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Development of Geometry Control System for Cable-Stayed Bridges and Application to the Incheon Bridge

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ABSTRACT: A geometry control system for cable-stayed bridge, which can effectively control the cable tension and geometry during construction, is developed. The geometry control system is consisted of two methods that are cable length adjustment and system identification. The cable adjustment system can bring the geometry and cable tension to remain within the allowable limits by adjusting the cable lengths for the error occurred during production, installation and tensioning process. And the system identification minimizes the differences between theoretically estimated values and real ones. The system parameter of system identification is chosen as the unstrained length of cable members. The sensitivity of displacement with respect to the unstrained length of cables is derived to solve the above problem. And the proposed method will be applied to the Incheon Bridge to verify the validity and effectiveness.

1 INTRODUCTION

Cable-stayed bridges have been recognized as one of the appealing structural type for large-scale structures due to the various structural advantages of cables as well as their aesthetic appearances. With the recent advances of structural analysis techniques and construction technologies in Korea, Incheon Bridge with a main span length reaching 800m emerges into reality. As cable-stayed bridges become larger and longer, more accurate and precise analysis techniques are required to control the cable tension and geometry during construction.

Structural system changes according to the progress of construction. Then errors in geometry and cable force may be accumulated and amplified through complicated construction steps. Although the design is accurate and correct construction is done, a certain amount of errors in cable tension and geometry is inevitable. Since the load intensity, modulus of elasticity and erection themselves have errors to some extent. Thus, the feedback process of measurement and control system is required for more accurate construction. In this study, the feedback process of measurement and control system is referred as geometry control system.

The purpose of the geometry control system is eliminating deviation between the object and actual structure. By applying the geometry control system to the cable-stayed bridge during the construction and completion, the structure satisfies the target configuration. To control the configuration and cable tensions under the construction of Incheon Bridge, two different methods will be applied.

The first one is cable length adjustment. Since the cable-stayed bridges resist the external force almost by the cable member force, adjusting the length of cables is one of the simplest ways to control the configuration of the bridge. The amount of cable length adjustment is calculated using an optimization method, which minimizes the errors between the measured configuration and target configuration. The adjustment for minimizing configuration errors may yield meaningless solutions in optimization process due to the instability, which is triggered the measured configuration polluted by noise. The regularization technique is considered to overcome the instability.

And the other one is system identification. In case the only cable adjustment system can not control the cable tension and geometry within the allowable limits due to the inaccuracy of the assumed analysis model of the bridge, the system identification scheme is utilized to update the properties of analysis model. The system parameters such as section moduli of cables, self-weight of girders are estimated by solving minimization problem. The error function is defined as the least square errors between calculated configuration by assumed analysis model and measured ones. A regularization scheme is adopted to alleviate the instability of minimization problem.

The sensitivity analysis for cable length adjustment and system identification is evaluated by the direct differentiation of the equilibrium equation of nonlinear structural system. The sensitivity of displacements with respect to the unstrained cable length is derived in this paper.

2 GEOMTRY CONTROL SYSTEM

2.1 *Cable length adjustment*

The cable length adjustment represents the method which eliminates deviation between the object and actual structure by changing the length of cables. After adjusting the cable length, the residual displacement error at the *i*-th node can be expressed like this:

$$e_i(\mathbf{L}_0 + \Delta\mathbf{L}_0) = x_i^c - x_i^A(\mathbf{L}_0 + \Delta\mathbf{L}_0) \tag{1}$$

where e_i , x_i^c and x_i^A represent the residual configuration error, the target configuration and the configuration of actual structure generated after adjusting the length of cables, respectively, and $\mathbf{L}_0$ and $\Delta\mathbf{L}_0$ denote the design values of unstrained cable length and the adjusting amount of unstrained cable length vector, respectively. The amount of unstrained cable length is the thickness of the shim plate to be added or removed at the end of the cables. In equation (1), the superscript *A* represents the unknown model that can describe the behavior of the actual structure.

The amount of the adjusting cable length that can minimizes the residual configuration errors overall is calculated by solving the following optimization problem.

$$\underset{\Delta\mathbf{L}_0}{\text{Min}}\,\Pi = \frac{1}{2}\sum_{i=1}^{nm}(x_i^c - x_i^A(\mathbf{L}_0 + \Delta\mathbf{L}_0))^2 \text{ subject to } (\Delta\mathbf{L}_0)_{\min} \le \Delta\mathbf{L}_0 \le (\Delta\mathbf{L}_0)_{\max} \tag{2}$$

where *nm*, $(\Delta\mathbf{L}_0)_{\min}$ and $(\Delta\mathbf{L}_0)_{\max}$ denote the number of measured points, lower bound and upper bound of the shim plate thickness, respectively.

The minimization problem given in equation (2) may yield meaningless solutions in optimization process due to the instability, which is triggered the measured configuration polluted by noise. In other words, the solution that minimize the residual configuration errors can give the tension of cables out of the target range. The regularization technique is considered to overcome the instability. In the regularization technique, adding a positive definite regularization function modifies the original objective function. In the next section, the regularization scheme will be introduced in detail. A modified minimization problem with the regularization function is defined as follows:

$$\begin{gathered}\underset{\Delta\mathbf{L}_0}{\text{Min}}\,\Pi = \frac{1}{2}\sum_{i=1}^{nm}(x_i^c - x_i^A(\mathbf{L}_0 + \Delta\mathbf{L}_0))^2 + \frac{\lambda^2}{2}\left\|\Delta\mathbf{L}_0\right\|_2^2 \\ \text{subject to } (\Delta\mathbf{L}_0)_{\min} \le \Delta\mathbf{L}_0 \le (\Delta\mathbf{L}_0)_{\max}\end{gathered} \tag{3}$$

where λ is the regularization factor and $\|\cdot\|_2$ .represents the 2-norm of a vector. The regularization effect is controlled by the magnitude of the regularization factor. Among the solutions obtained from the some regularization factors, the engineer’s decision is required to choose the best solution.

Since the optimization problem is the non-linear problem, the quadratic sub-problem of (3) is defined as

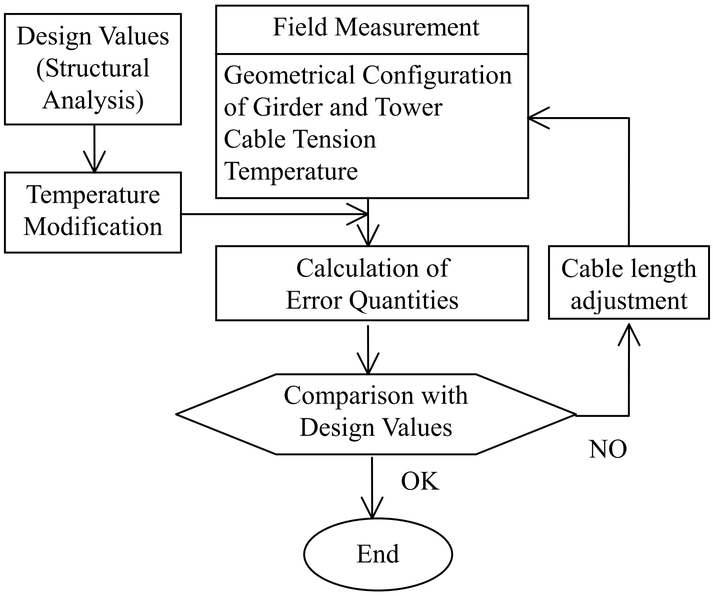

Figure 1. The procedure of the cable length adjustement

$$\underset{\Delta\mathbf{L}_0}{\mathrm{Min}}\,\Pi = [\frac{1}{2}\Delta\mathbf{L}_0^T(\mathbf{H}+\lambda\mathbf{I})\Delta\mathbf{L}_0 + \mathbf{G}\Delta\mathbf{L}_0] \;\text{ subject to}\; (\Delta\mathbf{L}_0)_{\min} \le \Delta\mathbf{L}_0 \le (\Delta\mathbf{L}_0)_{\max}$$
$$\mathbf{H} = \frac{\partial^2\Pi}{\partial\mathbf{L}_0\partial\mathbf{L}_0} = (\frac{\partial x_i^A}{\partial\mathbf{L}_0})(\frac{\partial x_i^A}{\partial\mathbf{L}_0}),\quad \mathbf{G} = \frac{\partial\Pi}{\partial\mathbf{L}_0} = (x_i^A(\mathbf{L}_0) - x_i^c)\frac{\partial x_i^A}{\partial\mathbf{L}_0} \tag{4}$$

where H and G are the Gauss-Newton Hessian matrix and the gradient vector of the error function, respectively.

The Gauss-Newton Hessian matrix in equation (4) consists of the sensitivity of displacement with respect to the current unstrained cable length. The sensitivity of displacement has to be obtained from the model that can express the actual response of the bridge, but the exact model of the bridge is unknown model. So, this study assumes that the displacement sensitivity of the actual model is almost same with the sensitivity of the analytical model. And the gradient vector in equation (4) requires the initial configuration before adjusting cable length. It can be obtained by measuring the actual behavior of the structure.

$$\frac{\partial x_i^A}{\partial\mathbf{L}_0} \approx \frac{\partial x_i}{\partial\mathbf{L}_0}, \qquad x_i^A(\mathbf{L}_0) = x_i^m \tag{5}$$

where x_i^m represents the measured configuration of the structure.

Because of the assumption in equation (5), the actual displacements after adjusting the cable length can have the difference with the ones that becomes to be. In that case, additional cable length adjustment is required. Figure 1 shows the series of the process to adjust the cable length.

2.2 *System identification*

In cable length adjustment, there is an assumption that the behavior of a bridge is almost same with the behavior of the analytical model. That assumption also contains the errors and the errors are accumulated and amplified through complicated construction steps. In case only the cable length adjustment can not control the configuration of a cable-stayed bridge because of the assumption in equation (5), error factors can be identified and quantified by the system identification method. This permits the prediction of the final construction state for the bridge and thereby makes cable tension adjustment precise, reducing camber error or member force error.

The unknown system parameters of the system identification are identified through the following minimization of the least-squared error between calculated and measured configuration at observation points.

$$\underset{\mathbf{X}}{\mathrm{Min}}\Pi = \frac{1}{2}\sum_{i=1}^{nm}(x_i(\mathbf{X}) - x_i^m)^2 \quad \text{subject to } (\mathbf{X})_{\min} \le \mathbf{X} \le (\mathbf{X})_{\max} \tag{6}$$

where x_i, $\mathbf{X}$, $(\mathbf{X})_{\min}$ and $(\mathbf{X})_{\max}$ are the configuration which is calculated by the analytical model, system parameter vector, the lower bound of the system parameters and the upper bound of the system parameters, respectively. Although the equation (6) seems to be similar with equation (2), there is definite difference. In equation (2), i.e. the cable length adjustment, the unknown variable is the cable length adjustment of actual cable-stayed bridge, not the cable length adjustment of analysis model. But in equation (6), i.e. the system identification, the unknown variable is the system parameter (e.g. the cable length) of analysis model. The system parameter will be modified to describe the actual behavior of the bridge by the system identification.

The parameter estimation method defined by a minimization problem as (6) is a type of ill-posed inverse problem, which suffers from instabilities such as non-existence, non-uniqueness and discontinuity of solutions. The instabilities are triggered when measured data are incomplete and polluted by noise. Because of the instabilities, the minimization problem given in equation (6) may yield meaningless solutions or diverge in optimization process. The regularization technique is considered to be a rigorous way to overcome the ill-posedness of inverse problems. In the regularization technique, the original objective function is modified by adding a positive definite regularization function. For successful system identification, a proper regularization function, which clearly defines characteristics of problems, should be selected.

The regularization can be interpreted as a process of mixing the a priori estimates of system parameters and the a posteriori solution. The baseline properties are selected as the a priori estimates of the system parameters in this study. The a priori estimates are taken into account in the problem statement of inverse problems by adding a regularization function with the a priori estimates of the system parameters to the error function. The regularity condition of the solution space can be weakly imposed by adding the following regularization function to the output error estimator of equation (6).

$$\underset{\mathbf{X}}{\mathrm{Min}}\Pi = \frac{1}{2}\left\|\mathbf{X} - \mathbf{X}_0\right\|_2^2 \quad \text{subject to } (\mathbf{X})_{\min} \le \mathbf{X} \le (\mathbf{X})_{\max} \tag{7}$$

where $\mathbf{X}_0$ denote the a priori estimates of system parameters. Equation (7) is referred as the standard Tikhonov regularization function.

By adding the regularization function normalized by the a priori estimates to the minimization problem of equation (6), a regularized system identification problem is written in the following form.

$$\underset{\mathbf{X}}{\mathrm{Min}}\Pi = \frac{1}{2}\sum_{i=1}^{nm}(x_i(\mathbf{X}) - x_i^m)^2 + \frac{\lambda^2}{2}\left\|\mathbf{X} - \mathbf{X}_0\right\|_2^2 \quad \text{subject to } (\mathbf{X})_{\min} \le \mathbf{X} \le (\mathbf{X})_{\max} \tag{8}$$

where λ is the regularization factor. The regularization effect diminishes for a small regularization factor while the regularization function dominates for a large regularization factor. In either case, the minimization problem (8) is unable to find meaningful system parameters due to instabilities or dominant regularization effects on the system parameters. Therefore, the selection of a proper regularization factor is very critical for the stability and accuracy of the solution of equation (8). The optimal regularization factor can be determined by the geometric mean scheme (GMS) proposed by Park et al. 2001. In the GMS, the optimal regularization factor is defined as the geometric mean between the largest and the smallest non-zero singular value of the sensitivity matrix.

It is important to decide which one will be the system parameter that is unknown variables in system identification problem of the cable-stayed bridge. The system parameter of a cable–stayed bridge can be the elastic modulus of tower and girder or the tension of stayed cable. By the sensitivity analysis of the displacement at the girder, it is identified that the elastic modulus of tower and girder has little effects on the behavior of the bridge. Since the cable structures like cable-stayed bridges resist the external force almost by the cable member force, the tension of cable is a dominant variable to define the configuration of a cable structure. And the tension of cables is affected by the unstrained length and elastic modulus of the cable. Because it is estimated that the unstrained length of cable has a little fabrication error, this study defines the system parameter as the elastic modulus of the cable in the system identification problem.

Now the system identification to estimate the elastic modulus of the cable is defined as following minimization problem.

$$\underset{\mathbf{L}}{\mathrm{Min}}\Pi = \frac{1}{2}\frac{1}{\left\|x_i^m\right\|_2^2}\sum_{i=1}^{nm}(x_i - x_i^m)^2 + \frac{\lambda^2}{2}\left\|\frac{\mathbf{L}}{\mathbf{L}_0} - \mathbf{1}\right\|_2^2 \qquad (9)$$
$$\text{subject to } (\mathbf{L})_{\min} \le \mathbf{L} \le (\mathbf{L})_{\max}$$

where $\mathbf{L}$, $\mathbf{L}_0$ and $\mathbf{1}$ represents the cable length to estimate, the design cable length and a column vector which has unit values in all the components, respectively, and $(\mathbf{L})_{\min}$ and $(\mathbf{L})_{\max}$ denote the constraints of the minimization problem in equation (9). $\| x_i^m \|_2^2$ is normalizing factor to make error function have non-dimensional value. Although the system parameter to estimate is the elastic modulus of the cable, the minimization problem in equation (9) is evaluated to estimate the length of cable members. This has an advantage to use the same sensitivity analysis results which is used to solve the problem of cable length adjustment. And the a priori estimates of the cable length are imposed as the design cable length.

The equation (9) is written in the following form by using the normalized quantity ξ.

$$\underset{\xi}{\mathrm{Min}}\Pi = \frac{1}{2}\frac{1}{\left\|x_i^m\right\|^2}\sum_{i=1}^{nm}(x_i - x_i^m)^2 + \frac{\lambda^2}{2}\left\|\xi - \mathbf{1}\right\|_2^2 \quad \text{subject to } (\xi)_{\min} \le \xi \le (\xi)_{\max} \qquad (10)$$

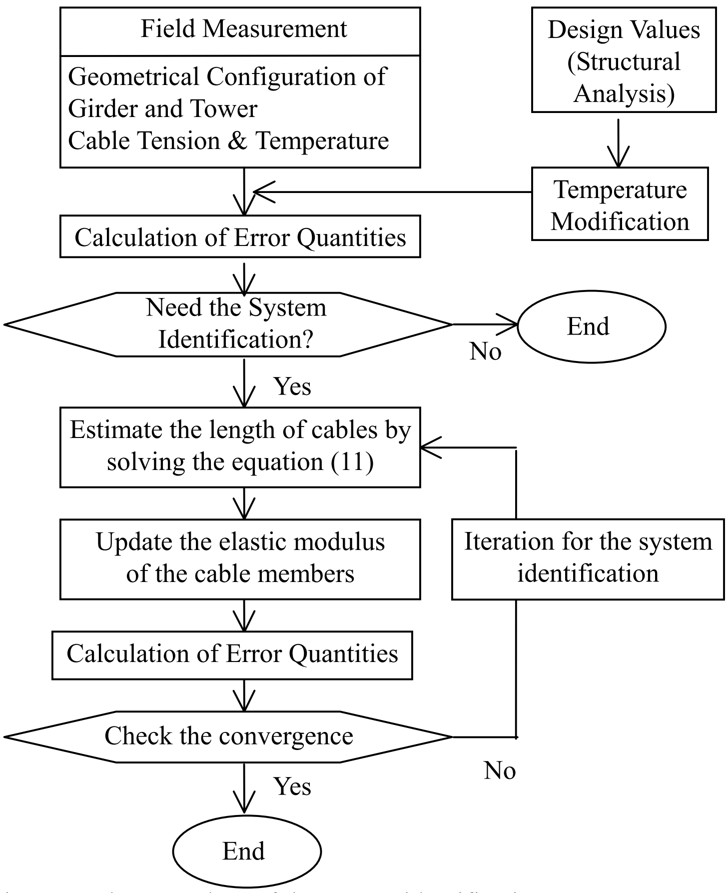

Figure 2. The procedure of the system identification

where ξ represents the $\mathbf{L}/\mathbf{L}_0$. And equation (10) can be expressed in matrix form as follows

$$\underset{\xi}{\mathrm{Min}}\Pi = \frac{1}{2}\frac{1}{\|\mathbf{x}^m\|^2}(\mathbf{x} - \mathbf{x}^m)^T(\mathbf{x} - \mathbf{x}^m) + \frac{\lambda^2}{2}(\xi - \mathbf{1})^T(\xi - \mathbf{1}) \tag{11}$$

subject to $(\xi)_{\min} \le \xi \le (\xi)_{\max}$

After calculating the length of cable by solving the optimization problem in equation (11), the elastic modulus of cable member is obtained from solving the equilibrium equation of cable element with calculated cable length. With updated elastic modulus of cable members, solve the equilibrium equation of the cable-stayed bridge and the optimization problem in equation (11) again until the convergence criteria enters the tolerance (see Fig. 2.).

The proposed system identification of cable-stayed bridge brings the modified elastic modulus of cable. After calculating the cable length to satisfy the target configuration with the modified properties of the cable, the difference with designed length of cable will be the amount of adjusting cable length.

2.3 *Sensitivity analysis*

To solve the above optimization problem, the sensitivity analysis has to be evaluated. The sensitivity of displacement with respect to the unstrained length of cable is evaluated by direct differentiation of equilibrium equation. The direct differentiation method will give the accurate sensitivity to help get the exact solution and converge rapidly in the optimization problem.

The equilibrium equation of steel cable-stayed bridge can be expressed by

$$\mathbf{K}_F\mathbf{u} + \mathbf{F}_c\left(\mathbf{L}^0, \mathbf{u}(\mathbf{L}^0)\right) = \mathbf{P} \tag{12}$$

where $\mathbf{K}_F$, $\mathbf{u}$, $\mathbf{F}_c$ and $\mathbf{P}$ are stiffness matrix of a frame structure, displacement vector, cable member force vector and the load vector, respectively. The tension of cable members is defined as an unstrained length of cable element and the displacement which is generated by the unstrained cable length. In case of the steel cable-stayed bridge, the stiffness matrix of the frame structure is not affected by the unstrained length of cables.

The sensitivity of the displacement is obtained by the direct differentiation of Eq. (12) with respect to the unstrained length of the cable.

$$\mathbf{K}_F\frac{\partial \mathbf{u}}{\partial \mathbf{L}^0} + \frac{\partial \mathbf{F}_c}{\partial \mathbf{L}^0} + \frac{\partial \mathbf{F}_c}{\partial \mathbf{u}}\frac{\partial \mathbf{u}}{\partial \mathbf{L}^0} = \frac{\partial \mathbf{P}}{\partial \mathbf{L}^0} = 0 \tag{13}$$

The sensitivity of the external load with respect to the unstrained cable length is zero because the external load is not a function of cable length. From the Eq.(13), the sensitivity of the displacement is expressed as follows:

$$\frac{\partial \mathbf{u}}{\partial \mathbf{L}^0} = -\left(\mathbf{K}_F + \frac{\partial \mathbf{F}_c}{\partial \mathbf{u}}\right)^{-1}\frac{\partial \mathbf{F}_c}{\partial \mathbf{L}^0} = -\left(\mathbf{K}_F + \mathbf{K}_c\right)^{-1}\frac{\partial \mathbf{F}_c}{\partial \mathbf{L}^0} \tag{14}$$

where $\mathbf{K}_c$ is stiffness matrix of a cable structure.

3 APPLICATION

The proposed method is applied to the construction of the cable-stayed bridge part of the Incheon Bridge do demonstrate its validity and effectiveness.

The Incheon Bridge, which will link the recently completed Incheon International Airport (based on Yongjong island) and the international business district of New Songdo City (second bridge crossing), started construction in June 2005. When the bridge is completed in 2009 (52 months) it will be longer than the current Seohae Bridge (the first bridge crossing) and will be among the five longest bridges of its kind in the world.

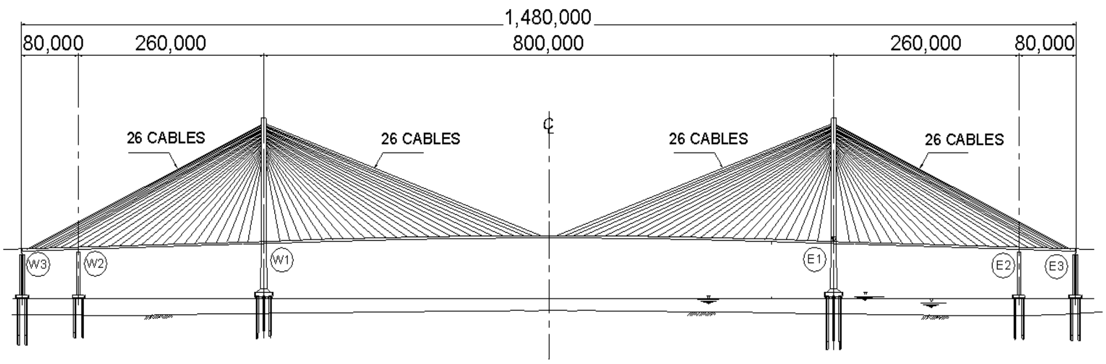

Figure 3. General view of the Incheon Bridge

The bridge will be a 12.3km (7.4-mile) toll bridge that will link the Seoul–Incheon expressway with the Seohaean expressway, to shorten the journey time from Incheon airport to the metropolitan districts of Seoul by 40 minutes. The bridge is expected to cost over $1.4 billion but is expected to stimulate economic development by improving logistics for Northeast Asia.

The most difficult part of the construction will be the cable-stayed bridge portion lying over the main sea route in and out of Incheon Port. The main span of the bridge will measure 800m and the height of the main tower will stand at 230.5m. The main bridge will have a vertical clearance of 74m (242ft) and five span lengths of 80m, 260m, 800m, 260m and 80m like Figure 3.

Under the construction of the cable-stayed bridge, the geometry of the bridge is controlled by applying the cable length adjustment. Every construction step will check the geometry of the bridge. At each side spans, there are 9 points to measure the configuration and there are 7 points to measure the configuration of tower. The tension of the 8 cables which is included the 4 cables which is constructed in this step and the 4 cables which is constructed lately before step will be measured. In case the error of geometry exceeds the tolerance, the 4 cables which is constructed in this step will be adjusted as much as the calculated amount by the equation (4).

In case the errors of the assumption in equation (5) are accumulated and amplified through the construction steps, the system identification will apply to update the analytical model. Accumulated information over the prior construction steps will be used for system identification.

The results applied the construction of the cable-stayed bridge part in Incheon Bridge will be introduced the presentation in the IABMAS'08.

4 CONCLUSION

An integrated system for the construction error control of cable-stayed bridges is developed in this study. The geometry control system is consisted of two methods which are cable length adjustment and system identification. Just by adjusting the length of cables, the configuration of the cable-stayed bridge can satisfy the target configuration easily. The amount of cable length adjustment is calculated by optimization method to minimize the errors between the designed and measured configuration overall. And system identification modifies the system parameter of the analysis model. Then, the updated analysis model can describe the behavior of the actual cable-stayed bridge.

The proposed method will be applied to the construction of the Incheon Bridge. The construction of the cable-stayed bridge part in the Incheon Bridge is in progress. Erecting girders and tensioning cables will starts at March 2008 and the proposed method will be applied under the construction.

The proposed method is formulated as optimization problems. And to solve the optimization problems, sensitivity analysis is evaluated. Therefore the proposed can be applied to the different type of bridges but also steel cable-stayed bridges once the sensitivity analysis of the bridge is evaluated.

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