Mid-term Exam 1, Theory of Elasticity

2016. 4. 12.

(Closed everything, and answers may be given in Korean or English.)

Prob.1 Define the surface traction and stress. Derive the Cauchy's relation. (10 pts.)

Prob.2 Explain why the potential theory is frequently adopted in various engineering problems on conservation or equilibrium. (10 pts.)

Prob.3 Derive the equilibrium equation for a three-dimensional solid in terms of stress from the physical point of view in case no body moment exists. Is it possible to determine the stress field of the given elastic body solely by the equilibrium equation? If not, propose a general approach to solve the equilibrium equation based on the potential theory. The symmetry of stress should be considered. (20pts)

Prob. 4 Show that the equilibrium equation derived in Prob. 3 is independent of the selection of the Cartesian coordinate system. (10 pts)

Prob. 5 Express the Green strain and the Cauchy's infinitesimal strain in terms of the displacement field. (10 pts.)

Prob.6 Explain the compatibility equation for the small deformation problems of 3-dimensional solids including the following points. (20 pts)

a) Why are the compatibility equations required?
b) The number of the required equations
c) Physical meaning
d) The reasons why the St. Venant's compatibility equations and the Bianchi's relations are not enough to describe the compatibility conditions completely.
e) The final requirement that is proven by Washizu. (You have to prove the final requirement!)

The St. Venant's compatibility equations and Bianchi's relations are given as follows.

$$R_1 = \frac{\partial^2 \varepsilon_{22}}{\partial x_3^2} + \frac{\partial^2 \varepsilon_{33}}{\partial x_2^2} - 2\frac{\partial^2 \varepsilon_{23}}{\partial x_2 \partial x_3} = 0$$

$$R_2 = \frac{\partial^2 \varepsilon_{33}}{\partial x_1^2} + \frac{\partial^2 \varepsilon_{11}}{\partial x_3^2} - 2\frac{\partial^2 \varepsilon_{13}}{\partial x_1 \partial x_3} = 0$$

$$R_3 = \frac{\partial^2 \varepsilon_{11}}{\partial x_2^2} + \frac{\partial^2 \varepsilon_{22}}{\partial x_1^2} - 2\frac{\partial^2 \varepsilon_{12}}{\partial x_1 \partial x_2} = 0$$

$$U_1 = -\frac{\partial^2 \varepsilon_{11}}{\partial x_2 \partial x_3} + \frac{\partial}{\partial x_1}\left(-\frac{\partial \varepsilon_{23}}{\partial x_1} + \frac{\partial \varepsilon_{13}}{\partial x_2} + \frac{\partial \varepsilon_{12}}{\partial x_3}\right) = 0$$

$$U_2 = -\frac{\partial^2 \varepsilon_{22}}{\partial x_1 \partial x_3} + \frac{\partial}{\partial x_2}\left(\frac{\partial \varepsilon_{23}}{\partial x_1} - \frac{\partial \varepsilon_{13}}{\partial x_2} + \frac{\partial \varepsilon_{12}}{\partial x_3}\right) = 0$$

$$U_3 = -\frac{\partial^2 \varepsilon_{33}}{\partial x_1 \partial x_2} + \frac{\partial}{\partial x_1}\left(\frac{\partial \varepsilon_{23}}{\partial x_1} + \frac{\partial \varepsilon_{13}}{\partial x_2} - \frac{\partial \varepsilon_{12}}{\partial x_3}\right) = 0$$

,

$$\frac{\partial R_1}{\partial x_1} + \frac{\partial U_3}{\partial x_2} + \frac{\partial U_2}{\partial x_3} = 0$$

$$\frac{\partial U_3}{\partial x_1} + \frac{\partial R_2}{\partial x_2} + \frac{\partial U_1}{\partial x_3} = 0$$

$$\frac{\partial U_2}{\partial x_1} + \frac{\partial U_1}{\partial x_2} + \frac{\partial R_3}{\partial x_3} = 0$$

Prob. 7 The properties of an isotropic material can be defined by three constants describing normal stress-normal strain, normal stress-other normal strain and shear stress-shear strain relation. Express the elasticity tensor in terms of the three constants using the Kronecker delta, and derive the relation between the three constants. (20 pts)