Final Exam - Theory of Elasticity

2017. 6. 14.

Prob. 1 Derive Cauchy's relation. (20 pts)

Prob. 2 Answer the following questions on the compatibility equations. Do not write down detailed equations. Just discuss. (30 pts./10 pts. each)

a) When and why are the compatibility equations required for a general 3D-elastic body.
b) Explain the required number of the compatibility equations.
c) Discuss the number equations derived by St. Venant, Bianchi and Washizu.

Prob. 3 Answer the following questions. (40 pts.)

a) Derive the equilibrium equations of the force resultants for a planar prismatic beam without torsion. (10 pts.)
b) List the assumptions on the displacement of the Timoshenko beam. (5pts.)
c) Derive the strain and stress components based on the assumptions. (5pts.)
d) Define the bending moment and shear force. (5 pts.)
e) Express the equilibrium equations in terms of the displacement field. (10 pts.)
f) What kind of difficulty can you expect to solve the final equation ? (5pts)

Prob. 4 Derive the torsional constant, *J*, for a rectangular section under the uniform torsion using the series solution. (40 pts.)

Prob. 5 The general solution of a rotating disk at an angular velocity of ω under plane stress condition is given as follows. The radius of the disk is *a*. (20pts.)

$$u_r = \frac{1}{E}[(1-\nu)Cr - (1+\nu)C_1\frac{1}{r} - \frac{1-\nu^2}{8}\rho\omega^2 r^3], \quad u_\theta = 0$$

Determine the integration constant for a solid disk with the radius of *a*, and calculate the stress components.

Prob. 6 A solid disk of a radius *a* rotates at an angular velocity of ω in a thick pipe. The outer and inner radii of the pipe are *b* and *a*, respectively. Calculate the displacement on the interface between the pipe and the disk. Assume the plane stress condition and no friction on the interface. The material properties of the pipe and the disk are the same. (30 pts.)

Prob. 7 Derive the dispaceement of a cantilever beam under the plane stress condition subject to a unoformly distributed vertical load. You may use idealized and/or statically equilvalent bounadry conditions for both displacement and traction. If so, you have to justify your boundary comditions. Explain physical meanings of each term in the vertical displacement. The Young's modulus and Poisson's ratio are *E* and ν, respectively. (20 pts.)