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Detection of sudden damages of structure by regularized autoregressive model using measured acceleration

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ABSTRACT: This paper presents a novel damage detection algorithm by regularized autoregressive model using measured acceleration. Damage is defined as a sudden change in some parameters of the considered structure. The autoregressive model is employed to analyze the characteristics of measured acceleration data statistically. A time windowing technique is utilized to overcome perturbation of environmental effects on measured acceleration data. A regularization technique is adopted to alleviate the ill-posedness and to stabilize autoregressive coefficients. The covariance between residuals and first order coefficients of autoregressive model is proposed as a new damage feature. The generalized extreme value distribution (GEV) is utilized to pick out outliers from distribution of damage features for more reliable statistical inference. The average method with normalization is utilized to draw integrated decision from various results of each sensor. A numerical simulation on a two-span continuous truss under normal operational condition will be demonstrated to verify the validity of proposed algorithm.

1 INTRODUCTION

Recently, there has been a significant increase in the health monitoring and safety management field of the civil structures. The primary goal of the structural health monitoring is to find changes of system parameters and to decide whether it is sound or not at earliest possible stage. There are two categories in structural health monitoring and damage assessment whether structural model, such as stiffness, damping and mass information is used or not. One is structural model based scheme and the other is non-structural model based scheme. In structural model based scheme, system parameters are estimated by inverse analysis based on the sensitivity method from a mathematical model. In non structural model based scheme, structural soundness is evaluated by transfer function model and statistical treatment using only measured signals without structural model information.

Structural model based system identification problem is a type of inverse problems, which are usually ill-posed problem. An ill-posed problem is characterized by the non-uniqueness, non-continuous and instability of solutions. Various regularization techniques have been developed to overcome this ill-posedness of inverse problem. In spite of ill-posedness can be alleviated by regularization techniques successfully, model-based system identification schemes are not applied in real situation because of modeling error that difference between mathematical model and real structure. Recently, a lot of structural health monitoring algorithms with statistical pattern recognition using purely measured signals has been attempted in the center of Los Alamos national laboratory in USA.

Non structural model based structural health monitoring procedure consists of four steps which is Data acquisition from sensors of structure, data transmission, data analysis using measured data for damage detection and decision making whether the considered structure is sound or not. In spite of rapid progress of sensor and IT technology, rigorous damage detection algorithm did not be proposed. Therefore so many measured signals obtained from various structures can not be used for structural health monitoring.

The autoregressive (AR) model is utilized to make non structural model based system. The autoregressive model was used by Yule for the first time to explain the changes and periods of sunspot from simple trigonometric identity. The autoregressive model can be extremely useful in the representation of certain practically occurring time series. In the autoregressive model, the current value of the process is expressed as a finite, linear aggregate of previous values of the process and a shock.

Various non structural based structural health monitoring algorithms using static or dynamic responses have been proposed. But the main problem of structural health monitoring is how to handle noises on measured signals, whereas measured signals contain a mix of information related to both the damage in the structure and the perturbations due to the environmental changes. The autoregressive model with time windowing technique is employed to overcome the perturbations of measured signals. In time windowing technique, the autoregressive model is estimated sequentially using measured data within a finite time period which called time window. The time window advances forward at each time step to update autoregressive model repeatedly. Perturbations due to environmental changes are commonly changed gradually during long time period and time window size is relatively very smaller than environmental perturbation period so it can be assumed that perturbations of environmental changes can be neglected within the time window.

Making decision whether the considered structure is sound or not using damage features from each sensor in every time step is also very important. The extreme value distribution is utilized to detect outliers because damage information almost lie in the tail of distribution and the extreme value distribution is well established for tail distribution. The generalized extreme value distribution (GEV) is utilized for simplicity. The average method with normalization is adopted to draw integrated decision whether the structure is sound or not using decision results from each sensor.

The validity and accuracy of the proposed damage detection algorithm will be verified through a numerical simulation studies on a two-span continuous truss. A normal operational condition is simulated randomly from three types of vehicle, car, bus and truck. The numerically generated acceleration data with proportional noise under normal operational condition are utilized as measured signals for the numerical simulation example.

2 AUTOREGRESSIVE MODEL AND DAMAGE FEATURE

2.1 *Autoregressive model*

The autoregressive (AR) model is utilized to evaluate structural health monitoring system using measured signals from sensors. The autoregressive model is widely used in time series analysis. Let us denote the values of a process at equally spaced times t, t-1, t-2, by $y(t)$, $y(t$-1), $y(t$-2).
Then

$$y(t) = -a_1 y(t-1) - a_2 y(t-2) - \cdots - a_p y(t-p) + e(t) \tag{1}$$

is called autoregressive process of order p. Where a is coefficients of autoregressive model, $e(t)$ is random error in the measured signal at time t, respectively. The autoregressive model is expressed with coefficients as weighted regressive form.

There are several methods to calculate coefficients of the autoregressive model which is least squared method, moment method, maximum likelihood, bayesian theory and so on. The least squared method to estimate autoregressive model is utilized because it is very simple and clear.

The prediction value from autoregressive model of order p can be defined as follows.

$$\begin{aligned}\hat{y}(t \setminus \theta) &= -a_1 y(t-1) - a_2 y(t-2) - \cdots - a_p y(t-p) \\ &= \theta^T \varphi(t) = \varphi^T(t)\theta\end{aligned} \tag{2}$$

Where $\hat{y}$ represents prediction value from autoregressive model, θ represents system parameter vector and φ represents regression vector, respectively.

$$\theta = [a_1 \quad a_2 \quad \dots \quad a_p]^T$$
$$\varphi(t) = [-y(t-1) \quad -y(t-2) \quad \dots \quad -y(t-p)]^T \tag{3}$$

The residuals can be defined as the difference between measured signals and prediction values using autoregressive model at each time step. The linear object function by least squared method can be obtained as shown in Equation 4.

$$V(\theta) = \frac{1}{2}\sum_{k=p+1}^{N}\left[y(k) - \varphi^T(k)\theta\right]^2 \tag{4}$$

Where N represents the total number of measured signals. The optimal solution of Equation 4 can be obtained by least squared method as shown in Equation 5.

$$\hat{\theta}_N^{LS} = \arg\min V(\theta) = \left[\sum_{k=p+1}^{N}\varphi^T(k)\varphi(k)\right]^{-1}\sum_{k=p+1}^{N}\varphi^T(k)y(k) \tag{5}$$

2.2 *Time windowing technique*

The key difficulty in structural health monitoring is perturbation of measured signals by unknown effects such as environmental and instrumental effects. Measurement errors can be reduced according to improvement of sensor technology but perturbation of environment cannot be reduced. Measured signals are gradually changed according to environment, especially temperature. Even if there is no damages in the considered structure, measured signals can be swayed by environment. Every previous method suffers from this difficulty of environmental factors. Though an damage detection algorithm is performed well in experimental data in laboratory, it cannot be applied in real structure because of perturbations of environment.

Various pattern recognition algorithms attempt to solve this problem, but they still have some limitations to apply in real structures. The reasons are accuracy and economical efficiency. Pattern recognition technique requires so many priori solutions. The more priori solutions we have, the more accuracy pattern recognition algorithm has. The learning process which called finding adequate solutions from the priori solutions takes so long time. Because the environmental conditions cannot be exactly same in the previous time step, the solutions obtained by learning process still have modeling errors. And the final goal of structural health monitoring is finding defects as soon as possible, so the process must be fast in order to apply in real time detection.

A time windowing technique is utilized to overcome this problem. Environmental perturbations are commonly changed during relatively long time period. In the autoregressive model with time windowing technique, the autoregressive model is estimated sequentially within a finite time period, which is called a time window. The time window overlaps and advances forward at each time step to update autoregressive model. A time window size is relatively very smaller than time period of environmental changes so it can be assumed that perturbations of measured signals from environmental changes within the time window cannot be happened.

In the autoregressive model with time windowing technique, the autoregressive model will be estimated sequentially not using all of the signals but measured signals within a finite time period which called time window as shown in figure 1. The estimated autoregressive coefficients represent the end time of the time window. The residuals can be obtained using estimated autoregressive model by the one step ahead prediction. The time window advances forward at each time step to estimate autoregressive coefficients and residuals repeatedly.

The optimal solution of coefficients of autoregressive model with time windowing technique can be obtained by following the same sequence in previous mentioned thus the final form is shown in Equation 6.

$$\hat{\theta}_t = \left[\sum_{k=(t-nw)+p+1}^{t}\varphi^T(k)\varphi(k)\right]^{-1}\sum_{k=(t-nw)+p+1}^{t}\varphi^T(k)y(k) \tag{6}$$

Where nw is the number of measured data within time window.

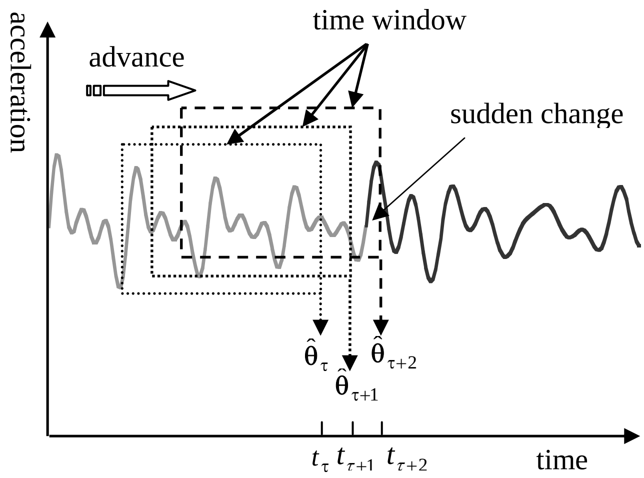

Figure 1. Outline of Time windowing technique.

2.3 *Regularization technique*

The autoregressive coefficients which are estimated by minimization of least squared errors are extremely unstable. Since the number of measured signals within time window cannot be increased, regularization technique must be adopted to alleviate instability of autoregressive coefficients. The regularized least square estimator is shown in Equation 7. The regularization function is added to the error function to overcome ill-posedness of inverse problems.

$$\underset{\theta}{\text{Min}}\ \Pi = \frac{1}{2}\sum_{k=(t-nw)+p+1}^{t}\left(\varphi^T(k)\,\theta_t - y(k)\right)^2 + \frac{\beta}{2}\left\|\theta_t - \theta^*{}_t\right\|_2^2 \tag{7}$$

Where θ^* represent the mean value of the autoregressive coefficients of previous time steps and β represents regularization factor.

The regularization factor has critical effect on the stability of the solution. The optimal regularization factor is determined by the geometric mean scheme (GMS) as shown in Equation 8.

$$\beta = \sqrt{S_{\max}\cdot S_{\min}} \tag{8}$$

Where S is a singular value obtained from singular value decomposition of system matrix.

2.4 *Damage feature*

There are two possible damage features in damage detection using autoregressive model. One is the residual and the other is the autoregressive coefficient. Residual has good characteristic which is very stable in any case and sensitive to both amplitude and frequency change of measured data, but it is difficult to identify the source of changes. The autoregressive coefficients are the system parameter of autoregressive model. The autoregressive coefficients are sensitive to frequency change and insensitive to amplitude change, but it suffers from ill-posedness in minimizing least squared errors.

The covariance between residuals and autoregressive coefficients is proposed as a new damage feature as shown in Equation 9 in order to use information of residuals and autoregressive coefficients instantaneously. The absolute value of residuals and autoregressive coefficients are used because the directional information of damage feature is not necessary.

$$\begin{aligned}\text{cov}[|e_t|, |\theta_t^1|] &= E[|(e_t - \mu_{e_t})||(\theta_t^1 - \mu_{\theta_t^1})|] \\ &= \frac{1}{nw-1}\sum_{k=t-nw+1}^{t}\left[|(e_k - \mu_{e_t})||(\theta_k^1 - \mu_{\theta_t^1})|\right] \qquad t \geq 2nw\end{aligned} \tag{9}$$

3 DECISION MAKING

3.1 *Extreme value distribution*

To decide whether the considered structure is sound or not using estimated damage features from autoregressive model is also very important. No matter how autoregressive model may work perfectly, it is useless without support of rigorous decision making algorithm. It is unreasonable to decide health of the structure by merely the magnitude of damage features. For more reliable decision making of structural health monitoring, statistical approach is inevitable. Distribution of the damage features must be found statistically from estimated damage features and pick out outliers from the distribution under normal condition in a given significant level.

The outlier of damage features always lie in the tail of the distribution of damage features. The extreme value distribution is utilized for a more accurate selection of outliers because extreme value distribution is well established for tail distribution.

3.2 *Generalized extreme value distribution*

Generally, it is known that tail distribution of any probability distribution follows one of three extreme value distributions, Gumbel, Weibull and Flechet distribution. It is very annoyed to find what the best distribution in three distributions to estimate the distribution of damage features is. The three types of distribution can be expressed in one single form, called von-mises form as shown in Equation 10. Equation 10 expresses distribution for maxima and minima, respectively. According to the value of *c*, von-mises form can change to the three extreme value distribution forms. In case $c > 0$, $c < 0$, $c = 0$, we get Frechet, Weibull and Gumbel distribution, respectively.

$$
\begin{aligned}
\overline{G}(x;\lambda,\delta,c) &= \exp\left\{-\left[1+c\left(\frac{x-\lambda}{\delta}\right)\right]^{-1/c}\right\} \quad \left(1+c\left(\frac{x-\lambda}{\delta}\right)\geq 0, \quad \delta>0\right) \\
\underline{G}(x;\lambda,\delta,c) &= 1-\exp\left\{-\left[1+c\left(\frac{\lambda-x}{\delta}\right)\right]^{-1/c}\right\} \quad \left(1+c\left(\frac{\lambda-x}{\delta}\right)\geq 0, \quad \delta>0\right)
\end{aligned}
\qquad (10)
$$

3.3 *Optimization*

Optimization process is utilized to find three parameters of the generalized extreme value distribution. The optimal three parameters can be obtained by minimizing difference between cumulative density function of the generalized extreme value distribution and empirical cumulative density function like Equation 11,.

$$
Min\frac{1}{2}\left[\mathbf{G}(x;\lambda,\delta,c)-\mathbf{p}\right]^{T}\mathbf{W}\left[\mathbf{G}(x;\lambda,\delta,c)-\mathbf{p}\right] \qquad (11)
$$

Where, **G** is cumulative density function of the generalized extreme value distribution, **p** is empirical cumulative density function of the extreme value of damage features and **W** is the weighting matrix.

3.4 *Outlier detection*

From the estimated distribution of the damage features, the threshold value of maxima can be found by one to one match from given significant level. A damage feature which is greater than threshold value of maxima is defined as outlier. It can be judged that some sudden changes are occurred in the structure when outlier of damage features is detected. A real-time structural health monitoring system can be consisted by repeating this sequence continuously.

The average method with normalization in the ratio of the damage feature to threshold value is utilized to make integrated decision from the results of each sensor.

4 EXAMPLE

4.1 *Two-span continuous truss*

The validity of the proposed structural health monitoring algorithm is verified through a two-span continuous truss shown in Figure 2. The sensors are cross located each other in order to avoid the loss of information because of symmetry. Typical material properties of steel (Young's modulus = 210 GPA, Specific mass = 7.85 Kg/m^3) are used for all truss members. The cross sectional areas of top, bottom, vertical and diagonal members are 112.5 cm^2, 93.6 cm^2, 62.5 cm^2 and 75.0 cm^2, respectively. The natural frequencies of the truss range from 6.6 Hz to 114.7 Hz. The damping characteristics are simulated by 5% Rayleigh damping. The sampling rate is 100 Hz and the duration of simulated acceleration is 1 hour (3600 seconds). 5% proportional noise is putted on measured data to consider measurement noise.

4.2 *Loading scenario and Damage scenario*

It is assumed that accelerations are measured under normal operational condition. The moving vehicles are classified into three types, car, bus and truck and assumed that distribution of weight follows normal distribution. The car, bus and truck follows $N(2.3,0.2^2)$, $N(13.5,3.2^2)$ and $N(33.8,2.9^2)$, respectively. The limit speed of universal road 60km/h is applied for speed of vehicle load and 20% reduction of speed for truck is applied. The car, bus and truck vehicle loads are generated by 77%, 15% and 7%, respectively. The overloading condition is also generated three times at 2244 second, 2474 second and 3401 second in order to compare with changes due to damage.

It is assumed that sudden change occurs twice in the considered structure at 2730 second and 3002 second. Damage is implemented as reduction of cross sectional area. The cross sectional areas of Upper member 9 and lower member 16 are reduced at first damage instant by 40% and 50%, respectively and lower member 16 are reduced additionally at second damage instant by 20%. Damaged members are represented as dotted line in Figure 2.

4.3 *Result of damage detection*

The damage detection results of representative sensor at both span S_{02} and S_{08} are shown in Figure 3 and Figure 4. The damage features estimated from regularized autoregressive model using measured accelerations with time windowing technique are shown in Figure 3 and Figure 4. Dotted line represents threshold values of outliers in given 99.9% significant level. The damage features at two damage instants are greatly distinguished from threshold value. It is definitely represented that some sudden changes occur in target structure at damage instants. There are a few false alarms and a little difference among the results of each sensor but sudden changes can be detected by proposed algorithm successfully.

In the damage detection result of sensor S_{08}, the sudden changes of truss cannot be detected at second damage instant. This result is concerned in load path of the considered structure. The results of sensor which is located in the load path through damage members show more accurate detection of sudden changes.

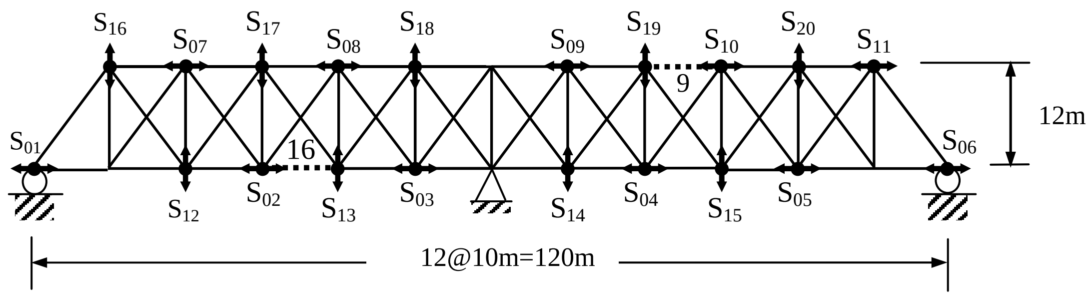

Figure 2. two-span continuous truss.

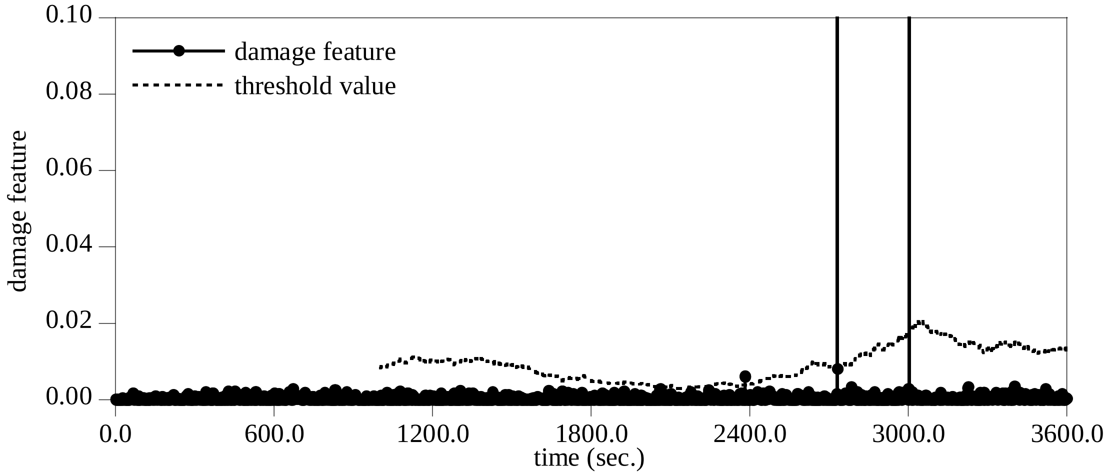

Figure 3. Damage detection result of sensor $S_{02.}$

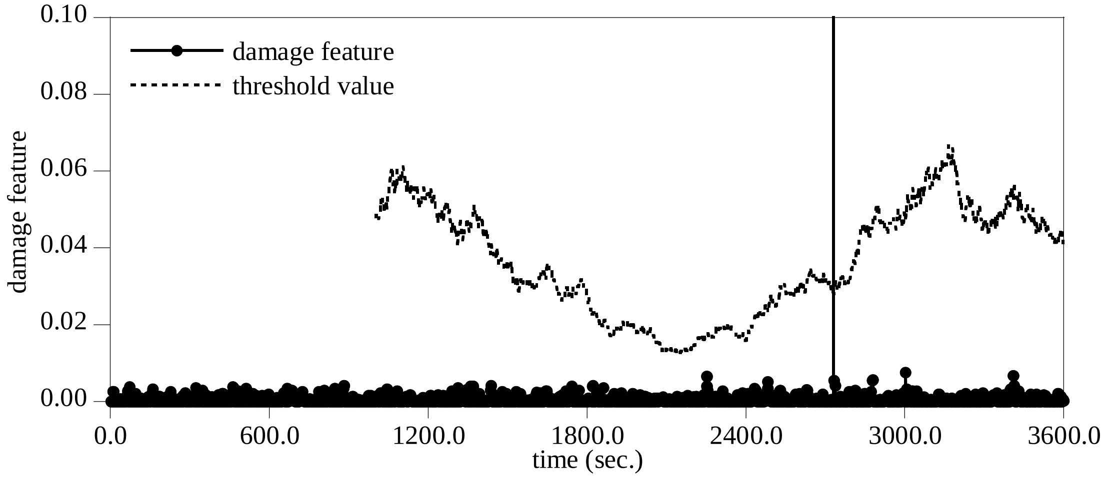

Figure 4. Damage detection result of sensor $S_{08.}$

4.4 *Integrated decision*

The result of integrated decision using the average method with normalization in the ratio of the damage feature to threshold value from the results of each sensor is shown in Figure 5. The sudden changes at both damage instants are detected successfully.

5 CONCLUSION & FURTHUR STUDY

New structural health monitoring algorithm using regularized autoregressive model with windowing technique is proposed. The perturbations due to environmental changes can be neglected within time window relatively smaller than time period of data acquisition. The covariance between residual and autoregressive coefficient is proposed as a new damage feature. The generalized extreme value distribution is utilized for more reliable decision making of soundness of the structure. The average method with normalization is utilized to make integrated decision from the damage detection results of each sensor. The validity of proposed algorithm is demonstrated by numerical simulation example in two-span continuous truss under normal operational condition.

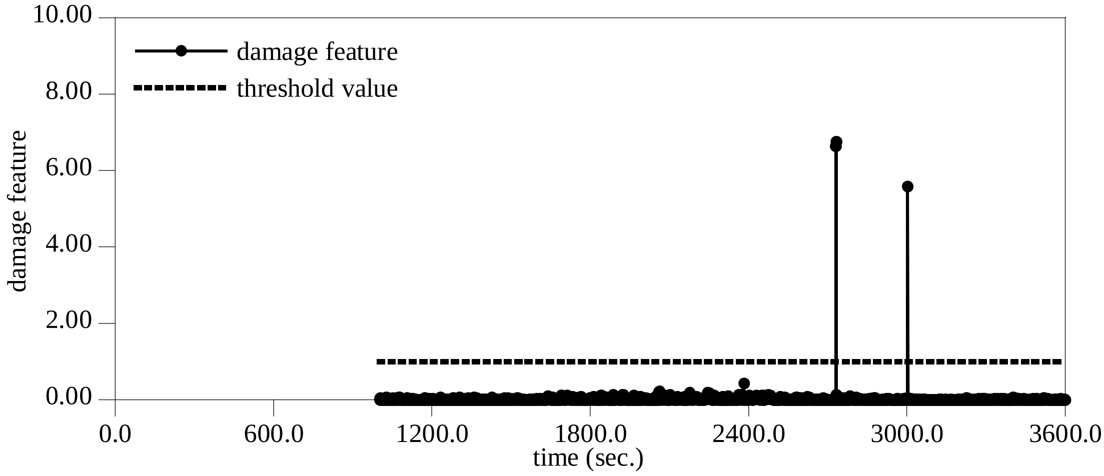

Figure 5. Integrated decision result.

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