Bridge Maintenance, Safety, Management, Health Monitoring and Informatics – Koh & Frangopol (eds)

Estimation of damping characteristics for cable using system identification scheme

S.-K. Park, K.W. Lyu & H. S. Lee
Seoul National University, Seoul, Korea

ABSTRACT: This paper proposes a method to estimate the damping ratio of the cable, based on a system identification scheme in time domain using measured acceleration data. The displacement is reconstructed by using measured acceleration data. The error function is defined as the time integral of the least square errors between the reconstructed displacement and the calculated displacement by a mathematical model. The Rayleigh damping model is adopted to estimate the damping parameters. Rayleigh damping yields a linear fit to the actual damping of a structure. A regularization technique is used to alleviate the instabilities of SI. The validity of the proposed method is demonstrated by an experimental study.

1 INTRODUCTION

Cable-supported structures are appropriate structural type for large-scale structures such as long-span suspension bridges and cable-stayed bridges. As cable-supported structures become larger and longer, dynamic problems caused by the rain-wind induced vibration and vortex vibration can be an important issue to design the bridges. This is because the inherent cable damping ratio is very low. Since the damping characteristics of the cable are different from other members, the accurate estimation of the cable damping ratio is important in designing damper and dynamic analyzing of cable-structures.

Logarithm decrement method and energy based estimation method are only applied in free vibration test. However, the damping ratio may be changed with respect to applied loading condition. Therefore, the damping ratio under forced vibration should be estimated accurate as well as the damping ratio under free vibration for designing cable structure.

This paper describes a study to estimate the damping ratio of the stay-cable, based on a system identification scheme in time domain using reconstructed displacement. Since the accelerations have instabilities with loading condition and are very sensitive compared with the displacements, the direct use of measured acceleration data to SI problem is difficult. However, the displacements of the cable are difficult to measure caused by support problem to fix the measuring device. The displacements are reconstructed using measured acceleration data through displacement reconstruction technique proposed by Hong et al. Displacement is calculated by linearization of equation of motion and discretization by finite element method. Linearized incremental equation of motion is integrated by Newmark-β method. Displacement is defined as the relative position between the current position and the position when the cable is loaded by its own self-weight. The validity of the proposed method are demonstrated through a laboratory test of the stay-cable. The damping ratios of force vibration and that of free vibration are estimated.

2 DYNAMIC ANALYSIS OF CABLE

2.1 *Dynamic equation of cable and variational statement*

Figure 1 illustrates two-dimensional cable element with unstrained length l_0. The Lagrangian coordinates *s* of undeformed shape moves to the Lagrangian coordinates *p*(*s*) of deformed shape when the cable element undergoes deformation due to the self weight *ws*. A particle located in Lagrangian coordinate *s* at undeformed state is located in Cartesian coordinate $\mathbf{x}^e$ after deformation. *p*(*s*) is defined as a cable length from origin to Lagrangian coordinate *s* after deformation.

$$p(s) = \int_0^s ((\frac{dx}{ds})^2 + (\frac{dy}{ds})^2)^{0.5} ds \tag{1}$$

The relation of *p* and *s* can be obtained by differentiation of Eq. (1).

$$\frac{dp}{ds} = ((\frac{dx}{ds})^2 + (\frac{dy}{ds})^2)^{0.5} \tag{2}$$

Figure 2 shows the free body diagram of cable segment. Equilibrium equation is expressed by cable tension as follows.

$$T\frac{dx}{dp} + F_x^1 - \int_0^s \rho\ddot{x}ds + \int_0^s f_x ds = 0, \quad T\frac{dy}{dp} + F_y^1 + qs - \int_0^s \rho\ddot{y}ds + \int_0^s f_y ds = 0 \tag{3}$$

where, *T* is a tension at *p*(*s*) and F_x^1, F_y^1 represent *x*- and *y*- component of $\mathbf{F}_1^e$, respectively. f_x, f_y and *q* represent external loads and self-weight of the cable per unit unstrained length. In case the cable strain is relatively small, the cable strain is defined as follows.

$$\varepsilon = \frac{dp^2 - ds^2}{2ds^2} \cong \frac{dp - ds}{ds} = \frac{dp}{ds} - 1 \tag{4}$$

By Hook's law, cable tension is expressed as follows.

$$T(s) = EA\varepsilon = EA(\frac{dp}{ds} - 1) = EA[(x'^2(s) + y'^2(s))^{0.5} - 1] \tag{5}$$

where *E* and *A* represent Young's Modulus and cross-sectional area of cable. $(\,)'$ represents a differentiation with respect to Lagrangian coordinate *s*. Substitution of Eq. (5) into the differentiate of Eq. (3) yields the following equilibrium equation expressed by tension and unstrained length.

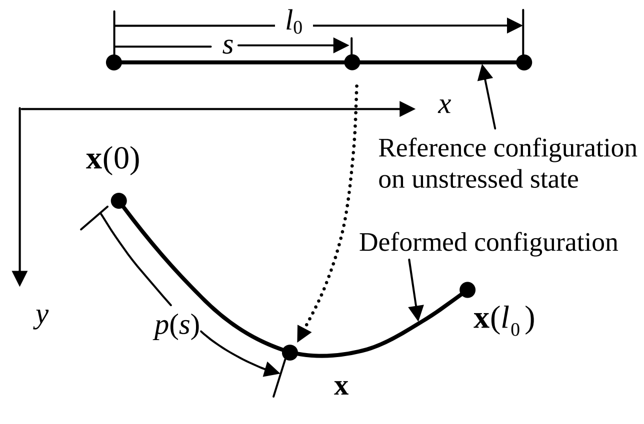

Figure 1. Coordinates of elastic catenary cable element.

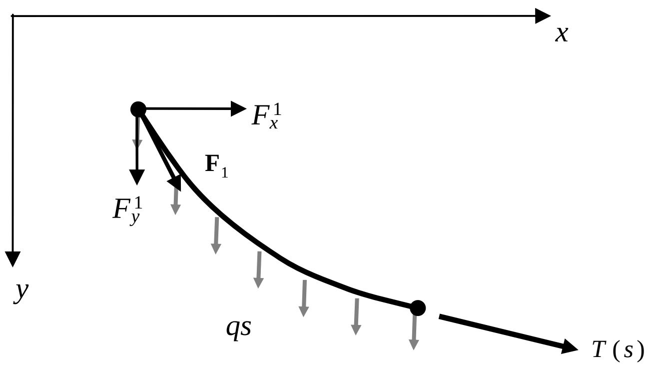

Figure 2. Free body diagram of cable segment.

$$\frac{d}{ds}\left(\frac{T}{1+T/EA}\frac{dx}{ds}\right)-\rho\ddot{x}+f_x=0\,,\quad \frac{d}{ds}\left(\frac{T}{1+T/EA}\frac{dy}{ds}\right)+(-\rho\ddot{y}+q)+f_y=0 \tag{6}$$

Using the variational approach, Eq. (6) can be expressed by weak form as follows.

$$\int_{l_0} dx\left(\frac{d}{ds}\left(\frac{T}{1+T/EA}\frac{dx}{ds}\right)-\rho\ddot{x}+f_x\right)ds=0\,,\quad \int_{l_0} dy\left(\frac{d}{ds}\left(\frac{T}{1+T/EA}\frac{dy}{ds}\right)+(-\rho\ddot{y}+q)+f_y\right)ds=0 \tag{7}$$

Integration by parts of the first term of Eq. (8) and boundary condition yields the following final variational statement of equation of motion as follows.

$$\begin{aligned}&\int_{l_0}\rho(\delta x)\ddot{x}ds+\int_{l_0}(\delta x')\frac{T}{1+T/EA}x'ds=\int_{l_0}(\delta x)f_x ds\\&\int_{l_0}\rho(\delta y)\ddot{y}ds+\int_{l_0}(\delta y')\frac{T}{1+T/EA}y'ds=\int_{l_0}(\delta y)(q+f_y)ds\end{aligned} \tag{8}$$

2.2 *Linearization and discretization of dynamic equation*

Dynamic responses of cable such as acceleration and coordinate are calculated by solving non-linear equation of motion Eq. (8) and using given initial condition. At time $t+\Delta t$, Eq. (8) is expressed as follows.

$$\begin{aligned}&\int_{l_0}\rho(\delta x)\ddot{x}^{t+\Delta t}ds+\int_{l_0}(\delta x')\frac{T^{t+\Delta t}}{1+T^{t+\Delta t}/EA}(x^{t+\Delta t})'ds=\int_{l_0}(\delta x)(f_x)^{t+\Delta t}ds\\&\int_{l_0}\rho(\delta y)\ddot{y}^{t+\Delta t}ds+\int_{l_0}(\delta y')\frac{T^{t+\Delta t}}{1+T^{t+\Delta t}/EA}(y^{t+\Delta t})'ds=\int_{l_0}(\delta y)(q+(f_y)^{t+\Delta t})ds\end{aligned} \tag{9}$$

Assuming all parameters of time t are already known. To derive linearized incremental form of equation of motion, coordinate and tension are expressed by incremental form.

$$\begin{aligned}&x_k^{t+\Delta t}=x_{k-1}^{t+\Delta t}+\Delta x=\underline{x}+\Delta x\,,\quad y_k^{t+\Delta t}=y_{k-1}^{t+\Delta t}+\Delta y=\underline{y}+\Delta y\\&T_k^{t+\Delta t}=T_{k-1}^{t+\Delta t}+\Delta T=\underline{T}+\Delta T\end{aligned} \tag{10}$$

The term related to tension of Eq. (9) can be linearized by Taylor expansion.

$$\frac{T^{t+\Delta t}}{1+T^{t+\Delta t}/EA}\approx\frac{\underline{T}}{1+\underline{T}/EA}+\frac{\partial}{\partial T}\frac{T}{1+T/EA}\bigg|_{T=\underline{T}}\Delta T=\frac{\underline{T}}{1+\underline{T}/EA}+\frac{1}{(1+T/EA)^2}\Delta T \tag{11}$$

Substitution Eq. (10) and Eq. (11) into Eq. (9) yield the linearized incremental equation of motion.

$$\int_{l_0} \rho(\delta\mathbf{x})^T \mathbf{I}\Delta\ddot{\mathbf{x}} ds + \int_{l_0} \frac{d(\delta\mathbf{x})^T}{ds} \mathbf{D}_c \frac{d\Delta\mathbf{x}}{ds} ds = \int_{l_0} (\delta\mathbf{x})^T (\mathbf{f}^{t+\Delta t} - \rho\underline{\ddot{\mathbf{x}}}) ds - \int_{l_0} (\delta\mathbf{x}')^T \frac{\underline{T}}{1+\underline{T}/EA} \underline{\mathbf{x}}' ds \tag{12}$$

where, $\mathbf{I}$ is a identity matrix, $\mathbf{x} = (x, y)$ and $\mathbf{D}_c$ represents tangent stiffness matrix.

$$\mathbf{D}_c = \frac{\underline{T}}{1+\underline{T}/EA} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{EA}{(1+\underline{T}/EA)^3} \begin{bmatrix} \frac{d\underline{x}}{ds}\frac{d\underline{x}}{ds} & \frac{d\underline{x}}{ds}\frac{d\underline{y}}{ds} \\ \frac{d\underline{x}}{ds}\frac{d\underline{y}}{ds} & \frac{d\underline{y}}{ds}\frac{d\underline{y}}{ds} \end{bmatrix} \tag{13}$$

Coordinates are interpolated by finite element method as follows.

$$\Delta\mathbf{x}^e \cong \mathbf{N}^e \Delta\mathbf{X}^e, \quad \delta\mathbf{x}^e \cong \mathbf{N}^e \delta\mathbf{X}^e \tag{14}$$

Final incremental equation is expressed by substitution Eq. (14) into Eq. (12) and assemble of all elements.

$$\mathbf{M}_c \Delta\ddot{\mathbf{X}} + \mathbf{K}_c \Delta\mathbf{X} = \Delta\underline{\mathbf{f}} \tag{15}$$

where, $\Delta\mathbf{X}$ and $\Delta\ddot{\mathbf{X}}$ represents a nodal coordinate and a nodal acceleration, respectively. $\mathbf{M}_c, \mathbf{K}_c$ and $\Delta\underline{\mathbf{f}}$ is mass matrix, tangent stiffness matrix and unbalanced force of cable as follows.

$$\begin{gathered} \mathbf{M}_c = \sum_e \int_{l_0^e} \rho(\mathbf{N}^e)^T \mathbf{N}^e ds, \quad \mathbf{K}_c = \sum_e \int_{l_0^e} (\frac{d\mathbf{N}^e}{ds})^T \mathbf{D}_c \frac{d\mathbf{N}^e}{ds} ds \\ \Delta\underline{\mathbf{f}} = \sum_e \int_{l_0^e} ((\mathbf{N}^e)^T \mathbf{f}^{t+\Delta t} - \rho(\mathbf{N}^e)^T \underline{\ddot{\mathbf{x}}} - (\frac{d\mathbf{N}^e}{ds})^T \frac{\underline{T}}{1+\underline{T}/EA} \underline{\mathbf{x}}') ds \end{gathered} \tag{16}$$

The equation of motion Eq. (15) is integrated with the Newmark-β method. The integration constants of the Newmark-β method, β=1/2, γ=1/4, are used.

3 SYSTEM IDENTIFICATION

3.1 *Displacement Reconstruction scheme*

In this research, reconstructed displacement is utilized to system identification scheme instead of measured acceleration. In case the cable is excited by sudden release of concentrated load, measured accelerations include many high frequency components caused by bending motion. Because the applied cable model in chapter 2 can not be considered the bending motion of cable, the accelerations calculated by mathematical model may not be accurate. In contrast, The measured acceleration only includes lower modes, the applied cable model can be proper. Since the displacements caused by bending motion are relatively small compared with accelerations, reconstructed displacement is used instead of measured acceleration in this study. Reconstructed displacement can be obtained by solving minimization problem as follows.

$$\underset{\bar{\mathbf{u}}}{\mathrm{Min}}\, \Pi(\bar{\mathbf{u}}) = \frac{1}{2}\left\|\mathbf{L}\bar{\mathbf{u}} - (\Delta t)^2 \mathbf{L}_a \bar{\mathbf{a}}\right\|_2^2 + \frac{\lambda^2}{2}\left\|\bar{\mathbf{u}}\right\|_2^2 \tag{17}$$

where, $\bar{\mathbf{u}}$, Δt, $\bar{\mathbf{a}}$ and $\|\cdot\|_2$ represents reconstructed displacement, time interval, measured acceleration and 2-norm of a vector, respectively. $\mathbf{L}_a$ denote linear algebraic operator matrix and $\mathbf{L} = \mathbf{L}_a \mathbf{L}_c$. $\mathbf{L}_a$, $\mathbf{L}_c$ and $\bar{\mathbf{u}}$ are shown in reference by Hong. λ represents a regularization factor, which adjusts the degree of the regularization in the minimization problem. Well-balanced regularization factor should be selected to obtain physically meaningful and accurate displacements. Optimal regularization is determined by the following power function.

$$\lambda_{\mathrm{opt}} = 46.81 N^{-1.95} \tag{18}$$

where N=n+1 is the number of data points.

The minimization problem in Eq. (17) forms a quadratic problem with respect to the unknown displacement vector, and thus the solution of Eq. (17) is given analytically as

$$\mathbf{u} = (\mathbf{L}^T\mathbf{L} + \lambda_{\text{opt}}^2\mathbf{I})^{-1}\mathbf{L}^T\mathbf{L}_a\bar{\mathbf{a}}(\Delta t)^2 \tag{19}$$

where **I** is the identity matrix of order (n+3).

The reconstruction of displacement is performed sequentially in each time window. The reconstructed displacement at the middle of each time window is taken as the solution of the current time window. Once the solution of the current time window is determined, the time window advances forward to reconstruct the displacement at the next time step.

3.2 *System Identification Scheme*

Damping coefficient can be estimated by solving optimization.

$$\underset{\mathbf{X}_c}{\text{Min}}\,\Pi(\tau) = \frac{1}{2}\int_0^\tau \left\|\tilde{\mathbf{u}}(\mathbf{X}_c, t) - \bar{\mathbf{u}}\right\|_2^2 dt + \Pi_R \quad \text{subject to} \quad \mathbf{R}(\mathbf{X}_c) \le 0 \tag{20}$$

where $\tilde{\mathbf{u}}$, $\bar{\mathbf{u}}$, $\mathbf{X}_c$ and Π_R denote calculated displacement by a mathematical model, reconstructed displacement with measured acceleration, system parameter vector and regularization function, respectively. System parameter vector consists of a_0 and a_1 in Rayleigh damping model. In general, displacement can not be defined because no unique undeformed configuration corresponding to the equilibrium configuration. In this study, displacement is defined as the relative position between the current position and the position when the cable is loaded by its own self-weight. The initial conditions and applied load should be known to calculate a displacement by a mathematical model.

First term of the Eq. (20) represents the error function and second term represents the regularization function. The SI scheme defined by the minimization problem Eq. (20) is a type of ill-posed inverse problem. In case measured data are polluted by noise, ill-posed problems suffer from instabilities. To overcome these instabilities, the regularization technique is employed. In this paper, Tikhonov regularization scheme is used and the regularization function is defined as follows.

$$\Pi_R = \frac{\beta^2}{2}\left\|\mathbf{X}_c - (\mathbf{X}_c)_0\right\|_2^2 \tag{21}$$

where λ is regularization factor, which adjust the degree of regularization effect. $(\mathbf{X}_c)_0$ is a baseline values of the system parameters. Determination of the regularization factor is very important in obtaining stable and physically meaningful solution. Regularization factor is determined by using geometric mean scheme as follows.

$$\beta_{\text{opt}} = \sqrt{S_{\max} \cdot S_{\min}} \tag{22}$$

where, $S_{\max}$ and $S_{\min}$ is the maximum singular value and the smallest non-zero singular value of sensitivity matrix of the object function.

The optimization problem Eq. (20) is nonlinear with respect to parameters, the recursive quadratic programming is utilized.

3.3 *Damping modeling*

It is a difficult task to model real damping properties of real structures. In Rayleigh damping model, a damping matrix is defined as a linear combination of mass matrix and stiffness matrix as follows.

$$\mathbf{C} = a_0\mathbf{M} + a_1\mathbf{K}_s\,. \tag{23}$$

where, **M** and $\mathbf{K}_s$ represents mass matrix and tangential stiffness matrix subjected to its own self-weight, respectively. Tangent stiffness matrix of cable structure is not uniquely defined because the cable shows nonlinear behavior with respect to position. In the case of free vibration, stiffness repeats increase and decrease based on stiffness subjected to self-weight. However the stiffness and the damping matrix can be assumed approximately constant in the case of taut cable. That's reason why it is reasonable that usage of tangential stiffness matrix subjected to self-weight.

Once a_0 and a_1 are estimated by system identification scheme, modal damping ratios of each modes can be calculated as follows.

$$\zeta_n = \frac{a_0}{2}\frac{1}{\omega_n} + \frac{a_1}{2}\omega_n \tag{24}$$

where, ω_n represents natural frequency of nth mode.

4 EXAMPLE

The validity of the proposed method is demonstrated through a laboratory test. The stay-cable for experiment and the experimental setups, the geometry and the boundary conditions of cable are shown in Figure 3. The material properties of cable are given in Table 1. The tension of approximately 300KN is applied to the cable, and the first natural frequency is 1.48Hz. The exciter generates vertical forces by two rotating masses. The mass of exciter and the rotating mass are 14.58Kg and 0.46Kg, respectively. The cable is excited by its fundamental frequency to induce the resonance of the cable for 41.25sec, and the cable is vibrated freely until 150sec. Three accelerometers and one LVDT are installed as Figure 3. The acceleration is measured at the sampling rate of 100Hz.

The estimation of damping ratio is performed by two part. The accelerations of excited vibration part are used in first estimation, and those of free vibration part are used in second estimation. It is because that damping ratio during excitation can be different from the damping ratio during free vibration. Once the damping parameters of force vibration part are identified, the displacement and velocity at t=41.25sec are evaluated, and used as the initial conditions of free vibration part. The experiment is carried out three times. The estimated modal damping ratio is compared with the damping ratio estimated by logarithmic decrement method as follows.

$$\delta = (1/j)\ln(u_1/u_{j+1}) \cong 2\pi\zeta \Rightarrow \zeta \cong (1/2\pi j)\ln(u_1/u_{j+1}) \tag{25}$$

where ζ and u_j represents the modal damping ratio and displacement of jth peak.
Figure 4 represents measured acceleration at the center of the cable and Figure 5 represents the reconstructed displacement and calculated displacement by using estimated damping coefficients at the middle point of cable. To distinguish two data, the calculated displacement is shown as the envelop curve. The calculated displacement agrees well with the reconstructed displacement.

Table 2 shows modal damping ratio calculated by Eq. (24) using estimated Rayleigh damping coefficients. The damping ratios for forced vibration are about 72% of those of free vibration. This fact should be considered carefully for the design of cables against wind. The damping ratios for free vibration is similar with the damping ratios estimated by logarithmic decrement method.

Table 1. Cable Property.

Young's Modulus (KN/mm^2)	Mass per unit length (Kg/m)	Effective sectional area (mm^2)	Unstrained length (m)
200	20.3	2348	44.304

Table 2. Modal damping ratio from estimated Rayleigh damping coefficients.

	Exciting (%)	Free Vibration (%)	Logarithmic Decrement Method (%)
test #1	0.108	0.139	0.140
test #2	0.084	0.147	0.155
test #3	0.082	0.138	0.139

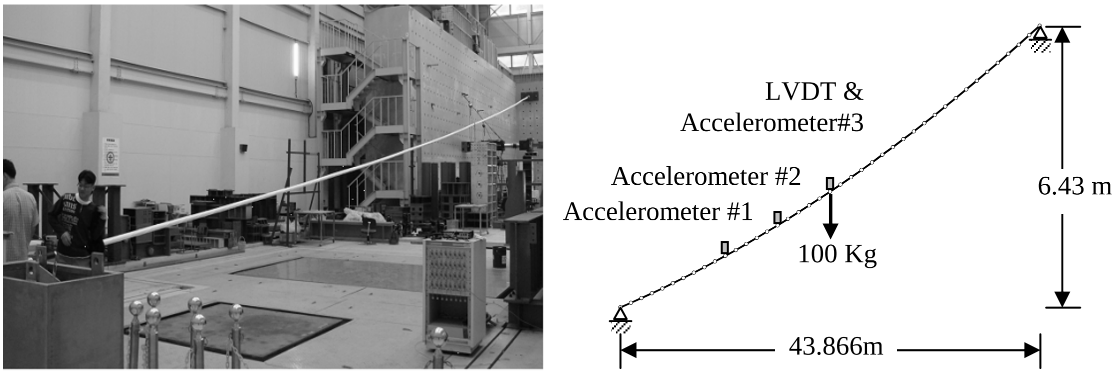

Figure 3. Cable experimental setup of a stay cable and its numerical model.

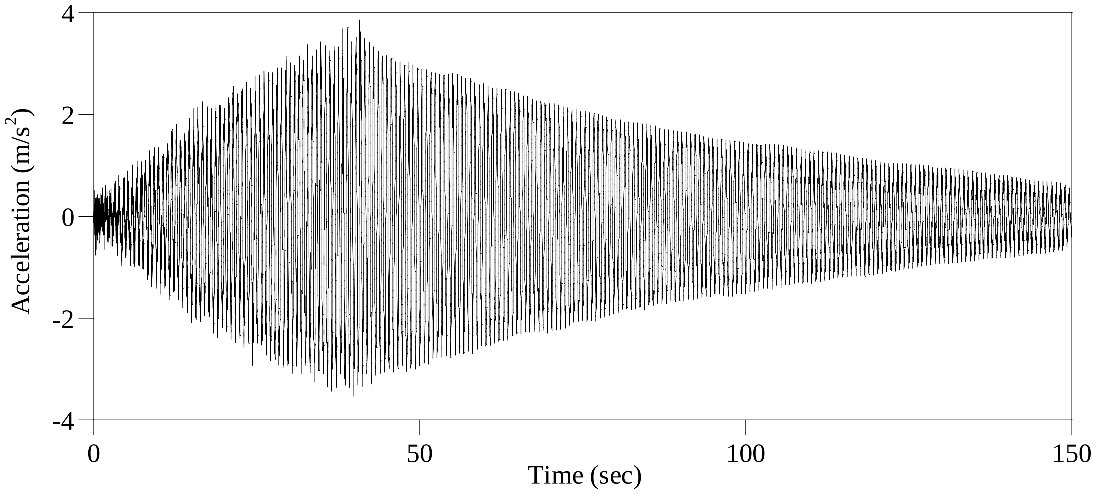

Figure 4. Measured acceleration at middle point of cable (test 2).

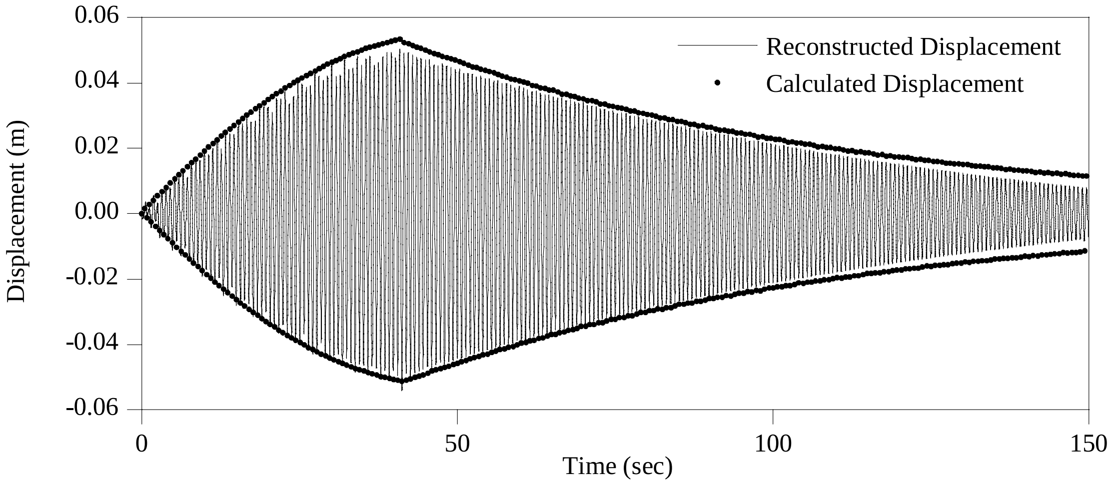

Figure 5. Reconstructed displacement and calculated displacement at middle point of cable (test 2).

5 CONCLUSION

This paper presents a system identification scheme to estimate the damping characteristics of cable using measured acceleration data. The damping properties are estimated through the minimization of the least squared errors between reconstructed displacement with measured acceleration and calculated displacement by a mathematical model. The displacement reconstruction scheme proposed by Hong is employed to reconstruct the displacement with the measured acceleration. Displacement is calculated by linearization of equation of motion and discretization by finite element method. Linearized incremental equation of motion is integrated by Newmark-β method. Displacement is defined as the relative position between the current position and the position when the cable is loaded by its own self-weight. Rayleigh damping is employed to approximate the damping properties of the cable. Regularization scheme is used to overcome the instabilities of inverse problems. The optimal regularization factor is determined by geometric mean scheme.

The validity of the propose method is demonstrated through laboratory experiment. The forced vibration test is performed. The estimated modal damping ratio is compared with the damping ratio by logarithmic decrement method. The damping ratio is estimated using displacement for forced vibration and free vibration, separately. The damping ratios for forced vibration are low compared to those of free vibration. This means that the damping ratio estimated by free vibration may be overestimated. This fact should be considered very carefully for the design of cables against wind.

REFERENCE

Hiroki Y & Loranjana J. 1992. Analytical estimation of structural damping in cable structures. *Journal of Wind Engineering and Industrial Aerodynamics* (41-44): 1961-1972

Yamaguchi H. & Adhikari R. 1995. Energy-based evaluation of modal damping in structural cables with and without damping treatment. *Journal of Sound and Vibration* 181 (1): 71-83

Kawashima K. & Unjoh S. Tunomoto M. 1993. Estimation of damping ratio of cable-stayed bridges for seismic design. *Journal of Structural Engineering* 119 (4): 1015-1031

Kim K. S. & Lee H. S. 2001. Analysis of target configurations under deal loads for cable-supported bridges. *Computers and Structures* 79 (29-30): 2681-2692

Park H. W. & Shin S. B. &Lee H. S. 2005. Determination of an optimal regularization factor in system identification with Tikhonov regularization for linear elastic continua. *International Journal for Numerical Methods in Engineering* 51 (10): 1211-1230

Kang J. S. & Park S.-K. & Shin S. B. & Lee H. S. 2005. Structural system identification in time domain using measured acceleration. Journal of Sound and Vibration 288(1-2): 215-234

Park Y. C. 2006. A stabilizing scheme for the dynamic analysis of slack cables. Master’s thesis. Seoul National University.

Hong Y. H. 2007. A regularization scheme for displacement reconstruction using acceleration data measured from structures. Master’s thesis. Seoul National University.

Irvine H. M. 1981. *Cable structures*. Cambridge: MIT press

Cook R. D. & Malkus D. S. & Plesha M. E. & Witt R. J. *Concept and applications of finite element analysis, 4^{th} ed*. New York, NY: John Wiley and Sons

Chopra A. K. Dynamics of structures, 2^{nd} ed. Upper Saddle River, NJ: Practice Hall