

# **DETECTION OF ABRUPT CHANGES DURING EARTHQUAKE BY AUTOREGRESSIVE MODEL**

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**Abstract** This paper presents a novel damage detection algorithm using measured acceleration during severe events, such as earthquake. Damage is defined as abrupt change which means change of system parameter occurs either instantaneously or at least very fast with respect to the sampling rate of the measurements. Autoregressive model is employed as a transfer function model of non-structural model based scheme (NMBS). Since measured acceleration contain a mix of information related to both the damage in the structure and unknown effects such as environmental perturbations, a time

windowing technique is utilized to deal with environmental changes. The covariance between residual errors and coefficients of autoregressive model is adopted as damage sensitive features. According to decreasing time window size and increasing the noise level, the coefficients of autoregressive model will be extremely unstable. A regularization technique is utilized to alleviate this effect and stabilize the coefficients of autoregressive model. A bilinear fitting method (BFM) is utilized to separate outliers from ordinary state distribution for more reliable statistical inference. A two-span continuous truss example is demonstrated to verify the validity of proposed algorithm.

## **INTRODUCTION**

Over the last few decades, there has been a significant increase in the health monitoring and safety management field of the complex structure. The primary objective of the structural health monitoring is to find changes of system parameters as soon as possible. There are two categories in structural health monitoring and damage assessment whether structural model, such as stiffness, damping and mass information exist or not. One is structural model based scheme and the other is non-structural model based scheme. In structural model based scheme, system parameters are estimated by inverse analysis based on the sensitivity method from a mathematical model. In non-structural model based scheme, structural soundness is evaluated by pattern recognition and statistical approach from only measured signals without a structural model.

Various algorithms for structural health monitoring using static or dynamic responses are proposed. But the main problem of structural health monitoring system is how to handle noises, whereas measured signals contain a mix of information related to both the damage in the structure and the perturbations due to the environment. A structural health monitoring algorithm with time windowing technique is employed.[4] In time windowing technique, the residual errors are predicted sequentially within a finite time period which called time window. The time window advances forward at each

time step to predict residual errors repeatedly. Perturbations of environment are commonly changed gradually during long time period and time window size is relatively very smaller than environmental perturbation period so it is assumed that perturbation of environment can be neglected within the time window.

Covariance between residual errors and coefficients of AR model is adopted as a new damage feature. Residual errors or coefficients are usually adopted as damage features in structural health monitoring using AR model. Residual errors can be influenced by changes of system parameter and external loading, so it is hard to decide the source of changes. The coefficient which is sensitive to damage information and not sensitive to external loading is very good damage feature, but it is highly unstable, so it should to need regularization technique to obtain meaningful coefficients.[3]

Decision whether the considered structure is sound or not using damage feature in every time step is also very important. Bilinear fitting method is utilized for making decision the soundness of the target structure after severe events.[2]

The validity and accuracy of the proposed algorithm is demonstrated through a numerical simulation studies on a two-span continuous truss bridge. The numerically generated acceleration data with noise under Kobe earthquake ground excitation are utilized as measured signals for the numerical simulation example.

## **DAMAGE DETECTION ALGORITHM**

### **Autoregressive (AR) Model**

AR model is utilized to evaluate structural health monitoring system using acceleration signals during a long period.[1] AR model is widely used stochastic model that can be extremely useful in the representation of certain practically occurring series.

In this model, the current value of the process is expressed as a finite, linear combination of previous values of the process and a random error  $e_t$ . Let us denote the values of a process at equally spaced times  $t, t-1, t-2, \dots$  by  $x_t, x_{t-1}, x_{t-2}, \dots$ . Then

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \dots + \phi_p x_{t-p} + e_t \quad (1)$$

is called AR model of order  $p$ . Where,  $\phi$  is coefficients of AR model,  $e_t$  is random error in the measured signal at time  $t$  and  $p$  is order of AR model.

### Least Square Method

AR model is expressed with coefficients as weighted regressive form. There are several methods to calculate coefficients of the AR model. Least square method is utilized because it is very simple and clear. From Eq.1, residual error between estimated value from AR model and measured value at time  $t$  is as follows.

$$e_t = x_t - (\phi_1 x_{t-1} + \phi_2 x_{t-2} + \dots + \phi_p x_{t-p}) \quad (2)$$

The first term in the right side of Eq.2 is a measured signal at time  $t$  and the second term is the estimated value from AR model at time  $t$ . After expansion of Eq.2 into considered time periods and minimize residual errors, the linear object function by least square method is obtained as shown in Eq.3.

$$\Pi = \text{Min}_{\phi} \sum_{t=p+1}^N \|e_t\| = \text{Min}_{\phi} \sum_{t=p+1}^N \{x_t - \Psi_t^T(x) \Phi(\phi)\}^2 \quad (3)$$

Where,  $\Psi_t(x) = [-x_{t-1} \dots -x_{t-p}]^T$ ,  $\Phi(\phi) = [\phi_1 \dots \phi_p]^T$  and  $N$  is total number of measured signals in considered time period.  $N$  must be greater than twice of the order  $p$  of the AR model. The optimal solution is obtained by solving minimization problem.

### Time Window Technique

The key difficulty in structural health monitoring is perturbation of measured signals by unknown effects such as environmental and instrumental effects. Measurement errors can be reduced according to improvement of sensor technology but perturbation of environment cannot be reduced. Measured signals are gradually changed according to various factors of environment such as day and night, season, temperature

and humidity and so on. Even if there is no changes in the considered structure, measured signals can be swayed by this environmental situation. Almost previous methods suffer from this difficulty of environmental factors. Though the algorithm is performed well in experimental data in laboratory, it cannot be applied in real structure because of perturbations of environment.

Time windowing technique is adopted to solve this phenomenon. Environmental factors are commonly gradually changed during very long time period. In autoregressive model with time window technique, the residual errors are predicted sequentially within a finite time period, which is called a time window. The time window overlaps and advances forward at each time step to predict residual errors step by step. Time window size is relatively very smaller than time period of environmental perturbations so it is assumed that changes of environment within the time window cannot happen.

### **Damage Feature**

Both residual errors and coefficients are possible as damage feature in structural health monitoring using AR model. Residual errors can be influenced by external loading as well as changes of system parameter, so it is hard to choose abrupt changes of system parameter. The coefficient which is sensitive to damage information and not sensitive to external loading is very good damage feature, but it is highly unstable. In order to use information of residual errors and coefficients instantaneously, covariance between residual errors and coefficients is suggested as a new damage feature.

### **Regularization**

Because coefficients are extremely unstable, it is need that increasing measured information or regularization of coefficients. Since the number of measured acceleration within time window cannot be increased, regularization technique must be adopted. The regularized least square estimator is shown in Eq.4.

$$\Pi = \underset{\phi}{\text{Min}} \sum_{t=p+1}^N \|e_t\| + \frac{\alpha}{2} \|\Phi - \bar{\Phi}\|^2 = \underset{\phi}{\text{Min}} \sum_{t=p+1}^N \{x_t - \Psi^T(x) \Phi(\phi)\}^2 + \frac{\alpha}{2} \|\Phi - \bar{\Phi}\|^2 \quad (4)$$

where  $\bar{\Phi}$  is the mean value of previously estimated coefficients of AR model and  $\|\cdot\|$  representing the Euclidean norm of a vector.

The regularization function, which represents the variance of system parameters in time, is added to the error function to overcome ill-posedness of inverse problems. The regularization factor has critical effect on the stability of the solution of Eq.4. The optimal regularization factor is determined by the geometric mean scheme (GMS). The recursive quadratic programming with a line search technique is employed to optimize Eq.4.

### Decision Making

To decide whether the considered structure is sound or not using estimated damage features from prediction model is also very important. No matter how prediction model may work perfectly, it is useless without support of rigorous decision making algorithm. It is unreasonable to decide health of the structure by merely the magnitude of damage features. For more reliable decision making of structural health monitoring, statistical approach is inevitable. Effect of variance from each damage feature can be separated by bilinear fitting method and pick up outliers from the boundary of each distribution.

### EXAMPLE

The validity of the proposed structural health monitoring algorithm is verified through a simulation study with a two-span continuous truss shown in Figure 1. Typical material properties of steel (Young's modulus = 210 GPA, Specific mass = 7.85 Kg/m<sup>3</sup>) are used for all truss members. The cross sectional areas of top, bottom, vertical and diagonal members are 112.5 cm<sup>2</sup>, 93.6 cm<sup>2</sup>, 62.5 cm<sup>2</sup> and 75.0 cm<sup>2</sup>, respectively. The

natural frequencies of the truss range from 6.6 Hz to 114.7 Hz. Sampling rate is 400 Hz to involve all of the high frequency modes information. The damping characteristics are simulated by modal damping ratio 3%~30% in each mode, continuously.

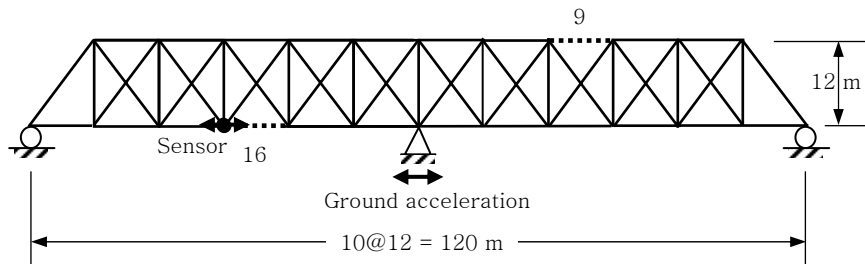


Figure 1: two-span continuous truss

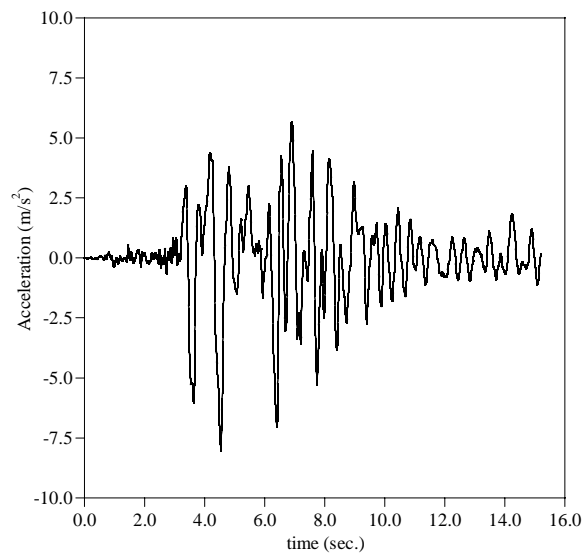


Figure 2: Input ground acceleration from Kobe earthquake

It is assumed that accelerations are measured with Kobe earthquake ground excitation at centered hinge support in the horizontal direction for simulating under

earthquake situation. Input ground acceleration is shown in Figure 2. This ground accelerations which is a maximum magnitude part are extracted from 40 second full data. Sensing points are center of the left span in bottom nodes of the truss in the horizontal direction. It is assumed that abrupt change occurs in the considered structure about 4.55 second. Damage is implemented as reduction of cross sectional area. The cross sectional areas of top member 9 and bottom member 16 are reduced by 40% and 50%, respectively. Damaged members are represented as dotted line in Figure 1. Horizontal direction accelerations are measured numerically in the time period from 0 sec to 15.2 sec. The measurement errors are simulated by adding 5% random proportional noise to accelerations calculated by the finite element model.

The covariance estimated from regularized least square estimator using measured accelerations with time window technique are shown in Figure 3.

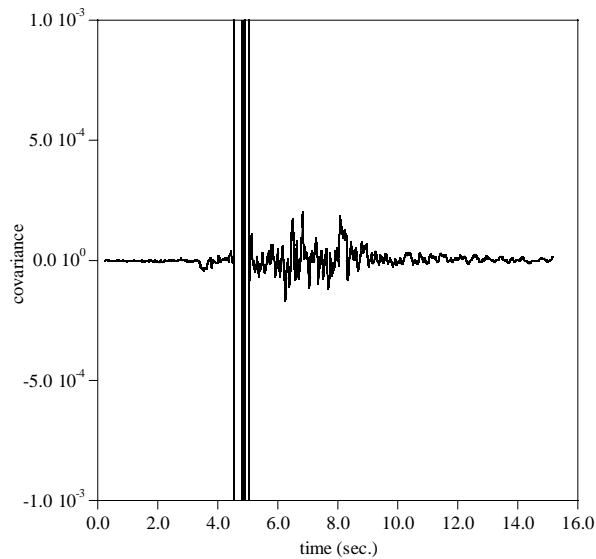


Figure 3: Estimated covariance from regularized AR model



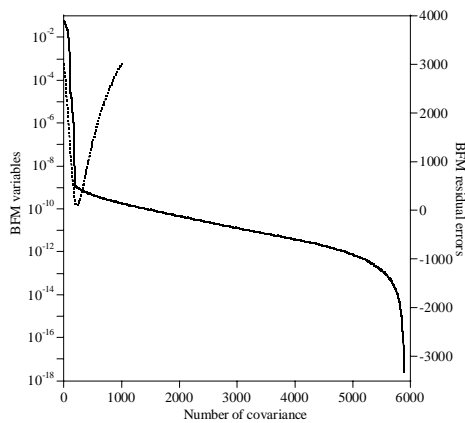


Figure 4: Bilinear Fitting result

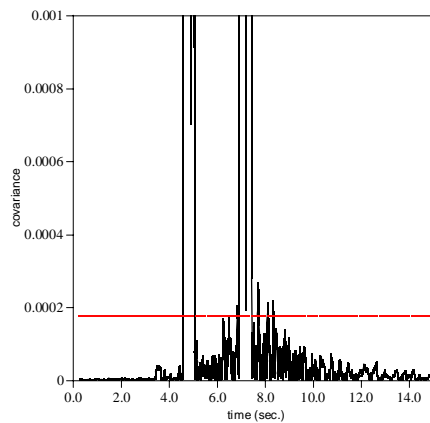


Figure 5: Outlier Detection result

Solid line in Figure 4 is the effect to variance from covariance estimated from AR model. Dotted line is the residual of the least squares fit of covariance in log scale. About two hundred damage features affect huge increase to variance. Therefore, it can be thought that left side of this boundary is distribution of outliers and right side is ordinary distribution. It is shown that outlier detection result in Figure 5. The solid line is covariance as damage feature and the dotted line is threshold value from bilinear fitting method. It is clearly shown that two damage situation around 4.5 second and 7.0 second.

## CONCLUSIONS

New structural health monitoring algorithm which is free from perturbations of environment is proposed. Covariance between residual errors and coefficients are estimated using an autoregressive model with a time window technique is suggested as damage feature. Perturbations of environment can be neglected within time window relatively smaller than time period of data acquisition. Bilinear fitting method is utilized for more reliable threshold value to make decision of soundness of the considered

structure and both damage situations are successfully detected.

## REFERENCES

1. George E. P. Box, Gwilym M. Jenkins and Gregory C. Reinsel, 1994, *Time Series Analysis forecasting and control*, Prentice-Hall, Englewood Cliffs, New Jersey 07632
2. Hyun Woo Park, Byung Kyu Ahn, Hae Sung Lee, 2007, "1-norm based regularization scheme for system identification of structures with discontinuous system parameters", *International Journal for Numerical Methods in Engineering*, Vol. 69, No. 3, pp. 504-523
3. Hyun Woo Park, Soobong Shin, Hae Sung Lee, 2001, "Determination of an optimal regularization factor in system identification with Tikhonov regularization for linear elastic continua." *International Journal for Numerical Methods in Engineering*, Vol. 51, No.10, pp.1211-1230
4. Joo Sung Kang, Seung-Keun Park, Soobong Shin, Hae Sung Lee, 2005, "Structural system identification in time domain using measured acceleration", *Journal of Sound and Vibration* 288, pp 215-234