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A regularization scheme for displacement reconstruction using measured structural acceleration data

Y.H. Hong
Dept. of Civil and Environmental Engineering, Seoul National University, Seoul, Korea

H.W. Park
Dept. of Civil Engineering, Dong-A University, Busan, Korea

H.S. Lee
Dept. of Civil and Environmental Engineering, Seoul National University, Seoul, Korea

ABSTRACT: This paper present a new displacement reconstruction scheme using only acceleration measured from a structure. For a given set of acceleration data, the reconstruction problem is formulated as a boundary value problem in which the acceleration is approximated by the second-order central finite difference of displacement. The displacement is reconstructed by minimizing the least squared errors between measured and approximated acceleration within a finite time interval referred to as a time window. An overlapping time window is introduced to improve the accuracy of the reconstructed displacement. The displacement reconstruction problem becomes ill-posed because the boundary conditions at both ends of each time window are not known *a priori*. Furthermore, random noise in measured acceleration causes physically inadmissible errors in the reconstructed displacement similar to the conventional time integration schemes. A Tikhonov regularization scheme is adopted to alleviate the ill-posedness. The validity of the proposed method is demonstrated through a stay cable experiment.

1 INTRODUCTION

Numerous attempts have been made to reconstruct displacement with measured acceleration based on the definition of acceleration, that is, acceleration is the second-order derivative of displacement in time domain. Time integration schemes based on time-marching algorithms such as the Newmark-β method are probably the most straightforward and easiest way to obtain displacement from measured acceleration. However, the time marching algorithms yield erroneous displacement. First of all, initial conditions on velocity and displacement required in the time marching algorithms are usually unavailable or inaccurate in real situations. Moreover, random noise in measured acceleration data causes physically inadmissible errors in the reconstructed displacement. Particularly, low-frequency spectral components in random noise are amplified during time marching procedures, which severely deteriorate the accuracy of the reconstructed displacement. This undesirable effect becomes a critical issue in the displacement reconstruction for large-scale civil infrastructures, which usually exhibit very low fundamental frequencies.

Several remedies to overcome the drawbacks of the time-marching algorithms have been proposed for the displacement reconstruction with measured acceleration. A baseline correction technique used in seismology applications is a well-known approach to eliminate the erroneous components in the reconstructed displacement by the time-marching algorithms. In this approach, polynomial functions approximately representing the inadmissible errors are constructed, and are subtracted from the reconstructed displacement. However, the baseline correction depends on an engineer's decision, and thus is inadequate to structural health monitoring (SHM) and structural control (SC) applications in which measured acceleration should be automatically processed in real-time. Moreover, this approach corrects erroneous results obtained by the time-marching algorithms, and is not completely free of the aforementioned drawbacks.

This paper formulates a new class of the displacement reconstruction scheme as a boundary value problem rather than an initial value problem using measured acceleration without any information on initial conditions. In case measured accelerations are given over a finite time interval referred to as a time window, the relation between the measured acceleration and the definition of acceleration forms a boundary value problem. As the second-order time derivative of displacement is acceleration, the displacement is reconstructed through the minimization of the least squared errors between measured acceleration and the second-order time derivative of displacement in a time window. The second-order time derivative is approximated by the central finite difference.

As the reconstruction problem of displacement is defined as a boundary value problem in a time window, boundary conditions at both ends of the domain should be specified to solve the minimization problem, but neither displacement nor velocity is known at the boundaries. Therefore, the minimization problem for the reconstruction of displacement becomes ill-posed or rank-deficient, and can not be solved for unknown displacement in a time window. Furthermore, a small amount of low-frequency spectral noise in measured acceleration data may significantly pollute the reconstructed displacement as in the time-marching algorithm. To overcome these two difficulties, the Tikhonov regularization scheme, which has been widely employed to alleviate the ill-posedness of inverse problems, is adopted. The 2-norm of the displacement to be reconstructed in a time window is chosen as the regularization function.

An overlapping time-window concept proposed by Part *et al*. is adopted to enhance the accuracy of reconstructed displacement. The reconstructed displacement only at the center of a time window is taken as the solution of the time window so that the error by inaccurate estimation of boundary conditions should be minimized. Considering the accuracy and computational effort of displacement reconstruction, the optimal time-window size is proposed through a parameter study of SDOF systems.

The validity of the proposed method is demonstrated through displacement reconstructions using raw acceleration data measured from laboratory vibration test of a stay cable. It is shown that the proposed displacement reconstruction scheme does not suffer from any instability caused by low-frequency spectral noise, and yields accurate and reliable results.

2 DISPLACEMENT RECONSTRUCTION SCHEME

2.1 *Displacement reconstruction scheme as an initial value problem*

Dynamic structural responses such as acceleration, velocity and displacement are calculated by solving the following discretized equation of motion of a structure with proper initial conditions.

$$\mathbf{Ma}(t) + \mathbf{Cv}(t) + \mathbf{Ku}(t) = \mathbf{p}(t)\,,\ \ \mathbf{v}(0) = \mathbf{v}_0 \ \text{ and } \ \mathbf{u}(0) = \mathbf{u}_0 \tag{1}$$

where **M, C**, **K,** and **p** represent the mass, damping, stiffness matrix of a structure and a load vector imposed on the structure, respectively, while **a**, **v** and **u** denote the acceleration, velocity and displacement of the structure, respectively. The prescribed initial conditions for velocity and displacement are given as $\mathbf{v}_0$ and $\mathbf{u}_0$, respectively. The equation of motion given in Eq. (1) is the system of an initial value problem in time domain, and represents a physical phenomenon that the specified initial conditions propagate through time.

To solve Eq. (1) numerically, a time integration scheme based on a time marching algorithm is employed to express displacement and velocity in terms of acceleration. The propagating characteristics of Eq. (1) should be properly considered in a time integration scheme. Several well-formulated time integration schemes have been proposed and successfully applied to various types of dynamic problems. Among them, the most popular one may be the Newmark-β method, which utilizes the following expressions.

$$\begin{aligned} v_i &= v_{i-1} + ((1-\gamma)a_{i-1} + \gamma a_i)\Delta t \\ u_i &= u_{i-1} + (\Delta t)v_{i-1} + ((0.5-\beta)a_{i-1} + \beta a_i)(\Delta t)^2 \end{aligned} \tag{2}$$

where subscript i denotes a discrete time step, and β and γ represent numerical parameters defining the variation of acceleration over a time step. Δt is a step length for the time marching algorithm, which is often referred to as a time increment. Once displacement and velocity are expressed in terms of acceleration using Eq. (2) for current time step *i*, Eq. (1) is solved for acceleration. As the above procedure is applied stepwise, the entire histories of dynamic responses of a structure are calculated.

To investigate propagating characteristics of Eq. (2), the velocity is eliminated from the equations, and the displacement is expressed in terms of the initial conditions and measured acceleration.

$$
\begin{aligned}
& u_k = u_0 + k\Delta t v_0 - k(\Delta t)^2 \gamma a_0 + (\Delta t)^2 \beta (a_k - a_0) + (\gamma + \frac{1}{2})(\Delta t)^2 S_{k-1} + (\Delta t)^2 \sum_{p=1}^{k-1} S_{p-1} \\
& \quad k \geq 1, \;\; S_k = \sum_{i=0}^{k} a_i
\end{aligned} \tag{3}
$$

From Eq. (3), it is clearly seen that noise in the initial displacement propagates though time while noise in the initial velocity and acceleration are amplified linearly and quadratically, respectively. In case noise components in measured acceleration are random with zero mean, noise in term S_{k-1} may vanish. However, the last term in Eq. (3) causes the accumulation of noise, which is explained by expressing the term for measured accelerations.

2.2 *Displacement reconstruction scheme as a boundary value problem*

A new approach to reconstruct displacement with measured acceleration is presented. Suppose acceleration at a fixed material point is completely measured during a time period $T_1 \leq t \leq T_2$, and known. By definition, the acceleration of a fixed material point is expressed in terms of displacement through a second order ordinary differential equation.

$$
a(t) \equiv \frac{d^2 u(t)}{dt^2} \approx \bar{a}(t) \qquad T_1 < t < T_2 \tag{4}
$$

where $\bar{a}(t)$ is the measured acceleration. In case proper boundary conditions on displacement or velocity at $t = T_1$ and $t = T_2$ are given, Eq. (4) becomes a boundary value problem, and displacement is easily obtained by integrating Eq. (4) twice and applying two boundary conditions. Since, however, the boundary conditions for Eq. (4) are generally not known, the displacement field cannot be determined by integrating Eq. (4) twice. Furthermore, random noise components included in the measurement should be taken care of in the displacement reconstruction with Eq. (4). This study utilizes the following minimization problem rather than attempts to solve Eq. (4) directly.

$$
\operatorname*{Min}_{u} \Pi_E(u) = \frac{1}{2} \int_{T_1}^{T_2} (a(u(t)) - \bar{a})^2 dt \tag{5}
$$

As acceleration is measured discretely by a uniform time interval Δt in actual, the object function in Eq. (5) is discretized by the trapezoidal rule.

$$
\begin{aligned}
\Pi_E(u) &= \frac{1}{2} \int_{T_1}^{T_2} (a - \bar{a})^2 dt \\
&\approx \frac{1}{2} \sum_{k=1}^{n} \frac{1}{2} ((\tilde{a}_{k-1} - \bar{a}_{k-1})^2 + (\tilde{a}_k - \bar{a}_k)^2) \Delta t \\
&= \frac{1}{2} (\frac{1}{2} (\tilde{a}_0 - \bar{a}_0)^2 + (\tilde{a}_1 - \bar{a}_1)^2 + \cdots + (\tilde{a}_{n-1} - \bar{a}_{n-1})^2 + \frac{1}{2} (\tilde{a}_n - \bar{a}_n)^2) \Delta t \\
&= \frac{1}{2} (\tilde{\mathbf{a}} - \bar{\mathbf{a}})^T (\mathbf{L}_a)^T \mathbf{L}_a (\tilde{\mathbf{a}} - \bar{\mathbf{a}}) \Delta t = \left\| \mathbf{L}_a (\tilde{\mathbf{a}} - \bar{\mathbf{a}}) \right\|_2^2 \Delta t
\end{aligned} \tag{6}
$$

where n, $\tilde{a}_k$, $\bar{a}_k$ and $\| \cdot \|_2$ are the number of the time intervals in period $T_1 \leq t \leq T_2$, the calculated acceleration, the measured acceleration at the k-th time step and the 2-norm of a vector, respectively, and the bold-faced variables denote the corresponding vectors. $\mathbf{L}_a$ is a diagonal weighting matrix of order $(n+1)$ defined as follows.

$$\mathbf{L}_a = \begin{bmatrix} 1/\sqrt{2} & & & & & & \\ & 1 & & & 0 & & \\ & & 1 & & & & \\ & & & \ddots & & & \\ & & & & 1 & & \\ & 0 & & & & 1 & \\ & & & & & & 1/\sqrt{2} \end{bmatrix} \tag{7}$$

The calculated acceleration, $\tilde{a}_k$, is discretized by the central finite difference of Eq. (4), which is the proper approximation of the second-order boundary value problems.

$$\frac{u_{k+1} - 2u_k + u_{k-1}}{(\Delta t)^2} = \tilde{a}_k(u) \text{ for } k = 0, \cdots, n \tag{8}$$

where u_k is displacement at the k-th time step. Eq. (8) is rewritten in a matrix form for all k.

$$\frac{1}{(\Delta t)^2}\mathbf{L}_c\mathbf{u} = \tilde{\mathbf{a}} \tag{9}$$

where $\mathbf{L}_c$ and u denote the linear algebraic operator matrix of order $(n+1)\times(n+3)$ and the vector of displacements at the discrete time steps, respectively, and are defined as follows.

$$\mathbf{L}_c = \begin{bmatrix} 1 & -2 & 1 & & & & \\ & 1 & -2 & 1 & & 0 & \\ & & & \ddots & & & \\ & 0 & & 1 & -2 & 1 & \\ & & & & 1 & -2 & 1 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} u_{-1} \\ u_0 \\ u_1 \\ \vdots \\ u_{n-1} \\ u_n \\ u_{n+1} \end{bmatrix} \tag{10}$$

Substitution of Eq. (9) into Eq. (6) leads to the following discretized minimization problem of Eq. (5).

$$\underset{\mathbf{u}}{\mathrm{Min}}\,\Pi_E(\mathbf{u}) = \frac{1}{2}\left\|\frac{1}{(\Delta t)^2}\mathbf{L}_a\mathbf{L}_c\mathbf{u} - \mathbf{L}_a\bar{\mathbf{a}}\right\|_2^2 \Delta t = \frac{1}{2}\left\|\mathbf{L}\mathbf{u} - (\Delta t)^2\mathbf{L}_a\bar{\mathbf{a}}\right\|_2^2 \frac{1}{(\Delta t)^3} \tag{11}$$

where $\mathbf{L} = \mathbf{L}_a\mathbf{L}_c$. As the time increment is considered as a constant in this study, the term on the time increment outside the 2-norm has no effect on the solution of the minimization problem, and thus is omitted from the object function in Eq. (11).

$$\underset{\mathbf{u}}{\mathrm{Min}}\,\Pi_E(\mathbf{u}) = \frac{1}{2}\left\|\mathbf{L}\mathbf{u} - (\Delta t)^2\mathbf{L}_a\bar{\mathbf{a}}\right\|_2^2 \tag{12}$$

The minimization problem of Eq. (12) is unable to yield a unique displacement for given measured acceleration due to the rank-deficiency in linear algebraic operator $\mathbf{L}$, which is caused by the fact that only $(n+1)$ finite difference equations are defined in Eq. (9) for $(n+3)$ unknown displacement. The two additional displacements at time step –1 and $(n+1)$ outside the time window are included in Eq. (10) to define the second-order central finite difference at the two boundaries. The time steps denoted by –1 and $(n+1)$ play the same role as fictitious nodes that are usually employed to solve elliptic partial differential equations by the finite difference method.

As proper boundary conditions are not specified for Eq. (4), the minimization problem of Eq. (12) becomes ill-posed. To solve ill-posed problems such as inverse problems, the regularization techniques, in which a priori estimates of solutions are defined by a regularity condition as additional information, are widely adopted. The reconstructed displacement with Eq. (12) should stay around the static displacement of a given system, which is expressed by the following equation.

$$\Pi_R = \frac{1}{2}\left\|\mathbf{u} - \mathbf{u}_{st}\right\|_2^2 \leq r^2 < \infty \tag{13}$$

where Π_R is a regularization function, and r defines a solution bound. As the static displacement has no effect on the acceleration defined in Eq. (4), only the dynamic component in the total displacement can be reconstructed. Therefore, the displacements in Eq. (12) and (13) represent the dynamic displacement measured from the static equilibrium position of a structural system, and the static displacement in Eq. (13) should be set to zero, which leads the following expression

$$\Pi_R = \frac{1}{2}\left\|\mathbf{u}\right\|_2^2 \leq r^2 < \infty \tag{14}$$

Since the solution bound is not known a priori, the regularity condition Eq. (14) is enforced as a penalty function to the original minimization problem.

$$\underset{\mathbf{u}}{\text{Min}}\,\Pi(\mathbf{u}) = \frac{1}{2}\left\|\mathbf{L}\mathbf{u} - (\Delta t)^2\mathbf{L}_a\overline{\mathbf{a}}\right\|_2^2 + \frac{\lambda^2}{2}\left\|\mathbf{u}\right\|_2^2 \tag{16}$$

The above minimization problem is generally known as the Tikhonov regularization. The penalty number in Eq. (16) is usually referred to as the regularization factor that adjusts the degree of the regularization in the minimization problem. As the regularization factor becomes larger, the solution bound approaches to zero, and zero displacements are reconstructed. Meanwhile a small regularization factor yields an ill-conditioned Hessian matrix for Eq. (16), which may result in a meaningless and/or unstable solution. Therefore, a well-balanced regularization factor should be selected to obtain physically meaningful and accurate displacements. The selection of the optimal regularization factor will be presented in the next section. The minimization problem in Eq. (16) forms a quadratic problem with respect to the unknown displacement vector, and thus the solution of Eq. (16) is given analytically as

$$\mathbf{u} = (\mathbf{L}^T\mathbf{L} + \lambda^2\mathbf{I})^{-1}\mathbf{L}^T\mathbf{L}_a\overline{\mathbf{a}}(\Delta t)^2 \tag{17}$$

where I is the identity matrix of order $(n+3)$. Note that the regularization function not only provides the minimization problem defined in Eq. (16) with the sufficient rank, but also suppresses noise-polluted solution components in the reconstructed displacement. The role of the regularization function for ill-posed problems is presented in detail in references.

3 TIME WINDOW TECHNIQUE AND OPTIMAL REGULARIZATION FACTOR

3.1 *Time-window technique*

The accelerations of a real structure are usually continuously monitored in real time by a time marching fashion, but the displacements have to be reconstructed within a finite time interval, which will be referred to as a time window hereafter, in the proposed method. After the reconstruction of displacement is completed for a time window, the time window advances forward by time increment Δt. The reconstruction of displacement is performed sequentially in each time window then the reconstructed displacement at the middle of each time window is taken as the solution of the current time window. The reconstructed displacements at the other time in the time window are discarded, and the time window advances to reconstruct the displacement at the next time step. In this way, the error caused by inaccurate estimation of boundary conditions is minimized in the reconstructed displacement, and the accuracy of the reconstructed displacement is maintained at the same level for all time steps.

To reduce computational effort, the time window size should be kept as small as possible. In case the time-window size is too small, however, the error in the estimated boundary conditions is not damped out sufficiently at the center of a time window, and thus inaccurate results are obtained. Based on aforementioned discussions, the optimal time-window size is defined as the smallest time interval that does not affect the accuracy of the reconstructed displacement. Intensive numerical simulation tests on various systems are performed in this study, and it is found that the time-window sizes larger than three times the longest period of a system do not improve the accuracy of solutions. The optimal time-window size is set to three times the longest period of a given system, which is obtained by the Fast Fourier Transform (FFT) of measured acceleration, throughout this study.

3.2 *Optimal regularization factor*

A robust and efficient scheme is formulated to select the optimal regularization factor for the proposed method. The compositions of the system matrices in Eq. (17) do not vary with problems, but only the orders of the system matrices, which are determined by the number of data points in a time window, depend upon specific problems. Therefore, the optimal regularization factor for Eq. (17) should be a function of the number of data points in a time window and the noise level in measurement, and is defined as the solution of the following minimization problem.

$$\lambda_{\text{opt}}^2(N, A_n) = \operatorname*{Min}_{\lambda} \frac{\left\| \mathbf{u}(\lambda) - \mathbf{u}_{\text{exact}} \right\|_2^2}{\left\| \mathbf{u}_{\text{exact}} \right\|_2^2} \tag{18}$$

where $N = n + 1$ is the number of data points in a time window, and A_n, $\mathbf{u}(\lambda)$ and $\mathbf{u}_{\text{exact}}$ are the noise level in measurement, reconstructed displacement for a regularization factor λ and the exact displacement, respectively. The number of the data points in a time window is calculated using the time-window size (three times the longest period) and the sampling rate of measurement.

To investigate fundamental characteristics of the optimal regularization factor, the solutions of Eq. (18) are calculated for the free vibrations of the SDOF systems with different frequencies and noise levels. Five SDOF systems with the natural frequencies of 1Hz, 2Hz, 4Hz, 6Hz, 8Hz and 10Hz are tested, and the sampling rate for measurement is fixed at 100 Hz. As the time-window size is fixed to three times the natural period, the numbers of data points for the five cases are 301, 151, 76, 51, 39 and 31, respectively. Proportional random noise is generated by the uniform probability density function with maximum amplitudes of 5%, 10% and 20% for each SDOF system, and added to the exact acceleration obtained by solving the exact governing equation. The initial condition for displacement and the velocity are set to 1m and 0 m/sec, respectively, for all cases. Since it is difficult to obtain the optimal solution of Eq. (18) directly, displacement is reconstructed for different regularization factors, and the regularization factor that minimizes the object function during the second period in the time window is selected as the optimal value. To simulate actual situations of non-zero initial conditions at the beginning of a time window, the measurement begins at $T/8$ in each case, where T denotes the natural period of a system. As a result of parameter study using SDOF system, the regularization factor is expressed in terms of window size.

$$\lambda_{\text{opt}} = 46.81 N^{-1.95} \tag{19}$$

4 EXPERIMENTAL VERIFICATION

4.1 *Forced vibration of a stay cable*

A forced vibration test of a stay cable is performed at Structural Laboratory of Hyundai Institute of Construction, Kyungki-do, Korea. The geometry and the boundary conditions of the cable are shown in Fig. 1.

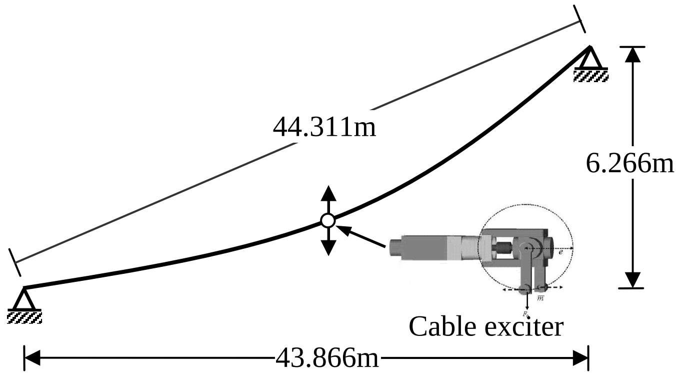

Figure 1. Setup of Forced vibration experiment of a stay cable.

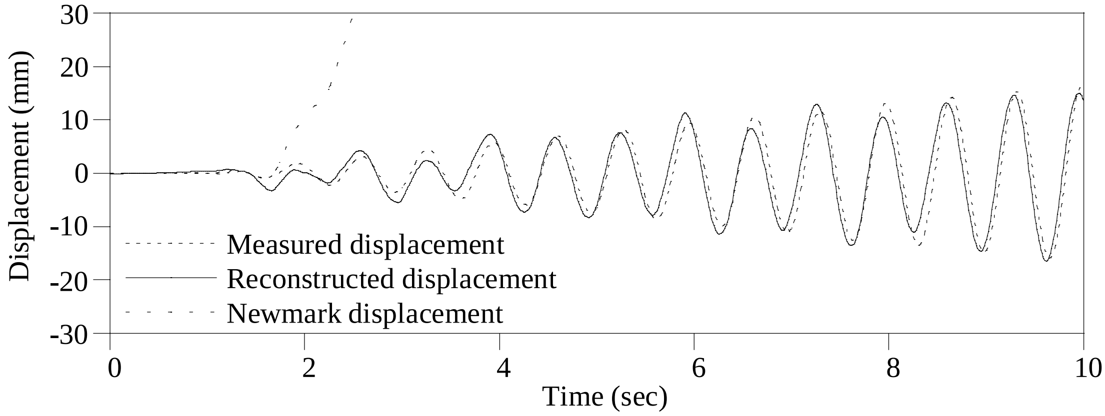

Figure 2. Setup of Forced vibration experiment of a stay cable.

The tension of approximately 300 kN is applied to the cable, and the fundamental period is calculated as 1.5Hz. The forced vibration of the cable is introduced with the cable exciter developed by Hyundai Institute of Construction at the center of the cable. The exciter generates vertical exciting forces by two rotating masses in the opposite direction. The total mass of the exciter and the rotating mass are 14.58 kg and 0.46 kg, respectively. The cable is excited by its fundamental frequency, i.e. 1.5Hz to induce the resonance of the cable for 40 sec. An accelerometer is installed at the center of the stay cable and the vertical acceleration is measured at the sampling rate of 100 Hz. A linear variable differential transformer (LVDT) is installed at 20cm away from the accelerometer to avoid interference between the exciter and the LVDT. The LVDT measured vertical displacement at the same sampling rate as the accelerometer. The FFT of the measured acceleration yields the dominant frequency of 1.48 Hz, which is slightly smaller than the excitation frequency. The window size is set to 2.04 sec and the optimal regularization factor is selected as 1.45×10^{-3}.

The reconstructed results are shown in Fig. 2 for the period around the beginning of the excitation. In the figure, the reconstructed displacement agrees with the measured displacement from the LVDT well except for a small, constant phase difference. It is believed that the phase difference is caused by the difference in positions between the accelerometer and the LVDT. In Fig. 2, the displacement reconstructed by the Newmark-β method is drawn together with the others. The Newmark-β method yields diverging displacement after 2 sec even though the exact initial conditions are specified.

5 SUMMARY AND CONCLUSION

This paper presents a new class of the displacement reconstruction scheme with the measured acceleration. The proposed method is formulated as a boundary value problem rather than an initial value problem. That is, the displacement is reconstructed in a finite time interval with two boundaries called as a time window through the minimization of the least squared errors between measured accelerations and calculated accelerations by displacement. The acceleration is approximated by the second-order central finite difference of displacement. To improve the accuracy of the reconstructed displacement, the overlapping time-window technique is adopted. As the boundary conditions at both ends of a time window are not known *a priori*, the minimization problem for the reconstruction of displacement becomes rank-deficient by 2. To overcome the rank-deficiency, the Tikhonov regularization scheme is employed. The regularization function is defined as the 2-norm of the discretized displacements to be reconstructed in a time window. An equation to determine the optimal regularization is proposed.

The validity of the proposed method is demonstrated through laboratory experiment of the stay cable. The reconstructed displacements with the measured acceleration agree well with the measured displacements in an overall sense. Even though small errors at the peaks are observed in the results, they neither propagate nor are amplified, and are in an acceptable range from the viewpoint of engineering. Moreover, the proposed method is numerically stable and efficient, and does not require any initial or boundary conditions at all.

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