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Identification of aeroelastic parameters for cable-supported bridges using measured accelerations

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ABSTRACT: After the flutter derivative-based aeroelastic formula had been proposed, great number of efforts have been made to estimate the flutter derivatives from the test of bridge model in wind tunnel. However, these methods are based on the response error estimation, which minimizes the relative error between measured displacement and predicted displacement. This paper propose a new approach to identify the flutter derivatives based on the equation error estimation which minimizes the relative error between the resisting forces (stiffness, damping and inertia) and the aeroelastic self-excitation forces. The displacement and velocity reconstruction scheme is used to calculate the displacement and velocity history from measured acceleration. The validity of the proposed method is demonstrated through the experimental free vibration test.

1 INTRODUCTION

1.1 *Type area*

After the flutter derivative-based aeroelastic formula had been by Scanlan, great number of efforts have been made to estimate the flutter derivatives from the test of bridge model in wind tunnel. Scanlan and Tomko proposed the extraction scheme for flutter derivatives from 2DOFs coupled motion tests (1971). Sarkar developed the Modified Ibrahim Time Domain (MITD) to estimate cross flutter derivatives along with direct flutter derivatives (1994). The procedures of these approaches are generally based on the response error estimation, which minimizes the relative error between measured displacement and predicted displacement.

However, the flutter derivatives are conceptually more closely related with the aerodynamic force equilibrium than with the predicted displacement itself in formulas. Therefore, this paper proposes a new approach to identify flutter derivatives based on the equation error estimation (EEE) which minimizes the relative error between the resisting forces (stiffness, damping and inertia) and the aeroelastic self-excitation forces.

The EEE approach requires not only displacement response but also velocity and acceleration history for system identification. In this approach, a displacement and velocity reconstruction scheme is used to calculate displacement and velocity history from measured acceleration. Hence, both these reconstructed responses and the measured acceleration is used for EEE method.

2 ESTIMATION OF FLUTTER DERIVATIVES

2.1 *Equations of motion for the free vibration test*

In this paper, a section model for the wind induced vibration is assumed to have two degrees of freedom in vertical (h) and rotational (α) direction. The equations of motion per unit length for the section model can be expressed by following equations.

$$m_h\ddot{h} + c_h\dot{h} + k_h h = L_h$$
$$m_\alpha\ddot{\alpha} + c_\alpha\dot{\alpha} + k_\alpha\alpha = M_\alpha \tag{1}$$

where m_h and m_α are mass and the mass moment of inertia per unit length, respectively; c_h and c_α are mechanical damping in bending and torsion, respectively; k_h and k_α are the mechanical stiffness coefficients in vertical and rotational direction; L_h and M_α are aeroelastic lift force and moment, respectively. Here, the material properties for each mode are indicated with the corresponding subscripts.

Omitting the coupling term between vertical and torsional oscillations, the aeroelastic lift and moment can be written as follows.

$$L_h = C_{Lh}\dot{h} + C_{L\alpha}\dot{\alpha} + K_{L\alpha}\alpha + K_{Lh}h$$
$$M_\alpha = C_{Mh}\dot{h} + C_{M\alpha}\dot{\alpha} + K_{M\alpha}\alpha + K_{Mh}h \tag{2}$$

where K_* and C_* are eight unknown aerodynamic parameters which will be identified from the free vibration test. The relation between these parameters and flutter derivatives is expressed as follows.

$$\mathbf{f}_h = \begin{bmatrix} C_{Lh} \\ C_{L\alpha} \\ K_{L\alpha} \\ K_{Lh} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \rho B^2\omega_1 H_1^* \\ \rho B^3\omega_2 H_2^* \\ \rho B^3(\omega_2)^2 H_3^* \\ \rho B^2(\omega_1)^2 H_4^* \end{bmatrix},$$
$$\mathbf{f}_\alpha = \begin{bmatrix} C_{Mh} \\ C_{M\alpha} \\ K_{M\alpha} \\ K_{Mh} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \rho B^3\omega_1 A_1^* \\ \rho B^4(\omega_2) A_2^* \\ \rho B^4(\omega_2)^2 A_3^* \\ \rho B^3(\omega_1)^2 A_4^* \end{bmatrix} \tag{3}$$

where $\mathbf{f}_h$ and $\mathbf{f}_\alpha$ are unknown coefficient vectors in vertical and horizontal direction, ρ is the air density, B is the deck width, ω_1 and ω_2 are the circular frequency for first and second modes, H_n^* and A_n^* (for $n = 1, 2, 3, 4$) are the flutter derivatives.

2.2 *Equation error estimation (EEE)*

To identify the flutter derivatives defined in Eq. (3) from the free vibration test of the deck system, a great number of approaches have been proposed. The general procedures of these approaches are based on the response error estimation which minimizes the relative error between measured displacement and predicted displacement. Among them, the prediction error minimization method (PEM) is one of the most simple and popular methods which is implemented in Matlab.

Since, however, the flutter derivatives are more closely related not to the displacement response but the force equilibrium in Eq. (1), this paper proposes the new identification scheme of the flutter derivatives based on the equation error estimation which minimize the relative error between the left and right sides of the Eq (1).

In case the acceleration, velocity and displacement history is given, the left sides of dynamic equation in Eq. (1) can be calculated from the known responses and material properties. Therefore, the first equation in Eq. (1) can be divided into the known and unknown force terms. Note that the acceleration is measured form the measuring equipment but the velocity and displacement are reconstructed by the FFIR filter which will be presented in the next section.

$$\bar{p}(t) = m_h\ddot{h} + c_h\dot{h} + k_h h$$
$$p(t) = L_h = C_{Lh}\dot{h} + C_{L\alpha}\dot{\alpha} + K_{L\alpha}\alpha + K_{Lh}h \tag{4}$$

where $\bar{p}$ and p are the known force history and unknown force history which will be referred as an measured and calculated force hereafter, respectively.

As mentioned above, this study utilizes the following discrete minimization problem to identify the aerodynamic unknown vector, ($\mathbf{f}_h$).

$$\underset{\mathbf{f}_h}{\mathrm{Min}}\,\Pi(\mathbf{f}_h) = \frac{1}{2}\sum_{k=1}^{N}(\bar{p}_k - p_k)^2 \quad \text{where} \quad p_k = p(k\Delta t) \tag{5}$$

where N, p_k and $\bar{p}_k$ are the number of data points used in the estimation, the calculated force and the measured force at the k-th time step, respectively. The minimization problem in Eq. (5) forms a quadratic problem with respect to the unknown coefficient vectors, and thus the solution of Eq. (5) is given analytically from the first order necessary condition of Eq. (5).

3 RECONSTRUCTION OF DISPLACEMENT AND VELOCITY HISTORY FROM MEASURED ACCELERATION

Lee et al. have proposed a new class of FEM-based finite impulse response (FFIR) filter to reconstruct displacement and velocity simultaneously. In their approach, the displacement is reconstructed by solving the following minimization problem defined in a time interval, $T_1 < t < T_2$.

$$\underset{u}{\mathrm{Min}}\,\Pi(u) = \frac{1}{2}\int_{T_1}^{T_2}\left(\frac{d^2u}{dt^2} - \bar{a}\right)^2 dt + \frac{\beta^2}{2}\int_{T_1}^{T_2} u^2 dt \tag{6}$$

where u and $\bar{a}$ are the displacement and the measured acceleration, respectively.

The direct discretization of the variation statement of Eq. (6) with the finite element method using $2k$-th elements leads to a FFIR filter.

$$\begin{aligned}\delta\Pi(u) &= \sum_{e=1}^{2k}\int_{\Delta t}\frac{d^2\delta u^e}{dt^2}\left(\frac{d^2u^e}{dt^2} - \bar{a}^e\right)dt \\ &+ \beta^2\sum_{e=1}^{2k}\int_{\Delta t}\delta u^e u^e dt = 0\end{aligned} \tag{7}$$

Here, u^e and $\bar{a}^e$ denote the displacement and acceleration in element, e, respectively.

The standard FEM formulation for a beam on an elastic foundation is adopted to derive the following matrix expression of Eq. (7).

$$(\mathbf{K} + \beta^2(\Delta t)^4\mathbf{M})\mathbf{u} = (\Delta t)^2\mathbf{Q}\bar{\mathbf{a}} \tag{8}$$

where $\mathbf{u}$ and $\bar{\mathbf{a}}$ denote the nodal unknown vector and the measured acceleration vector associated with all sampling points of measurement. The nodal unknown vector consists of the nodal displacements and the nodal velocities. The matrixes in Eq. (8) are defined as

$$\begin{aligned}\mathbf{K} &= \sum_e\int_0^1\frac{d^2\mathbf{N}_H^T}{d\xi^2}\frac{d^2\mathbf{N}_H}{d\xi^2}d\xi\ , \\ \mathbf{M} &= \sum_e\int_0^1\mathbf{N}_H^T\mathbf{N}_H dt\ , \\ \mathbf{Q} &= \sum_e\int_0^1\frac{d^2\mathbf{N}_H^T}{d\xi^2}\mathbf{N}_L d\xi\end{aligned} \tag{9}$$

where $\sum_e$ is the assembly operator of the FEM, and ξ is the natural coordinate for the time variable ranging from 0 to 1, $\mathbf{N}_H$ and $\mathbf{N}_L$ are Hermitian shape function and the linear shape function, respectively. The nodal unknown vector is obtained by solving Eq. (8).

$$\mathbf{u} = (\Delta t)^2(\mathbf{K} + \beta^2(\Delta t)^4\mathbf{M})^{-1}\mathbf{Q}\bar{\mathbf{a}} = (\Delta t)^2\mathbf{C}\bar{\mathbf{a}} \tag{10}$$

where $\mathbf{C}$ is the coefficient matrix of order $2(2k+1)\times(2k+1)$.

Assuming the time step at the center of a time window represents time *t*, the reconstructed displacement and velocity are expressed as following equations

$$\begin{aligned}u(t) = u_{k+1} &= (\Delta t)^2\sum_{p=1}^{2k+1}C_{2k+1,p}\bar{a}_p \\ &= (\Delta t)^2\sum_{p=-k}^{k}c_{p+k+1}\bar{a}(t+p\Delta t)\end{aligned} \tag{11}$$

$$\begin{aligned}v(t) = v_{k+1} &= \sum_{p=1}^{2k+1}C_{2k+2,p}\bar{a}_p \\ &= \Delta t\sum_{p=-k}^{k}\hat{c}_{p+k+1}\bar{a}(t+p\Delta t)\end{aligned} \tag{12}$$

4 EXPERIMENTAL VERIFICATION

The flutter derivatives of a section model representing a bridge deck system are evaluated using the measured acceleration and reconstructed displacement and velocity using proposed method, and compared with the result of the prediction error minimization method (PEM) with measured displacement.

The experiment was performed by King at el. at the Boundary Layer Wind Tunnel Laboratory of the University of Western Ontario in Ontario, Canada. Experimental setups are shown in Fig. 1. To estimate flutter derivatives from free oscillation test, the section model is mounted on the four soft springs, which permit the simulation of the vertical and torsional frequencies of vibration.

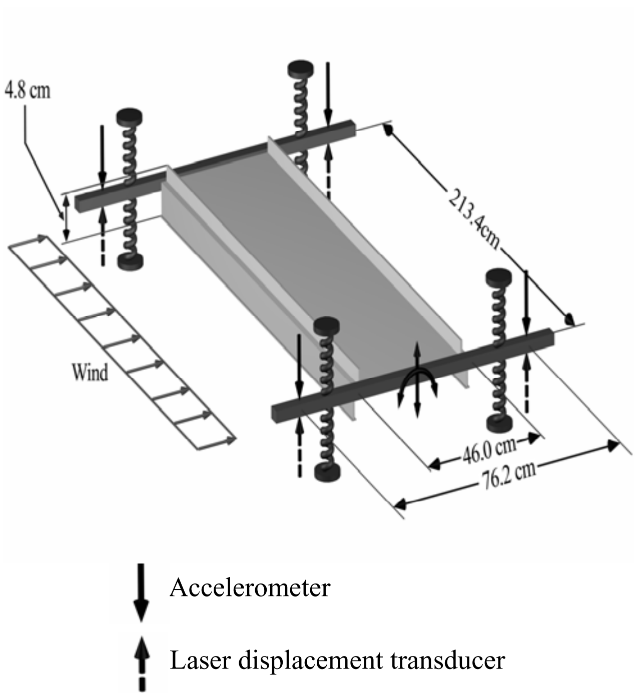

Figure 1. Experimental setup of the plate girder section for wind tunnel test.

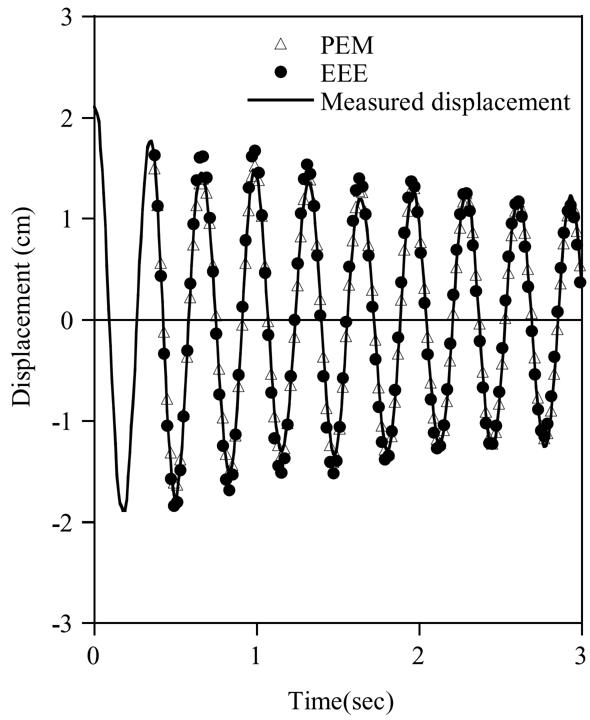

Figure 2. Calculated displacement history in vertical direction

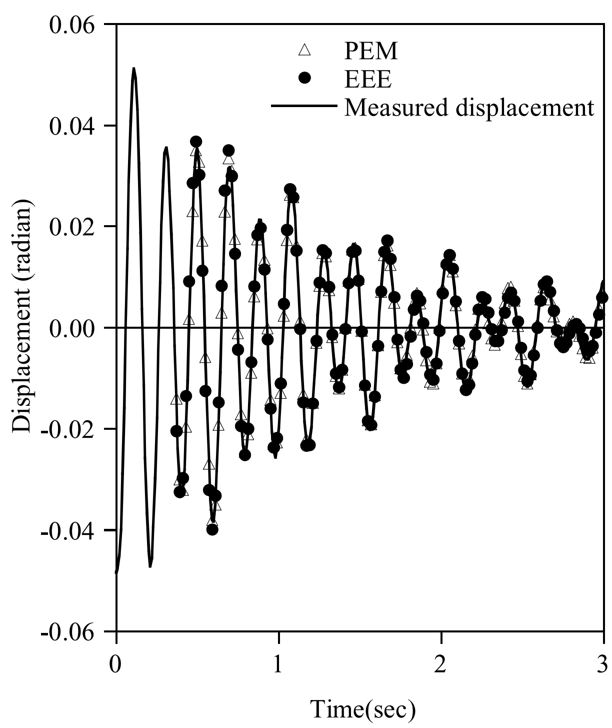

Figure 3. Calculated displacement history in torsionall direction

During the test, the model deck suddenly release for sixteen different wind speeds, and twenty trials are carried out for each wind speed. The response is measured in discrete manner with the sampling frequency of 100 Hz.

Using the identified flutter derivatives form EEE and PEM, the predicted displacement response are calculated and compared with the measured displacement. Comparisons of the results in vertical and torsional direction in Fig. 2 and 3 shows that the fit obtained by using the system parameters identified from PEM with measured displacement is slightly superior to that from EEE method.

However, it is common characteristic of the PEM because the PEM aimed to optimize the measured displacement response but the EEE focuses on the forcing equation. Moreover, the PEM sometimes fails to converge or yield physically meaningful results due to the ill-posedness of the inverse problem but the EEE always yield reasonable result.

All eight flutter derivatives are defined in Eq. (3) are identified. Among them, the direct components, H_1^* and A_2^* are presented in Fig. 4 and 5. The centered symbols and the lines represent the extracted derivatives and the cubic spline fits of them, respectively. In the figures, the flutter derivatives are plotted against the non-dimensional reduced velocity defined as follows. Though there is slight discrepancy, overall shape of both the identified flutter derivatives agrees well with each other.

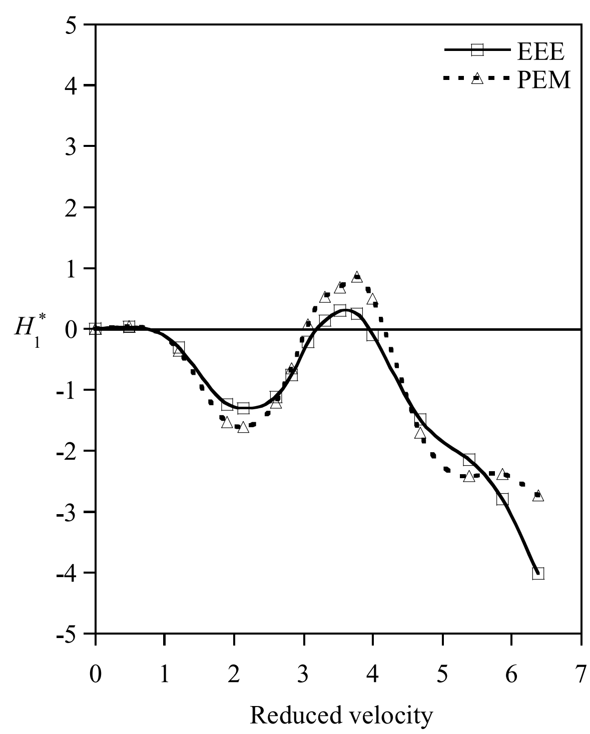

Figure 4. Identified direct flutter derivative in vertical direction with EEE and PEM, (H_1^*).

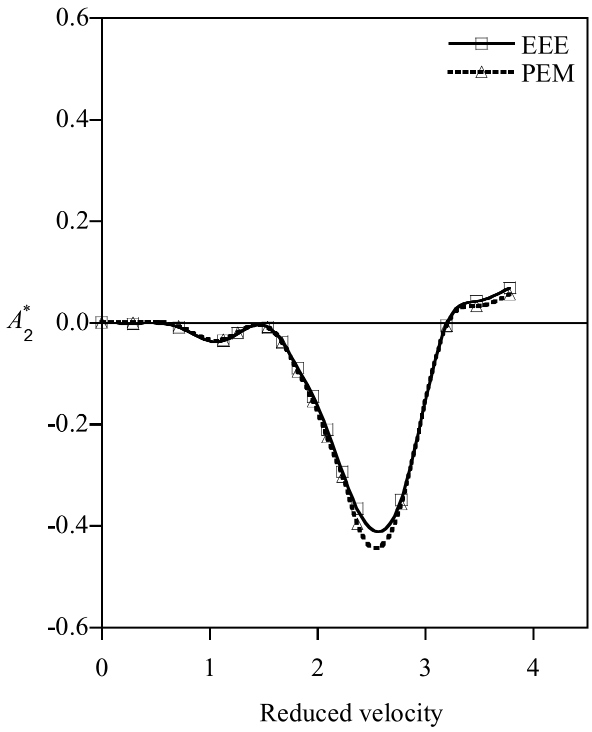

Figure 5. Identified direct flutter derivative in tosional direction with EEE and PEM, (A_2^*).

5 SUMMARY AND CONCLUSION

This paper proposed a new identification method for flutter derivatives base on the EEE, and compared with conventional PEM.

Though EEE results in less compatible results between predicted and measured displacements than PEM, the authors believe that the flutter derivatives identified by EEE contain more physically reasonable meanings based on the governing equilibrium equation. Moreover, the EEE is free from the convergence of ill-posed problem, and give very stable identification results.

6 ACKNOWLEDGEMENT

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