School of Civil & Environmental Engineering
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Mid-term Exam II, Theory of Elasticity

2015. 5. 13.

(Closed everything, and all answers should be given in English.)

Prob. 1 The compatibility equations for general 3D bodies consist of 3 *U*-type and 3 *R*-type equations. Identify the relations between two types of equations. Show that if one type of the equations is satisfied in a domain and the other type of the equations is satisfied in the boundary of the given domain, all compatibility equations are satisfied in both domain and boundary simultaneously. (20 pts.)

$$R_1 = \frac{\partial^2 \varepsilon_{22}}{\partial x_3^2} + \frac{\partial^2 \varepsilon_{33}}{\partial x_2^2} - 2\frac{\partial^2 \varepsilon_{23}}{\partial x_2 \partial x_3} = 0 \qquad U_1 = -\frac{\partial^2 \varepsilon_{11}}{\partial x_2 \partial x_3} + \frac{\partial}{\partial x_1}\left(-\frac{\partial \varepsilon_{23}}{\partial x_1} + \frac{\partial \varepsilon_{13}}{\partial x_2} + \frac{\partial \varepsilon_{12}}{\partial x_3}\right) = 0$$

$$R_2 = \frac{\partial^2 \varepsilon_{33}}{\partial x_1^2} + \frac{\partial^2 \varepsilon_{11}}{\partial x_3^2} - 2\frac{\partial^2 \varepsilon_{13}}{\partial x_1 \partial x_3} = 0 \qquad U_2 = -\frac{\partial^2 \varepsilon_{22}}{\partial x_1 \partial x_3} + \frac{\partial}{\partial x_2}\left(\frac{\partial \varepsilon_{23}}{\partial x_1} - \frac{\partial \varepsilon_{13}}{\partial x_2} + \frac{\partial \varepsilon_{12}}{\partial x_3}\right) = 0$$

$$R_3 = \frac{\partial^2 \varepsilon_{11}}{\partial x_2^2} + \frac{\partial^2 \varepsilon_{22}}{\partial x_1^2} - 2\frac{\partial^2 \varepsilon_{12}}{\partial x_1 \partial x_2} = 0 \qquad U_3 = -\frac{\partial^2 \varepsilon_{33}}{\partial x_1 \partial x_2} + \frac{\partial}{\partial x_3}\left(\frac{\partial \varepsilon_{23}}{\partial x_1} + \frac{\partial \varepsilon_{13}}{\partial x_2} - \frac{\partial \varepsilon_{12}}{\partial x_3}\right) = 0$$

Prob. 2 If the following mathematical strain components are a strain field of 2D body, what kind of relation among the coefficient should hold? Derive the displacement field corresponding to the strain field based on the relation that you derived. Neglect the body force. (10 pts.)

$$\varepsilon_{xx} = a_1 x^2 + a_2 y^2, \quad \varepsilon_{yy} = b_1 x^2 + b_2 y^2, \quad \varepsilon_{xy} = c_1 xy$$

Prob. 3 There exist 81 independent material properties in a general elastic medium. The number of the independent material properties can be reduced to only three considering symmetry and isotropy. Identify the relations between stress and strain components that the three material properties represent. It is not necessary to derive the relations from the beginning if you know the relations. If not, you have to identify the three material properties first. Derive the relation between the three material properties, and express the equilibrium equations for general 3D elastic bodies in terms of displacements using the material properties you identified. (20 pts.)

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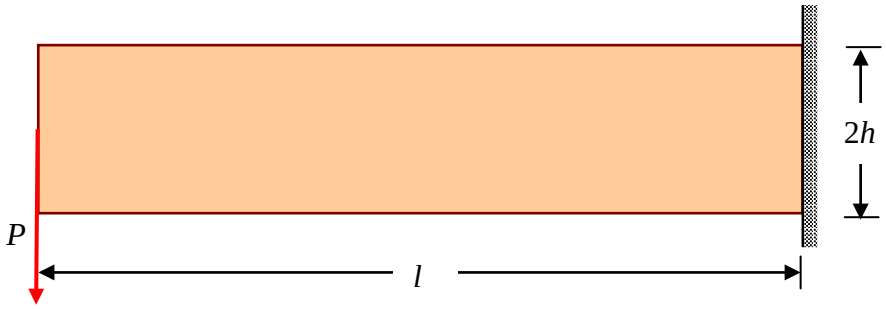
Prob. 4 Consider the cantilever beams shown in the following figure.

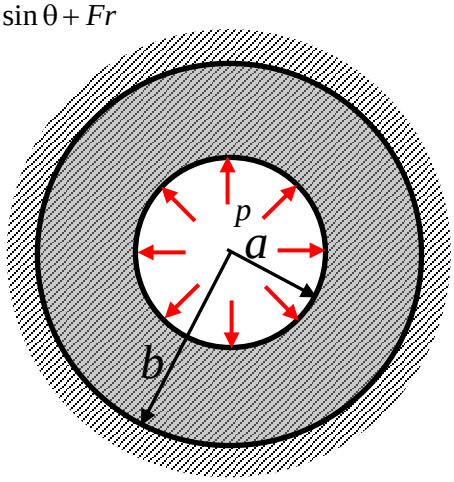


a) Identify boundary conditions. You may replace the concentrate load by a statically equivalent traction. (5 pts.)
b) Assume proper stress fields and calculate strain fields using elementary solutions. (5 pts)
c) Obtain the displacement field corresponding to the assumed stress field. (10 pts)
d) Check your displacement boundary condition. If the displacement boundary condition is not satisfied exactly, discuss why. Under what conditions is the displacement boundary condition satisfied exactly? (10pts.)

Prob. 5. Derive stress components of the pipe shown below. The displacements on the outer boundary of the pipe are fixed ($u_r = u_\theta = 0$). Assume the Possion's ratio is zero. Use the following displacement field of an axis-symmetry problem under the plane stress condition. (20 pts.)

$$u_r = \frac{1}{E}[-\frac{(1+\nu)A}{r} + 2(1-\nu)Br\ln r - (1+\nu)Br + 2(1-\nu)Cr] + H\sin\theta + K\cos\theta$$

$$u_\theta = \frac{4Br\theta}{E} + H\cos\theta - K\sin\theta + Fr$$



Answers given in languages other than English are not considered at all for scoring!

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