}(\mathbf{x}(t)) \le 0 \tag{3}$$

Here, t and d_w is the initial time and the window size of a given time window. It is assumed that system parameters are constant in a time window, and that system parameters estimated by (3) represent the system parameters at time t. As the time window advances forward sequentially in time, the variations of system parameters in time are identified.

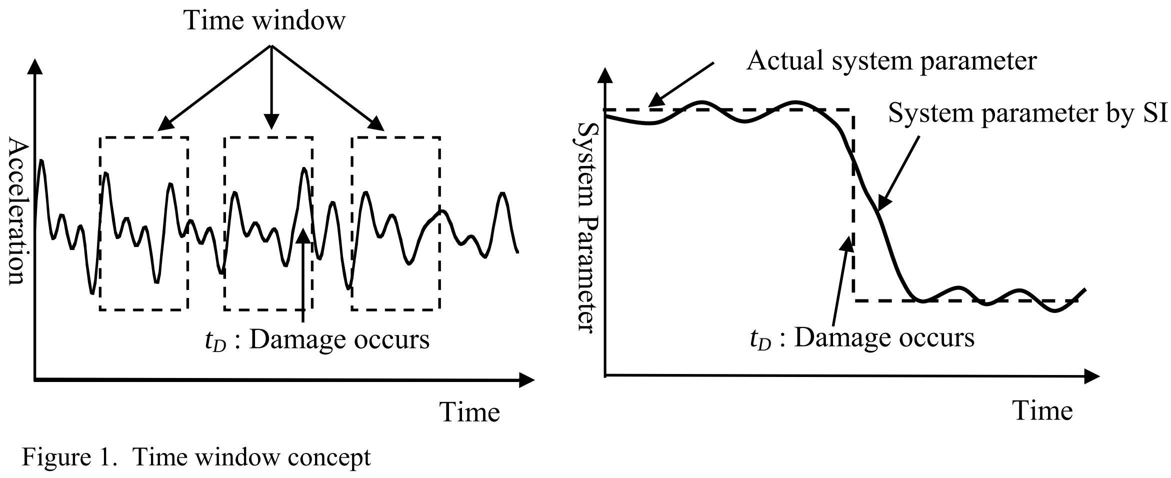

Figure 1. Time window concept

2.2 L_1-Regularization scheme

The parameter estimation defined by the minimization problems is a type of ill-posed inverse problems. Ill-posed problems suffer from three instabilities: nonexistence of solution, non-uniqueness of solution and discontinuity of solution when measured data are polluted by noise. Because of the instabilities, the optimization problem given in (2) and (3) may yield meaningless solutions or diverge in optimization process. Attempts have been made to overcome instabilities of inverse problems merely by imposing upper and lower limits on the system parameters. However, it has been demonstrated by several researchers that the constraints on the system parameters are not sufficient to guarantee physically meaningful and numerically stable solutions of inverse problems.

The regularization technique proposed by Tikhonov is widely employed to overcome the ill-posedness of inverse problems. In the Tikhonov regularization technique, a positive definite regularization function is added to the original optimization problem.

$$\underset{\mathbf{x}}{\text{Min}}\ \Pi(t) = \Pi_E(t) + \lambda \Pi_R(t) \text{ subject to } \mathbf{R}(\mathbf{x}) \le 0 \tag{4}$$

where Π_R and λ are a regularization function and a regularization factor, respectively. Various regularization functions are used for different types of inverse problems. Kang et al proposed the following regularization function defined by the L_2-norm for the SI in time domain.

$$\Pi_R(t) = \frac{1}{2}\int_0^t \left\| \frac{d\mathbf{x}}{dt} \right\|_2^2 dt \tag{5}$$

The regularization function defined in (5) is able to represent continuously varying system parameters in time. Since, however, the system parameters may vary abruptly (Fig.1) with time during earthquakes due to damage, a regularization function that can accommodate piecewise continuous functions in time is required to access damage that occurs during an earthquake. To represent discontinuity of system parameters in time, this paper proposes an L_1-regularization function of the first derivative of system parameters with respect to time.

$$\Pi_R(t) = \frac{1}{2}\int_t^{t+d_w} \left\| \frac{d\mathbf{x}}{dt} \right\|_1 dt \tag{6}$$

where $\|\cdot\|_1$ representing the 1-norm of a vector.

Since the error function is nonlinear with respect to stiffness parameters, a Newton-type optimization algorithm, which requires gradient information of an objective function, is usually employed in SI. As the L_1-regularization function is non-differentiable, the objective function in the Tikhonov regularization scheme defined in (4) contains a non-differentiable function, and thus a Newton-type optimization algorithm cannot be applied. To avoid this difficulty, this paper employs the L_1-TSVD to impose the L_1-regularization function in the optimization of the error function. In the proposed method, the incremental solution of the error function is obtained by solving the quadratic sub-problems without the constraints. The noise-polluted solution components are truncated from the incremental solution. Finally, the regularization function is imposed to restore the truncated solution components and the constraints. The above procedure is defined as follows.

$$\underset{\mathbf{x}}{\text{Min}}\ \Pi_R(t) = \frac{1}{2}\int_t^{t+d_w} \left\| \frac{d\mathbf{x}}{dt} \right\|_1 dt \text{ subject to } \mathbf{R}(\mathbf{x}) \le 0 \text{ and } \underset{\mathbf{x}}{\text{Min}}\ \Pi_E(t) = \frac{1}{2}\int_t^{t+d_w} \left\| \tilde{\mathbf{a}}(\mathbf{x}) - \bar{\mathbf{a}} \right\|_2^2 dt \tag{7}$$

The error function and the regularization function are easily discretized in time domain using simple numerical methods. The truncated solution of the minimization problem of the error function is obtained by the truncated singular value decomposition, while the simplex method is

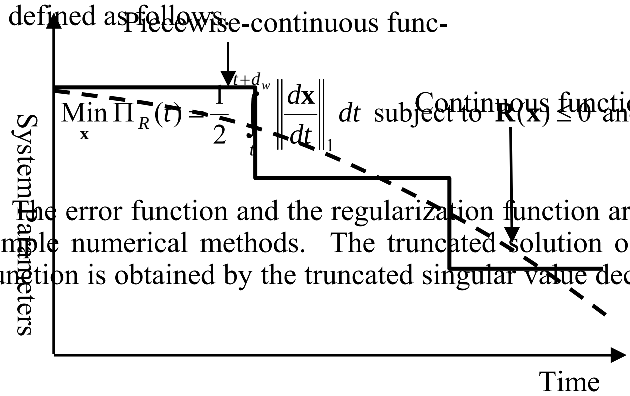

Figure 2. Continuous and piecewise-continuous function

employed to solve the minimization problem of the L_1-regularization function with constraints. Detailed solution procedures are presented in References.

2.3 *Bilinear fitting method*

It is crucial to determine a proper truncation number so that the TSVD produces a numerically stable and physically meaningful solution. The truncation number plays a similar role to the regularization factor in the Tikhonov regularization technique. In case a truncation number is too small, most of the useful information on a structure is lost while too large a truncation number yields noise-polluted, meaningless solutions. Therefore, the truncation number should be determined so that useful information of a structure is retained as much as possible while noise-polluted solution components are truncated.

The optimal truncation number may be defined as the truncation number associated with the smallest singular value that does not amplify noise in measurement. The bilinear fitting method (BFM) is proposed to determine the optimal truncation number for the TSVD in this paper. Figure 3 illustrates schematically the variation of the residual of the error function with truncation numbers. As the truncation number approaches an optimal truncation number, the residual of the converged error function decreases very quickly since the number of useful solution compo-

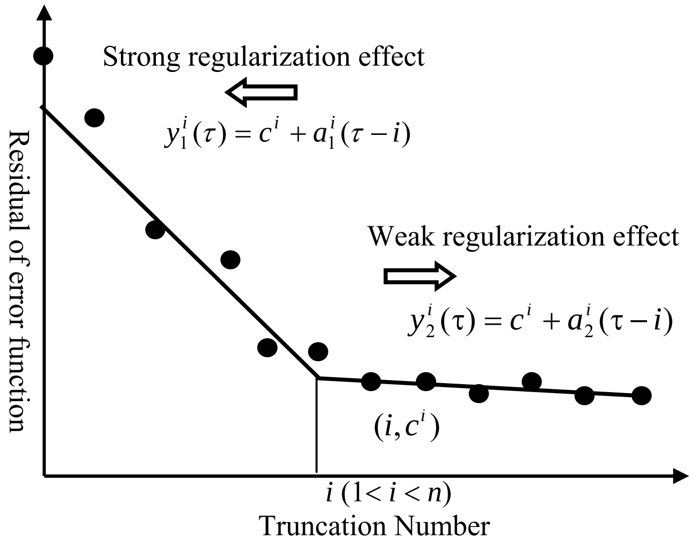

Figure 3. A schematic illustration of the bilinear fitting method

nents without amplified noise increases. Once the truncation number becomes larger than the optimal truncation number, the decreasing rate of the error function suddenly reduces because noise-polluted solution components associated with smaller singular values are included in solution components. Based on this observation, the plot of the residuals of the error function versus the truncation number is represented by two straight lines, i.e., a bilinear function as shown in Figure 3.

The bilinear function with the breaking point at the i-th truncation number is defined as follows:

$$\begin{aligned} y_1^i(\tau) &= a_1^i(\tau - i) + c^i \quad 1 \le \tau \le i \\ y_2^i(\tau) &= a_2^i(\tau - i) + c^i \quad i \le \tau \le p \end{aligned} \tag{8}$$

where τ denotes a truncation number, and a_1^i, a_2^i, c^i are the unknown coefficients of the bilinear function. The line segment with a steep slope denoted as $y_1^i(\tau)$ represents a region of strong regularization effect, while the line segment with a gentle slope denoted as $y_2^i(\tau)$ represents a region of week regularization effect. The coefficients a_1^i, a_2^i, c^i in (8) are determined by the least-squares method as follows:

$$\underset{(a_1^i, a_2^i, c^i)}{\text{Min}} \ \pi^i = \sum_{k=1}^{i} (y_1^i(k) - \Pi_E^k)^2 + \sum_{k=i}^{p} (y_2^i(k) - \Pi_E^k)^2$$
$$\Pi_E^k = \int_{t_i}^{t_f} \left\| \widetilde{\mathbf{a}}(\mathbf{x}(t)) - \overline{\mathbf{a}} \right\|_2^2 dt \tag{9}$$

where Π_E^k is the converged residual of the error function for truncation number k. The bilinear function is determined for each breaking point $1 \le i \le p$ by (9), and then the least-squares error between each best-fitting bilinear function and the residuals of the error function is evaluated.

$$\hat{\pi}^i = \sum_{k=1}^{i} (y_1^i(k) - \Pi_E^k)^2 + \sum_{k=i}^{p} (y_2^i(k) - \Pi_E^k)^2 \tag{10}$$

The breaking point of the bilinear function that minimizes (10) is chosen as an optimal truncation number, which yields the best trade-off between the regularization effect and the minimization of the error function. The bilinear function formed by the optimal truncation number represents the best-fit for the residual of the converged error function for each truncation number.

The proposed BFM can also be applied to 2-norm based regularization schemes to determine an optimal regularization factor or truncation number. Since the only difference between 1-norm and 2-norm based regularization schemes is the type of the regularization function, the proposed BFM is applicable to the 2-norm based regularization scheme with a simple modification. For example, the procedure is modified slightly for the Tikhonov regularization scheme because the regularization factor instead of the truncation number controls the regularization effect. The BFM procedure is applied to the residuals of the error function computed by taking each singular value as the regularization factor.

3 DAMPING MODEL

It is a difficult task to model damping properties of real structures. In fact, existing damping models cannot describe actual damping characteristics exactly, and are approximations of real damping phenomena to some extents. Since the damping has an important effect on dynamic responses of a structure, the damping properties should be considered properly in the parameter estimation scheme. In most of previous studies on the parameter estimation, the damping properties of a structure are assumed as known properties, and only stiffness properties are identified. However, the damping properties are not known a priori and should be included in system parameters in the SI.

Among various classical damping models, the modal damping and the Rayleigh damping are the most frequently adopted model. In the modal damping, a damping matrix is constructed by using generalized modal masses and mode shapes. In Rayleigh damping, a damping matrix is defined as a linear combination of the mass matrix and stiffness matrix as follows.

$$\mathbf{C} = a_0\mathbf{M} + a_1\mathbf{K} \tag{11}$$

The damping coefficients of the Rayleigh damping can be determined when any two modal damping ratios and the corresponding modal frequencies are specified.

In case the modal damping is employed in the parameter estimation, the number of the system parameters associated with the damping is equal to that of the total number of DOFs, which increases the total number of unknowns in the optimization problem given in (11). Since neither modal damping nor Rayleigh damping can describe actual damping exactly, and the modal damping requires more unknowns than the Rayleigh damping in the parameter estimation, this study employs the Rayleigh damping for the SI. The Rayleigh damping yields a linear fit to the exact damping of a structure.

In case either the regularization scheme or damping estimation is not included in the SI, the optimization procedure does not converge or converges to meaningless solutions. Therefore, only the results with the regularization scheme and damping estimation are presented here.

4 EXAMPLE

Numerical simulation studies are presented to illustrate validity of proposed method. Numerical simulation study is performed through a two-span continuous truss in Fig. 4. Newmark integration is used for obtaining the acceleration. The integration constants of the Newmark β-method, $\beta=1/2$, $\gamma=1/4$, are used for all cases.

The validity of the proposed method is examined through a simulation study with a two-span continuous truss shown in Fig. 4. Typical material properties of steel (Young's modulus = 210 GPa, Specific mass = 7.85×10^3Kg/m^3) are used for all members. The cross sectional areas of top, bottom, vertical and diagonal members are 250 cm^2, 300 cm^2, 200 cm^2 and 220 cm^2, respectively. The natural frequencies of the truss range from 6.6 Hz to 114.7 Hz. Damage of the truss is simulated with 40% and 50% reductions in the sectional areas of member 7 and 16, respectively. The damaged members are depicted by dotted lines in Fig. 4. It is assumed that the damage suddenly occurs at t=0.5 sec. Accelerations of the truss are measured from a vibration induced by a sudden release. The measurement errors are simulated by adding 3% random noise generated from a uniform probability function to accelerations calculated by the finite element model. Observation points and directions are represented in Fig. 5. The total numbers of selected DOF are 22.

Both x- and y- direction accelerations are measured in the time period from 0 sec to 2 sec

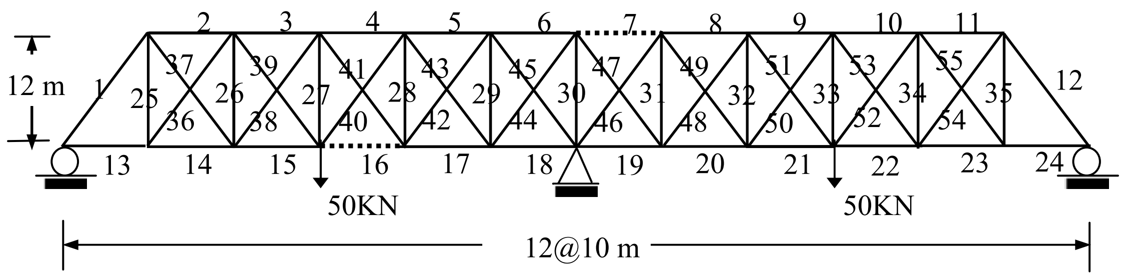

Figure 4. Two span continuous truss

with the interval of 1/200 sec. To filter high frequency mode, the interval of inverse analysis is 1/100 sec. The size of time window is 0.2 sec.

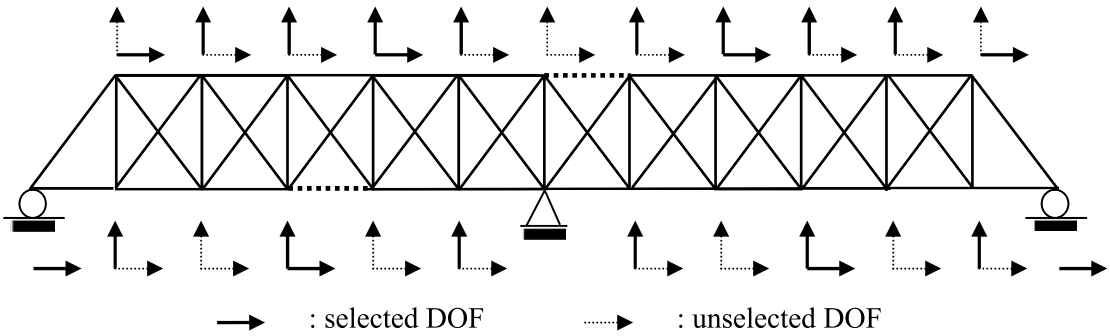

Figure 5. Observation points and directions

The error function values of each trucation number are drawn in Fig. 6. The optimal truncation number is determined as 5 by bilinear fitting method. But the identification results of the truncation number 7 are better than those of truncation number 5. Because the identification results are very sensitive to truncation number, the truncation number must be selected through the results of several truncation numbers. The variations of axial rigidities of the two damaged members and one undamaged member with time are drawn in Figure 7. From the figure, it is clearly seen that the damage occurs at t=0.5 sec, and that the estimated stiffness parameters of damaged members 7 and 16 converge to the actual values as time steps proceed. Figure 7 shows the axial rigidity of each member identified at the final time t=2.0 sec. The vertical axes of both Figure 7 and Figure 8 represent the normalized axial rigidity with respect to the initial value of each member. Since axial rigidities of the damaged members are reduced prominently compared with those of the other members, the damaged members are clearly distinguished from undamaged members.

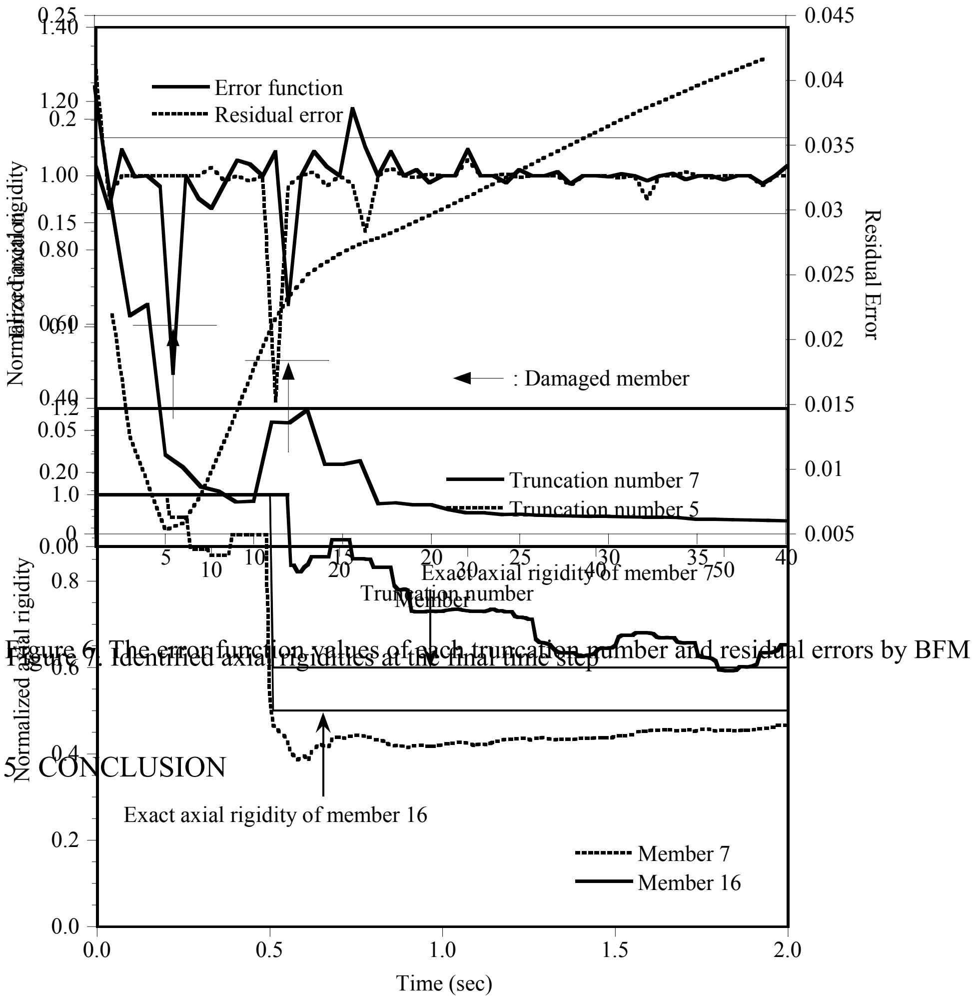

Figure 6. The error function values of each truncation number and residual errors by BFM

Figure 7. Identified axial rigidities at the final time step

5 CONCLUSION

Figure 8. Variations of axial rigidities of two damaged members with time

The time windowing technique are proposed for SI in time domain using measured acceleration data is proposed. The system parameters include the damping parameters as well as the stiffness parameters of a structure. The Rayleigh damping is used to estimate the damping characteristics of a structure. The least square errors of the difference between calculated acceleration and measured acceleration is adopted as an error function. The regularization technique is employed to alleviate the ill-posedness of the inverse problem in SI. The L_1-TSVD is utilized to optimize a non-differential object function. Optimal truncation number is determined by bilinear fitting method.

The proposed method exhibits very compromising characteristics in detecting damage, and is able to estimate the stiffness properties accurately even though the damping characteristics are approximated by the Rayleigh damping. The measurement noise and modeling are well filtered out through the regularization technique. The example presented in this paper shows capabilities of the time window technique for the identification of damage caused by earthquakes.

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