School of Civil and Environmental Engineering
Seoul National University
Kwanak-Gu, Seoul 151-742, Korea

Introduction of Finite Element Method - Final Exam

2015. 12. 14.

(Closed book and note)

Prob.1 Present the integral expression defining the curve length of a function, $f(x)$, for $a \le x \le b$, and obtain $f(x)$ that minimizes the curve length using the Euler equation. The boundary conditions are given as $f(a) = y_1$ and $f(b) = y_2$. (20 pts.)

Prob. 2 Answer the following questions on a beam with shear deformation. (30 pts.)

a) Define the displacement field for the beam. (5 pts.)

b) Derive the total potential energy for the beam. (5 pts.)

c) Derive the governing equations and boundary conditions for the beam. (10 pts.)

d) Obtain an element stiffness matrix for the beam using linear interpolation functions (10pts.)

Prob. 3 Derive the virtual work expression for general 3-D elasticity problems, and prove the uniqueness of the solution obtained by the virtual work expression. (20 pts.)

Prob. 4 Describe a procedure to obtain continuous stress distribution in a given domain for finite element analysis and Loubignac iteration. (20 pts.)

Prob. 5 Derive the Gauss points and the corresponding weighting values for 3 points Gauss quadrature. (20 pts.)

Prob. 6 Drive the shape functions of the third order polynomial for a triangular element in the area coordinate system. Discuss whether the reduced integration is required for the triangular element to solve the 2-D elasticity problems or not. (20 pts)

Prob. 7 Solve the following first-order differential equation using the finite element method based on the Galerkin method. Discretize the given domain with three elements, and utilize a constant shape function in each element. It is not necessary to solve the finally-derived algebraic equations. (30 pts.)

$$\frac{dw}{dx} = f \qquad 0 < x < l \qquad w(0) = 1$$

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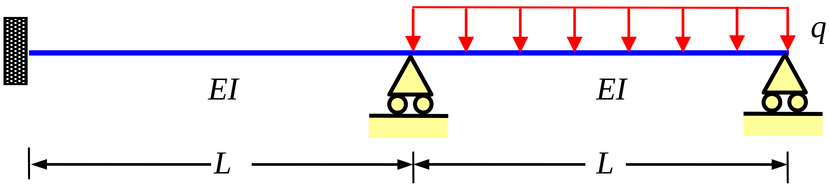
Prob. 8 The displacement of the Bernoulli beam shown below is assumed with the following trial function. (40 pts)

$$w = a\sin\frac{\pi}{L}x + b\sin\frac{2\pi}{L}x$$



a) Define the boundary condition for the rotational angle at the fixed support as an equality constraint. Apply the derived constraint to the principle of minimum potential energy in a strong sense, and obtain the deflection of the beam. (15 pts)

b) Apply your constraint derived in (a) to the principle of minimum potential energy by the Lagrangian multiplier method. It is not required to solve the final algebraic equation Discuss the physical meaning of the Lagrangian multiplier.(10 pts)

c) Apply your constraint derived in (a) to the principle of minimum potential energy by the penalty method. Identify an appropriate penalty number, and obtain the deflection of the beam with the identified penalty number. Compare your solution with that obtained in a). (15 pts)